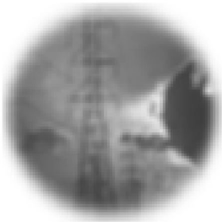
# Basic Electrical and Electronics Engineering-I

## (ES101)

**Fifth Edition**

**WBUT-2015**



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# PREFACE

Basic Electrical Engineering and Basic Electronics Engineering are the two fundamental subjects of most engineering disciplines. It is extremely important to ensure that the fundamentals of these two courses are well understood by all engineering students since these subjects have applications in all streams. Though numerous textbooks on these two subjects are already available, we felt that there was still a need for another book which would present the basics of electrical and electronics engineering in a comprehensive manner. It is also true that there is hardly any comprehensive textbook available on this subject that precisely covers the prescribed syllabus of WBUT. Moreover, following the new syllabus framed by West Bengal University of Technology (WBUT), there was a need to bring out a textbook that would cover the entire syllabus of Basic Electrical and Electronics Engineering. An attempt has therefore been made to present an exhaustive material in the form of a textbook to the students studying basic electrical and electronics engineering in WBUT. We hope that after going through this book, the undergraduate engineering students of WBUT will find that their learning and understanding of the subject 'Basic Electrical and Electronics Engineering' has increased progressively.

Our main aim was to equip the students with the fundamental knowledge of Electrical and Electronics Engineering, the theory backed up by illustrative solved problems. We have emphasized on building of fundamental concepts, mathematical derivations coupled with applications and solution of problems. We have attempted to keep the language in the textbook as lucid as possible. Relevant examples have been selected to supplement the theory so that students have a thorough idea of the applicability of the theoretical concepts.

The salient features of this book are as follows:

## Salient Features

- Full coverage of the latest WBUT syllabus (2010 Regulation)
- Fundamentals of Electrical as well as Electronics Engineering covered in a single book
- Stepwise explanation of theories and derivations along with relevant examples
- Diagrammatic and elaborate representation of circuits and phases
- Thoroughly updated chapter-end exercises
- Solutions to previous WBUT examination questions at the end of each chapter
- Updated with solutions of 2013 and 2014 WBUT examination questions

***Pedagogy***

- Illustrations: 460
- Solved Examples: 365
- Exercises with Answers: 160
- Multiple Choice Questions: 410
- WBUT Examination Questions with Answers: 268

The book comprises 8 chapters designed as per WBUT syllabus. The first three chapters deal with the fundamentals of basic electrical engineering, and the next five chapters deal with the fundamentals of basic electronics engineering. Care has been taken regarding the chronology of presentations, mathematical treatments and in representing the theory in a lucid manner. Each chapter has sufficient number of theoretical questions and unsolved problems with hints and answers. For the benefit of students, we have also included answers of questions and detailed solutions of problems from question papers of WBUT.

A number of experts took pains to provide valuable feedback on various chapter of  the book. Our heartfelt gratitude goes out to those whose names are given below:

**Karim** *Academy of Technology, Hooghly*

**S K Nath** *Calcutta Institute of Technology, Howrah*

We acknowledge the assistance and encouragement of our colleagues and staff and express sincere gratitude to the entire team at McGraw Hill Education in bringing out this book in a very short time. We will be extremely thankful to the readers for their constructive suggestions and views to enhance the utility of this book.

**ABHIJIT CHAKRABARTI**
**SUDIPTA DEBNATH**

## Publisher's Note

McGraw Hill Education India would like to acknowledge the contributions of Prof. Prasanta Mukhopadhyay towards this edition, which we will hopefully be able to incorporate fully in later editions of this extremely popular book.

**Remember to write to us.** We look forward to receiving your feedback, comments and ideas to enhance the quality of this book. You can reach us at *info.india@mheducation.com*. Kindly mention the title and the author's name as the subject. In case you spot piracy of this book, please do let us know.

# ROADMAP TO THE SYLLABUS

# BASIC ELECTRICAL AND ELECTRONICS ENGINEERING-I (ES101)

## PART I: BASIC ELECTRICAL ENGINEERING-I

**DC Network Theorems:** Definition of electric circuit, network, linear circuit, non-linear circuit, bilateral circuit, unilateral circuit; Dependent source; Kirchhoff's law; Principle of superposition

Source equivalence and conversion; Thevenin's theorem; Norton's theorem; Nodal analysis; Mesh analysis, Star-delta conversion; Maximum power transfer theorem with proof

Go to *CHAPTER 1:* **DC NETWORK ANALYSIS**

**Electromagnetism:** Biot-Savart law; Ampere's circuital law; Field calculation using Biot-Savart and Ampere's circuital law

Magnetic circuits; Analogous quantities in magnetic and electric circuits; Faraday's law; Self- and mutual inductance

Energy stored in a magnetic field; B-H curve; Hysteretic and Eddy current losses; Lifting power of electromagnets

Go to *CHAPTER 2:* **ELECTROMAGNETISM AND MAGNETIC CIRCUITS**

**AC Fundamentals:** Production of alternating voltage; Waveforms; Average and RMS values; Peak factor; Form factor; Phase and phase difference; Phasor representation of alternating quantities; Phasor diagram; Behaviour of ac series; Parallel and series parallel circuits; Power factor; Power in ac circuit; Effect of frequency variation in RLC series and parallel circuits; Resonance in *RLC* series and parallel circuit; Q-factor; Bandwidth of resonant circuit

Go to **_CHAPTER 3:_ AC FUNDAMENTALS**

## PART II: BASIC ELECTRONICS ENGINEERING-I

**Introduction:** Crystalline material—mechanical properties; energy band theory; Fermi levels

Conductors, Semiconductors and Insulators—electrical properties; band diagrams Semiconductors—intrinsic and extrinsic; energy band diagram; electrical conduction phenomenon; *P*-type and *N*-type semiconductors; drift and diffusion carriers; mass action law and continuity equation

Formation of *P-N* junction; energy band diagram; built-in-potential forward and reverse biased *P-N* junction; formation of depletion zone; *V-I* characteristics; Zener breakdown; Avalanche breakdown and its reverse characteristics; junction capacitance and varactor diode

Simple diode circuits; load line; linear piecewise model; rectifiers: half wave; full wave; its PIV; dc voltage and current; ripple factor; efficiency

Go to **_CHAPTER 1:_ BASIC CONCEPTS IN SEMICONDUCTOR PHYSICS**
**_CHAPTER 2:_ SEMICONDUCTOR DIODES AND THEIR ANALYSIS**

**Introduction to Transistors:** Formation of *PNP / NPN* junctions; energy band diagram; transistor mechanism and principle of transistors; CE; CB; CC configuration, Transistor characteristics—cut-off active and saturation mode; Early effect

Biasing and Bias Stability—calculation of stability factor; CE; CB; CC and their properties; small signal low frequency operation of transistors; equivalent circuits *h* parameters as a two-port network

Transistors as Amplifier—expression of voltage gain; current gain; input impedance and output impedance; frequency response for CE amplifier with and without source impedance

Go to **_CHAPTER 3:_ INTRODUCTION TO TRANSISTORS**
**_CHAPTER 4:_ JUNCTION TRANSISTORS: MODELLING, AMPLIFICATION AND BIASING**

**Introduction to Field Effect Transistor:** Structure and characteristics of MOSFET; depletion and enhancement type; CS; CG; CD configurations; CMOS—basic principles

**Go to** ***CHAPTER 5:*** **FIELD EFFECT TRANSISTOR**

# PART-I

# BASIC ELECTRICAL ENGINEERING-I


# DC NETWORK ANALYSIS

## 1.1 INTRODUCTION

When a number of network elements* are connected together to form a system that consists of set(s) of interconnected elements performing specific or assigned functions, it is called a "network". An electrically closed network is a "circuit". An electrical network is a combination of numerous electric elements (e.g., resistance $R$, inductance $L$, capacitance $C$, etc.).

Some important definitions related to an electrical network are as follows:

***Node:*** It is the junction in a circuit where two or more network elements are connected together.

***Branch:*** It is that part of the circuit which lies between two junctions in a circuit.

***Loop:*** It is a closed path in a circuit in which no element or node is encountered more than once.

***Mesh:*** It is such a loop that contains no other loop within it.

## 1.2 CHARACTERISTICS OF NETWORK ELEMENTS

### 1.2.1 Linear and Non-linear Elements

A *linear element* shows linear characteristics of voltage vs current. Thus the parameters of linear elements remain constant (i.e., the parameters do not change with voltage or current applied to that element). Resistors, inductors and capacitors are linear elements.

On the other hand, for a *non-linear element*, the current passing through it does not change linearly with the linear change in applied voltage across it, at a particular temperature and frequency. In a non-linear element the parameters change with applied voltage and current changes. Semiconductor devices like diodes, transistors, thyristors, etc. are typical examples of non-linear elements. Ohm's law is not valid for non-linear elements.

*A network element is a component of a circuit having different characteristics like linear, non-linear, active or passive etc. and will be defined shortly.

### 1.2.2 Active and Passive Elements

If a circuit element has the capability of enhancing the energy level of an electric signal passing through it, it is called an *active element*, viz., a battery, a transformer, semiconductor devices, etc. Otherwise the element that simply allows the passage of the signal through it without enhancement is called *passive element* (viz., resistors, inductors, thermistors and capacitors). Passive elements do not have any intrinsic property of boosting an electric signal.

### 1.2.3 Unilateral and Bilateral Elements

If the magnitude of the current passing through an element is affected due to change in polarity of the applied voltage, the element is called a *unilateral element*. On the other hand if the current magnitude remains the same even if the polarity of the applied voltage is reversed, it is called a *bilateral element*. Unilateral elements offer varying impedances with variation in the magnitude or direction of flow of the current while bilateral elements offer same impedance irrespective of the magnitude or direction of flow of current. A resistor, an inductance and a capacitor, all are bilateral elements while diodes, transistors, etc. are unilateral elements.

## 1.3 SERIES RESISTIVE CIRCUITS

Resistors are said to be in *series* when they are connected in such a way that there is only one path through which current can flow. Therefore the current in a series circuit is the same at all parts in the circuit. The voltage drop across each component in a series circuit depends on the current level and the magnitude resistance.

### 1.3.1 Currents and Voltages in a Series Circuit

The circuit diagram for three series connected resistors and a d.c. voltage source is shown in Fig. 1.1.

The total resistance connected across the voltage source is $R = R_1 + R_2 + R_3$. ($R$ is called the *equivalent resistance* in ohms for the given circuit.)

For a series circuit with $n$ resistors, the equivalent resistance $R$ is thus

$$R = R_1 + R_2 + R_3 + \cdots + R_n \qquad (1.1)$$

The equivalent circuit for the series resistance circuit is shown in Fig. 1.2.

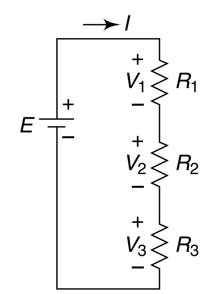


**Fig. 1.1** ***A simple series resistive circuit***

The equivalent circuit consists of the voltage source $E$ and the equivalent resistance $R$. The current $I$ flows from the positive terminal of the voltage source. Using Ohm's law the current through the series circuit in ampere is obtained as

$$I = \frac{E}{R} = \frac{E}{R_1 + R_2 + R_3 + \ldots + R_n} \qquad (1.2)$$

*There is only one path for current flow in a series circuit.*

The current flow causes a voltage drop $V$ or *potential difference* across each resistor in the circuit of Fig. 1.1. Using Ohm's law, the voltage drops across each resistor in volts are obtained as

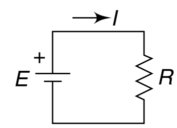



**Fig. 1.2** ***Equivalent of a simple series resistive circuit***

$V_1 = IR_1,\ V_2 = IR_2$ and $V_3 = IR_3$.

Since the sum of the resistive voltage drops is equal to the applied emf, for any series circuit,

$$E = V_1 + V_2 + V_3 + \cdots + V_n$$

or
$$E = I(R_1 + R_2 + R_3 + \cdots + R_n). \tag{1.3}$$

Next we consider series connection of voltage sources instead of series connection of resistors.

If three voltage sources are connected in series as shown in Fig. 1.3, the resultant voltage in volt is

$$E = E_1 + E_2 + E_3.$$

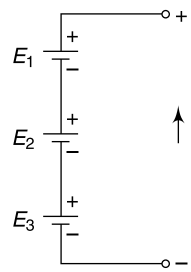



**Fig. 1.3** ***Series connection of three-voltage sources***

In Fig. 1.4 the lowermost voltage source $E_3$ has its negative terminal connected to the negative terminal of the middle cell. The resultant voltage in this case is

$$E = E_1 + E_2 - E_3$$

In Fig. 1.3 the voltage sources assist one another to produce the circuit current, so they are said to be in "series aiding". In Fig. 1.4 the bottommost voltage source will attempt to produce current in the opposite direction to that formed by the other two. Therefore this bottommost source is connected in "series opposing" with the top two cells.

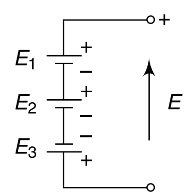



**Fig. 1.4** ***Series connections of three voltage sources with the polarity of one source reversed***

## 1.3.2 Voltage Divider

In Fig. 1.5 two series connected resistors are used as a *voltage divider* or *potential divider*.

Here, $V_1 = IR_1 = \dfrac{E}{R_1 + R_2} \cdot R_1 \quad \left[\because I = \dfrac{E}{R_1 + R_2}\right]$

Also $V_2 = I_2R_2 = \dfrac{E}{R_1 + R_2} \cdot R_2$

If $R_1 = R_2$ then $V_1 = V_2 = \dfrac{E}{2}$.

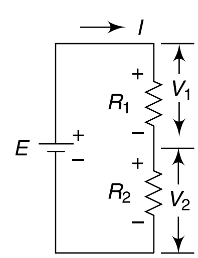



**Fig. 1.5** ***Voltage divider (or potential divider) circuit***

When $n$ number of resistors are connected in series then voltage drop ($V_i$) across any resistance ($R_i$) is given by

$$V_i = E \times \frac{R_i}{R_1 + R_2 + R_3 + ... + R_n} \quad (1.4)$$

$V_i$ and $E$ are expressed in volt and resistors are given in ohms.

***Voltage Divider Theorem***

*In a series circuit, the portion of applied emf developed across each resistor is the ratio of that resistor's value to the total series resistance in the circuit.*

### 1.3.3 Potentiometer

The circuit diagram of a variable resistor employed as a *potentiometer* is shown in Fig. 1.6. *The potentiometer is essentially a single resistor with terminals at each end and a movable contact that can be set to any point on the resistor.* Terminals $A$ and $B$ are the end terminals and terminals $C$ is the adjustable contact (Fig. 1.6).

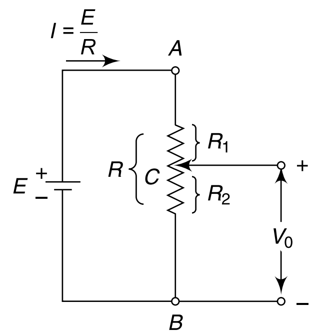



**Fig. 1.6** ***A simple potentiometric circuit***

The output voltage ($V_o$), in volt, is given as

$$V_o = E \times \frac{R_2}{R_1 + R_2} \quad (1.5)$$

If the moving contact is half way between the two end terminals then

$$R_1 = R_2 = \frac{R}{2}$$

or $$V_o = E \times \frac{1}{2}$$

When $R_2 = R$, $V_o = E$

and when $R_2 = 0$, $V_o = 0$

*Thus it is seen that the potentiometer can be adjusted to give an output voltage ranging linearly from 0 to E.*

### 1.3.4 Power in a Series Circuit

In Fig. 1.5, the power (VA) dissipated in $R_1$ is given by

$$P_1 = V_1 I = \frac{V_1^2}{R_1} = I^2 R_1$$

∴ For any series circuit containing $n$ number of resistors the power dissipated is

$$\begin{aligned} P &= P_1 + P_2 + P_3 + \cdots + P_n \\ &= V_1 I + V_2 I + V_3 I + \cdots + V_n I \\ &= I(V_1 + V_2 + V_3 + \cdots + V_n) \\ &= IE \end{aligned}$$

$$\therefore \qquad P = \frac{E^2}{R_1 + R_2 + R_3 + \ldots + R_n} \tag{1.6}$$

In dc circuit volt-ampere power (VA) is same as power expressed in watts. Thus $P$ is usually expressed in watts in dc circuits.

### 1.3.5 Current-limiting Resistor

Sometimes a resistor is included in series with an electrical circuit or electronic device to drop the supply voltage down to a desired level. This resistor can be treated as a *current-limiting resistor*.

In Fig. 1.7, $R_s$ provides a voltage drop to the series connected lamps $L_1$ and $L_2$. The lamps operate to a voltage level lower than the source voltage even in series connection. Also the resistor $R_s$ limits the current $I$ to the level required by the lamps.

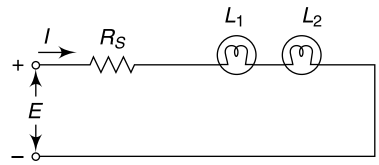



**Fig. 1.7** ***Application of current-limiting resistor $R_S$***

Here circuit current in ampere is obtained as

$$I = \frac{E}{R_S + (\text{sum of resistances of lamps})}$$

or $$R_s = (E/I) - (\text{sum of resistances of lamps}) \tag{1.7}$$

### 1.3.6 Open Circuits and Short Circuits in a Series Circuit

An *open circuit* occurs in a series resistive circuit when one of the resistors (or any series network element) becomes disconnected from the adjacent one. Open circuit can also occur when one of the resistors (or an element) has been destroyed by excessive power dissipation.

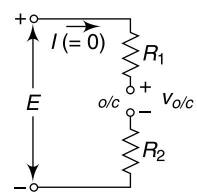



**Fig. 1.8** ***An open circuit***

In the circuit shown in Fig. 1.8, the open circuit can be thought of another resistance in series with value "infinity". Therefore the current,

$$I = \frac{E}{R_1 + R_2 + \infty} = 0.$$

The voltage drop across the open circuit ($V_{O/C}$) in volts is obtained as

$$V_{O/C} = E - IR_1 = E - 0 = E$$

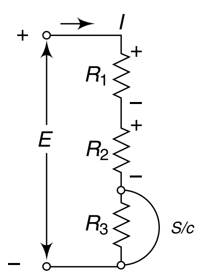



**Fig. 1.9** ***Deactivation of a resistance ($R_3$) in a series circuit by shorting terminals of $R_3$***

Figure 1.9 shows a series resistance $R_3$ short circuited in the series circuit. Here the resistance between the terminals of $R_3$ becomes zero after short circuit. Therefore, the circuit current $I$ in ampere is given by

$$I = \frac{E}{R_1 + R_2 + 0} = \frac{E}{R_1 + R_2}$$

**1.1** Find the current that flows through the resistors 10 Ω, 20 Ω, and 30 Ω connected in series across a 240 V supply.

***Solution***

$$\text{Current } I = \frac{V}{10 + 20 + 30}\text{ A} = \frac{240}{10 + 20 + 30} = 4 \text{ A}$$

**1.2** Determine the voltage drops across each resistor of the circuit shown in Fig. 1.10.

***Solution***

The current flowing through each resistor is given as

$$I = \frac{100}{5 + 2 + 3}\text{ A} = 10 \text{ A}$$

Voltage drop across the 5 Ω resistor = $10 \times 5 = 50$ V

Voltage drop across the 2 Ω resistor = $10 \times 2 = 20$ V

Voltage drop across the 3 Ω resistor = $10 \times 3 = 30$ V

Polarities are marked in Fig. 1.10(a). Check that total voltage drop is 100 V, same as the supply voltage.

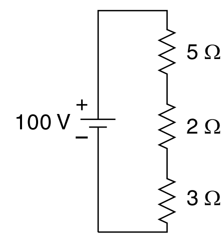



**Fig. 1.10** *Circuit of Ex. 1.2*

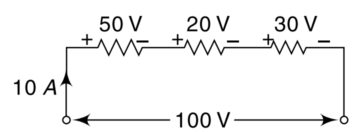



**Fig. 1.10(a)** *Voltage drops for the series circuit shown in Fig. 1.10*

**1.3** In the circuit shown in Fig. 1.11, if $E_1 = 10$ V and $E_2 = 7$ V, find the current through the resistors.

***Solution***

The current through the resistors

$$I = \frac{E_1 + E_2}{R_1 + R_2 + R_3} = \frac{10 + 7}{2 + 7 + 3} = 1.4167 \text{ A}$$

(Note that $E_1$ and $E_2$ are in series aiding connection.)

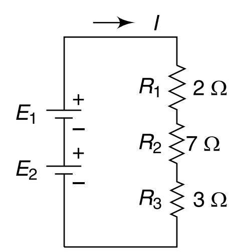



**Fig. 1.11** *Circuit of Ex. 1.3*

**1.4** Determine the current through the resistors in the circuit shown in Fig. 1.11 when the polarity of $E_2$ is reversed.

***Solution***

$$\text{Current } I = \frac{E_1 - E_2}{R_1 + R_2 + R_3} = \frac{10 - 7}{2 + 7 + 3}\text{ A} = 0.25 \text{ A}$$

(this time $E_1$ and $E_2$ are in series opposition).

**1.5** Calculate the minimum and maximum values of $V_o$ that can be obtained from the circuit shown in Fig. 1.12. $P$ is the moving contact and can slide linearly along a 300 Ω resistor.

***Solution***

By inspection it is evident that, if $P$ is at the bottommost point of 300 Ω resistor, $V_o$ is minimum.

$$\therefore \quad V_{o(\min)} = 240 \times \frac{800}{800 + 300 + 500} = 120 \text{ V.}$$

On the other hand, if $P$ is at the topmost point of 300 Ω resistor, $V_o$ is maxium.

$$V_{o(\max)} = 240 \times \frac{800 + 300}{800 + 300 + 500} = 165 \text{ V.}$$

It is possible to obtain values of ($V_o$) between 120 V and 165 V by sliding $P$ suitably across the 300 Ω resistor.

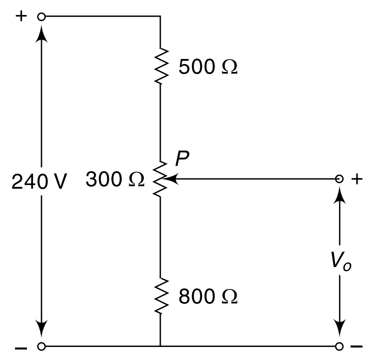


**Fig. 1.12** ***Circuit of Ex. 1.5***

**1.6** Determine the power dissipated in each resistor of Fig. 1.11 and also find the total power.

***Solution***

Power dissipated across the 2 Ω resistor in Fig. 1.11 is $I^2 \times 2$, i.e., $(1.4167)^2 \times 2$, i.e., 4.014 W. (The value of circuit current has been obtained in Ex 1.3 as 1.4167 A.)

Power dissipated across the 7 Ω resistor is $I^2 \times 7$, i.e., $(1.4167)^2 \times 7$ or 14.05 W.

Similarly, power dissipated in the 3 Ω resistor is $(1.4167)^2 \times 3$, i.e., 6.02 W.

Total power is $(E \times I)$, i.e., $(E_1 + E_2) \times I$.

This gives $(10 + 7) \times 1.4167$, i.e., 24.084 W.

[Check: Total power is $I^2(2 + 7 + 3)$, i.e., $(1.4167)^2 \times 12$ or 24.084 W.]

**1.7** In the circuit shown in Fig. 1.13 find the value of the resistor $R$ so that the lamps $L_1$ and $L_2$ operate at rated conditions. The rating of each of the lamps is 12 V, 9 W. If $L_2$ becomes short circuited find the current through the circuit and the power dissipated in each of the lamps.

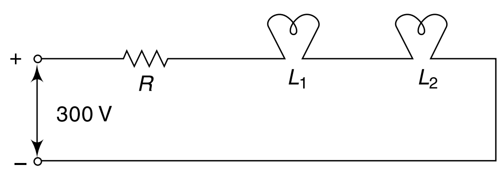


**Fig. 1.13** ***Circuit of Ex. 1.7***

***Solution***

Voltage rating of the lamps is 12 V each, while power rating of each of the lamps is 9 W.

If $I$ be the rated current through the lamps then

$$VI = P$$

or

$$I = \frac{P}{V} = 0.75 \text{ A}$$

If $R_L$ be the resistance of each lamp,

$$I^2 R_L = 9 \text{ or, } R_L = \frac{9}{(0.75)^2}\,\Omega$$

$$= 16\ \Omega.$$

Supply voltage = 300 V (given)

∴ Voltage across resistor ($R$) is $(300 - 2 \times 12)$ i.e., 276 V

Also, current through $R$ is 0.75 A

$\therefore \quad R = \frac{276}{0.75}\ \Omega = 368\ \Omega.$

If $L$ becomes short circuited, resistance across terminals of $L_2$ is 0.

If the current now is $I'$, we can write

$$300 = I'(R + R_L) = I'(368 + 16)$$

or $\quad I' = 0.78$ A.

Power dissipated in $L_1$ is now $(0.78)^2 \times 16 = 9.73$ W ($L_1$ will glow brighter)

Power dissipated in $L_2$ is obviously 0.

## 1.4 PARALLEL RESISTANCE CIRCUITS

*Resistors are said to be connected in parallel when equal voltages appear across each resistor (i.e., network element)*. The total current taken from the supply is the sum of all the individual resistor (or network elements) currents.

### 1.4.1 Currents and Voltages in Parallel Circuits

Resistors are said to be connected in parallel when the circuit has two terminals which are common to each resistor. Figure 1.14 represents a circuit having three resistors connected in parallel.

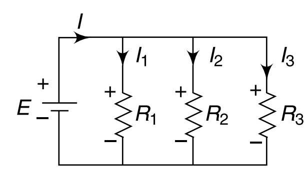



**Fig. 1.14** ***The resistors in parallel connection***

The voltage across each resistor is $E$ volts and the current through $R_1$ is $I_1$, through $R_2$ is $I_2$ and through $R_3$ is $I_3$.

$$\therefore \quad I_1 = \frac{E}{R_1},\ I_2 = \frac{E}{R_2} \quad \text{and} \quad I_3 = \frac{E}{R_3}$$

The current supplied by the battery in ampere is $(I) = I_1 + I_2 + I_3$

If $R$ be the equivalent resistance (in ohms) of the circuit in Fig. 1.14,

$$I = \frac{E}{R} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3}$$

or
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

or
$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}. \qquad (1.8)$$

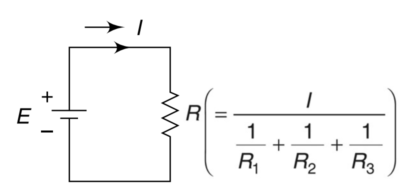



**Fig. 1.15** ***Equivalent circuit of three resistances in a parallel circuit***

The equivalent circuit is shown in Fig. 1.15.

If $n$ resistors are connected in parallel then we have

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n} \qquad (1.9)$$

where $R$ is the *equivalent resistance*.

*Therefore, the reciprocal of the equivalent resistance of resistors in parallel connection is equal to the sum of the reciprocals of the individual resistances.*

### 1.4.2 Conductances in Parallel

In dc circuits *conductance* is the reciprocal of resistance and its unit is "Siemens" (S) in SI units or "mho" in cgs units. If $G_1$, $G_2$, $G_3$, …, $G_n$ be the conductances of the resistors connected in parallel then the equivalent conductance ($G$) in Siemens is given by

$$G = G_1 + G_2 + G_3 + \cdots + G_n \tag{1.10}$$

According to Ohm's law, $I = \frac{V}{R} = VG$, where $V$ is the applied voltage, $G$ is the equivalent conductance of a parallel circuit, and $I$ is the source current.

### 1.4.3 Current Divider

Parallel resistance circuits are often called *current divider circuits* because the supply current is divided among the parallel branches.

The circuit in Fig. 1.16 can be called as a current divider circuit. Here

$$I_1 = \frac{E}{R_1} \quad \text{and} \quad I_2 = \frac{E}{R_2}$$

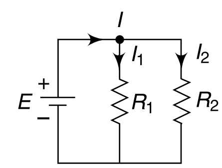



**Fig. 1.16** ***Concept of current division***

Again, $$I = I_1 + I_2$$

$$= \frac{E}{R_1} + \frac{E}{R_2} = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

or $$I = E\,\frac{R_1 + R_2}{R_1 \times R_2}$$

or $$E = I \times \frac{R_1 R_2}{R_1 + R_2} = IR.$$

$$\left[\text{If } (R) \text{ be the equivalent resistance then, } R = \frac{R_1 R_2}{R_1 + R_2}\right]$$

Now, $$I_1 = \frac{E}{R_1} = \frac{1}{R_1}(I) \times \frac{R_1 R_2}{R_1 + R_2} = I\,\frac{R_2}{R_1 + R_2}. \tag{1.11}$$

Similarly $$I_2 = I\,\frac{R_1}{R_1 + R_2}. \tag{1.12}$$

$I_1$ and $I_2$ are the currents in the branches of this current divider circuit in amperes.

These two equations (1.11 and 1.12) can be used to determine how a known supply current is divided into two individual currents through parallely connected resistors or network elements.

If $G_1$ and $G_2$ be the conductances of the resistors $R_1$ and $R_2$,

$$I_1 = I\,\frac{\frac{1}{G_2}}{\frac{1}{G_1} + \frac{1}{G_2}} = \frac{G_1}{G_1 + G_2} \cdot I \tag{1.13}$$

and $$I_2 = \text{I}\,\frac{\dfrac{1}{G_1}}{\dfrac{1}{G_1}+\dfrac{1}{G_2}} = \frac{G_1}{G_1+G_2}\cdot I. \tag{1.14}$$

If there are $n$ number of resistors with conductances $G_1$, $G_2$ …, $G_n$ connected in parallel across a voltage source then current in any resistor with conductance $G_i$ is

$$I_i = \frac{G_i}{G_1+G_2+G_3+\ldots+G_n}\cdot I \tag{1.15}$$

[$I$ being the supply current in ampere while $I_i$ is the currrent through $G_i$].

### 1.4.4 Power in Parallel Circuits

For the circuit in Fig. 1.14, the power (in VA) across resistor $R_1$ is given by

$$P_1 = EI_1 = \frac{E^2}{R_1} = I_1^2 = R_1$$

Total power $P = E\,(I_1 + I_2 + I_3)$

$$= E^2\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) = I_1^2R_1 + I_2^2R_2 + I_3^2R_3$$

$$= P_1 + P_2$$

When $n$ resistors are connected in parallel

$$P = P_1 + P_2 + P_3 + \cdots + P_n \tag{1.16}$$

($P$) can be expressed in VA or in Watts in dc circuits.

### 1.4.5 Open Circuits and Short Circuits in Parallel Circuits

When one of the components in a parallel resistive circuit is open circuited, as shown in Fig. 1.17, no current flows through that branch. The other branch currents are not affected by the open circuit as they still have the normal supply voltage applied across each of them. In Fig. 1.17, $I_1 = 0$. Supply current $I = I_2 + I_3$. All currents are expressed in amperes.

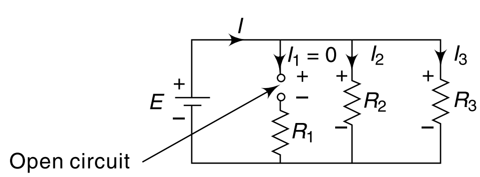



**Fig. 1.17** ***Open circuit in a branch in a parallel resistive circuit***

Figure 1.18 shows a short circuit across resistor ($R_3$).

As there is a short circuited path across $R_3$, i.e., across one of the resistors in the parallel circuit, no current will flow through resistors $R_1$, $R_2$ and $R_3$. Total current will flow from the battery through the short circuited path and the current ($I_{S/C}$) = $I$ = $E/0 = \infty$. However, in practice this current is limited by the internal resistance of

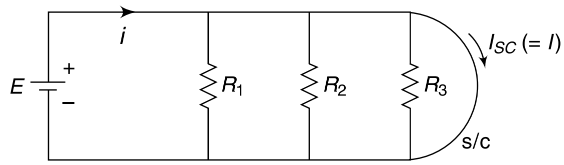



**Fig. 1.18** *Short circuit in a branch in a parallel circuit*

the battery and lead resistances of the wires. If the internal resistance of the battery be taken only and is equal to $R_i$, then current $I = E/R_i$ which is also very high (as the internal resistance of a battery is very small).

**1.8** Calculate the total current supplied by the battery in Fig. 1.19.

***Solution***

$$I_1 = \frac{24}{2} = 12 \text{ A}, I_2 = \frac{24}{3} = 8 \text{ A and}$$

$$I_3 = \frac{24}{6} \text{ A} = 4 \text{ A}$$

$\therefore$ The total current $I = I_1 + I_2 + I_3 = (12 + 8 + 4)$ A

$= 24$ A

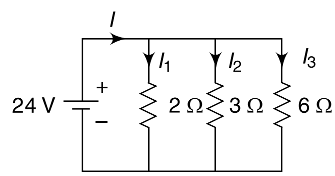



**Fig. 1.19** *Circuit of Ex. 1.8*

**1.9** Determine the equivalent resistance of the four resistances connected in parallel across a 240 V supply. Also find the total current. The resistances are of 10 Ω, 15 Ω, 25 Ω and 40 Ω.

***Solution***

The equivalent resistance

$$R = \frac{1}{\frac{1}{10} + \frac{1}{15} + \frac{1}{25} + \frac{1}{40}} = \frac{1}{0.1 + 0.067 + 0.04 + 0.025}$$

$$= 4.31 \ \Omega$$

Total current $I = \frac{E}{R} = \frac{240}{4.31 \text{ A}} = 55.68$ A.

**1.10** Three resistors of conductances 0.1 Siemens, 0.2 Siemens and 0.5 Siemens are connected in parallel. Calculate the equivalent resistance of the circuit.

***Solution***

Equivalent conductance $(G) = G_1 + G_2 + G_3$

$= 0.1 + 0.2 + 0.5 = 0.8$ Siemens

Equivalent resistance $R = \frac{1}{G} = \frac{1}{0.8} \ \Omega = 1.25 \ \Omega.$

**1.11** Using the current divider rule find the current in the resistors $R_1$ and $R_2$ connected in parallel across a voltage source. The supply current is 50 A, $R_1 = 10 \ \Omega$ and $R_2 = 20 \ \Omega$.

***Solution***

Total current $I = 50$ A

Current through resistor $R_1$ is $(I_1) = I \times \frac{R_2}{R_1 + R_2}$

$$= 50 \times \frac{20}{20 + 10}$$

$$= 33.33 \text{ A}$$

Current through resistor $R_2$ is $(I_2) = I \times \frac{R_1}{R_1 + R_2}$

$$= 50 \times \frac{10}{10 + 20}$$

$$= 16.67 \text{ A}$$

**1.12** In the circuit shown in Fig. 1.19 find the power dissipated across each resistor and the total power.

***Solution***

Power dissipated across 2 Ω resistor $(P_1) = I_1^2 \times 2 = (12)^2 \times 2$
$= 288$ W.

Power dissipated across 3 Ω resistor $(P_2) = I_2^2 \times 3 = (8)^2 \times 3$
$= 192$ W.

Power dissipated across 6 Ω resistor $(P_3) = I_3^2 \times 6 = (4)^2 \times 6$
$= 96$ W.

Total power $(P) = P_1 + P_2 + P_3$
$= 288 + 192 + 96 = 576$ W.

[The values of $I_1$, $I_2$, and $I_3$ have been obtained as 12 A, 8 A and 4 A in Ex 1.8]. Also, $(P) = EI = 24 \text{ V} \times 24\text{A} = 576$ W (check).

## 1.5 SERIES-PARALLEL CIRCUITS

Series-parallel resistive circuits consist of combinations of series connected and parallel connected resistors (or other passive network elements).Figure 1.20 represents a simple series-parallel resistive circuit. In this circuit $R_2$ and $R_3$ are connected in parallel. The parallel combination of $R_2$ and $R_3$ is $R_2R_3/(R_2 + R_3)$ $(= R_{eq})$.

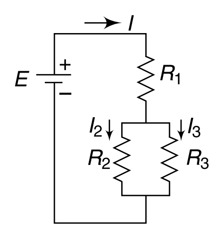



**Fig. 1.20** ***A series parallel circuit***

The equivalent circuit is shown in Fig. 1.21.

Since $R_1$ and $R_{eq}$ are connected in series, therefore the equivalent resistance of the whole circuit is $[(R_1 + R_{eq})\ \Omega]$.

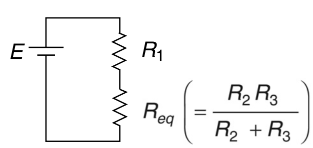



**Fig. 1.21** ***Equivalent of series-parallel circuit***

### 1.5.1 Currents and Voltages in Series–parallel Circuits

In Fig. 1.20 the supply current $I$ flows through resistance $R_1$. Then $I$ splits into $I_2$ and $I_3$ flowing through $R_2$ and $R_3$ respectively.

Obviously, $I = I_2 + I_3$, $I$ being expressed in amps

The currents $I_2$ and $I_3$ can easily be calculated using the current divider rule.

The voltage across resistor $R_1$ is given by $V_1 = IR_1$

The voltage across resistors $R_2$ and $R_3$ are equal as they are connected in parallel. Here

$$V_2 = V_3 = I_2R_2 = I_3R_3$$

Also, $E = V_1 + V_2 = V_1 + V_3$.

Once the branch currents are known, the voltages across each resistor can easily be calculated.

### 1.5.2 Open Circuits and Short Circuits in a Series–Parallel Circuit

The effect of open circuit in a series-parallel circuit is shown in Fig. 1.22(a) and Fig. 1.22(b).

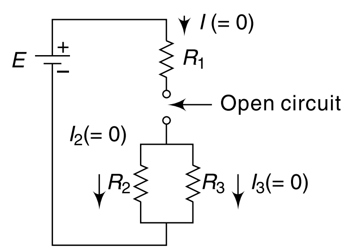



**Fig. 1.22(a)** ***Open circuit in series-parallel circuit***

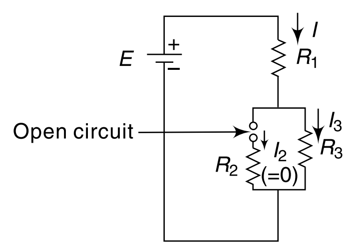



**Fig. 1.22(b)** ***Open circuit in a branch of series-parallel circuit***

In Fig. 1.22(a), open circuit occurs at one terminal of $R_1$. This has the same effect as an open circuit in the supply line, so that the main current flowing in any part of the circuit is zero. Also as the main current is zero there is no voltage drop across the resistors and the supply voltage $E$ appears across the open circuit.

When open circuit occurs at one end of one of the parallel resistors, as shown in Fig. 1.2(b), the current through that resistor only is zero. Here, $I_2 = 0$.

Also, $R_1$ and $R_3$ can be assumed to be connected in series.

Hence $$I = I_3 = \frac{E}{R_1 + R_3}.$$

As there is no current through $R_2$ so there is no voltage drop across it and the voltage across the open circuit is equal to the voltage across $R_3$, i.e. $V_3$.

When short circuit occurs across the terminals of $R_1$ as shown in Fig. 1.23(a), the resistance across the terminals of $R_1$ is 0.

The total current is obtained as, $I = \dfrac{E}{R_2 \parallel R_3} = \dfrac{E}{R_2 R_3/(R_2 + R_3)}$

$$I_2 = I \cdot \frac{R_3}{R_2 + R_3} \quad \text{and} \quad I_3 = I \cdot \frac{R_2}{R_2 + R_3}$$

When short circuit occurs across the terminals of $R_2$, as shown in Fig. 1.23(b), the resistance across the terminals of $R_2$ is 0. Therefore no current will pass through $R_3$ as there is a short circuited path in parallel with it.

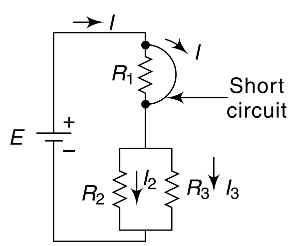



**Fig. 1.23(a)** ***Short circuit in series part of series-parallel circuit***

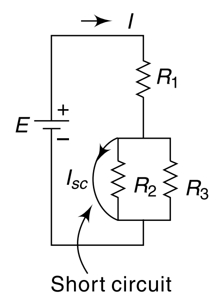



**Fig. 1.23(b)** ***Short circuit in a parallel branch of a series-parallel circuit***

Hence $\quad I = \dfrac{E}{R_1}$

also, $\quad I_3 = 0 = I_2$

If $I_{s/c}$ be the current in amps through the short circuited path then $(I_{s/c})(= I) = \dfrac{E}{R_1}$.

### 1.5.3 Analysis of a Series–Parallel Circuit

The following are the steps for solving series-parallel circuits.

1. Draw a circuit diagram identifying all components by number and showing all currents and resistor voltage drops.
2. Convert all series branches of two or more resistors into a single equivalent resistance.
3. Convert all parallel combinations of two or more resistors into a single equivalent resistance.
4. Repeat procedures 2 and 3 until the desired level of simplification is achieved.

The final circuit should be simple series or parallel circuit. Once the current through each equivalent resistance or the voltage across it is known, the original circuit can be used to determine individual resistor currents and voltages.

**1.13** Find the supply current and the currents in the parallel branches in the circuit shown in Fig. 1.24.

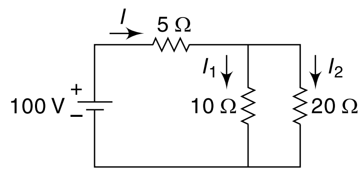



**Fig. 1.24** *Circuit of Ex. 1.13*

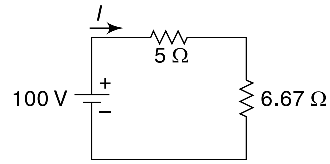



**Fig. 1.24(a)** *Equivalent circuit of the series-parallel circuit of Fig. 1.24*

***Solution***

In the circuit shown in Fig. 1.24, 10 Ω and 20 Ω are in parallel. The equivalent resistance of the parallel combination is $\frac{10 \times 20}{10 + 20} = \frac{200}{30}$ Ω = 6.67 Ω. 5 Ω and 6.67 Ω are now in series as shown in Fig. 1.24(a)

The supply circuit is $I = \frac{100}{5 + 6.67} = 8.57$ A

From Fig. 1.24,

$$I_1 = 8.57 \times \frac{20}{10 + 20} = 5.71 \text{ A}$$

$$I_2 = 8.57 \times \frac{10}{10 + 20} = 2.857 \text{ A}$$

**1.14** Find all resistor currents and voltages in the circuit shown in Fig. 1.25.

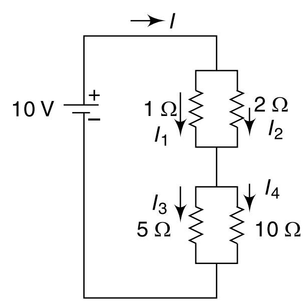



**Fig. 1.25** *Circuit of Ex. 1.14*

***Solution***

The parallel combination of 1 Ω and 2 Ω is $\frac{1 \times 2}{1 + 2} = \frac{2}{3}$ Ω.

The parallel combination of 5 Ω and 10 Ω is = $\frac{5 \times 10}{5 + 10}$ = $\frac{50}{15}$ Ω = $\frac{10}{3}$ Ω

Also, $\frac{2}{3}$ Ω and $\frac{10}{3}$ Ω are in series as shown in Fig. 1.25(a)

Figure 1.25(a) represents a simple series circuit. The supply current is given by

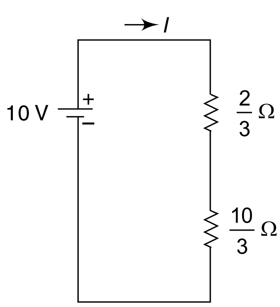



**Fig. 1.25(a)** *Simplified equivalent of circuit of Fig. 1.25*

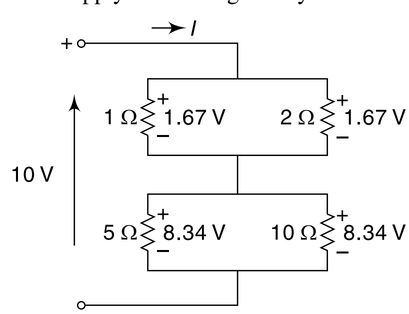



**Fig. 1.25(b)** *Polarity of voltage drops of the circuit of Fig. 1.25*

$$I = \frac{10}{\frac{2}{3} + \frac{10}{3}}\text{ A} = \frac{10 \times 3}{12}\text{ A} = 2.5\text{ A}$$

From Fig. 1.25,

Current through 1 Ω resistor $I_1 = 2.5 \times \frac{2}{1+2}\text{ A} = 1.67\text{ A}$

Current through 2 Ω resistor $I_2 = 2.5 \times \frac{1}{1+2}\text{ A} = 0.833\text{ A}$

Current through 5 Ω resistor $I_3 = 2.5 \times \frac{10}{10+5}\text{ A} = 1.667\text{ A}$

Current through 10 Ω resistor $I_4 = 2.5 \times \frac{5}{10+5}\text{ A} = 0.833\text{ A}$

Therefore

Voltage across 1 Ω resistor is 1.67 × 1 = 1.67 V

Voltage across 2 Ω resistor is 0.833 × 2   1.67 V

Voltage across 5 Ω resistor is 1.667 × 5 = 8.34 V

Voltage across 10 Ω resistor is 0.833 × 10 = 8.34 V

[Polarities of voltage drops are shown in Fig. 1.25(b)].

**1.15** Find the current through 5 Ω resistor in Fig. 1.26 when the terminals across a 10 Ω resistor is (i) open circuited and (ii) short circuited. Also find the current through the short circuited path.

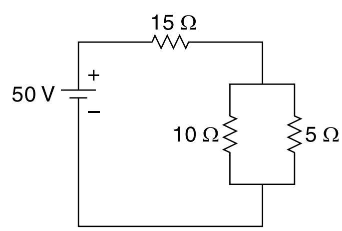



**Fig. 1.26** ***Circuit of Ex. 1.15***

***Solution***

(i) When terminals across 10 Ω resistor is open circuited as shown in Fig. 1.26(a), 15 Ω and 5 Ω are in series. Hence current $I$ flows through both 15 Ω and 5 Ω. The current through the 5 Ω resistor is obtained as

$$I = \frac{50}{15+5}\text{ A} = \frac{50}{20}\text{ A} = 2.5\text{ A}$$

(ii) When terminals across 10 Ω resistor is short circuited as shown in Fig. 1.26(b), no current will pass through the 5 Ω resistor as there is a short-circuited path in parallel with it. Therefore current through the 5 Ω resistor is 0.

The supply current $I$ will pass through 15 Ω and through the short circuited path. Hence the current through the short circuited path is 50/15 A = 10/3 A = 3.33 A.

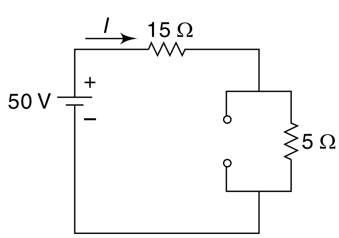



**Fig. 1.26(a)** ***One resistor in circuit of Fig. 1.26 open***

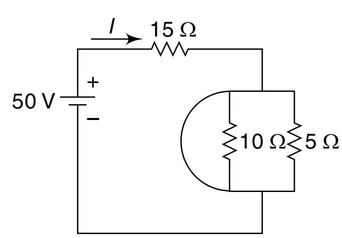



**Fig. 1.26(b)** ***10 Ω resistor is shorted in circuit of Fig. 1.26***

## 1.6 KIRCHHOFF'S LAWS

A German physicist Gustav Kirchhoff developed two laws enabling easier analysis of circuits containing interconnected impedances. The first law deals with flow of current and is popularly known as *Kirchhoff's current law* (KCL) while the second one deals with voltage drop in a closed circuit and is known as *Kirchhoff's voltage laws* (KVL).

### 1.6.1 Kirchhoff's Current Law (KCL)

*It states that in any electrical network the algebraic sum of currents meeting at any node of a circuit is zero.*

In Fig. 1.27, $i_1$ and $i_2$ are the inward currents towards the junction 0 and are assumed as negative currents. Currents $i_3$, $i_4$ and $i_5$ are outward currents and taken as positive. As per KCL,

$$-i_1 - i_2 + i_3 + i_4 + i_5 = 0$$

i.e., $$i_1 + i_2 = i_3 + i_4 + i_5 \tag{1.18}$$

*i.e., the algebraic sum of currents entering a node must be equal to the algebraic sum of currents leaving that node.*

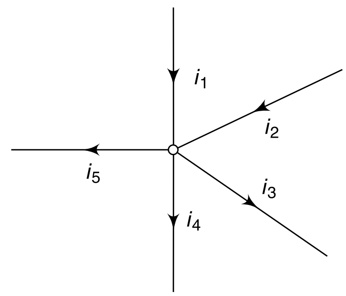



**Fig. 1.27** *Explanation of KCL*

### 1.6.2 Kirchhoff's Voltage Law (KVL)

*It states that the algebraic sum of voltages (or voltage drops) in any closed path, in a network, traversed in a single direction is zero.*

In Fig. 1.28, if we travel clockwise in the network along the direction of the current, application of KVL yields

$$-V_1 + iR_1 + V_2 + iR_2 + iR_3 = 0$$

or $$V_1 = i(R_1 + R_2 + R_3) + V_2 \tag{1.19}$$

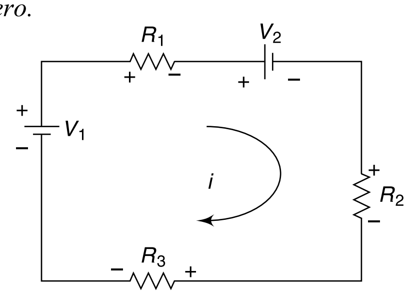



**Fig. 1.28** *Explanation of KVL*

[We can also write equation (1.19) as follows:

$$V_1 - V_2 = i(R_1 + R_2 + R_3)$$

or $$i = \frac{V_1 - V_2}{R_1 + R_2 + R_3}. \tag{1.20}$$

We consider the voltage drop as positive when current flows from positive to negative potential. Hence $V_1$ is negative while $V_2$ is positive in the first step of equation (1.19).

### 1.6.3 Network Analysis Procedure using Kirchhoff's Laws

1. Convert all current sources to voltage sources.
2. Letter or number all junctions on the network as *A*, *B*, *C* or 1, 2, 3 etc.
3. Identify current directions and voltage polarities and number them according to the resistor involved.
4. Identify each current path according to the lettered junctions and applying Kirchhoff's voltage law, write the voltage equations for the paths.
5. Applying Kirchhoff's current law, write the equations for the currents entering and leaving all junctions where more than one current is involved.
6. Solve the equations by substitution to find the unknown currents and or voltages.

**1.16** Find the magnitude and direction of the unknown currents in Fig. 1.29. Given $i_1 = 20$ A, $i_2 = 12$ A and $i_5 = 8$ A.

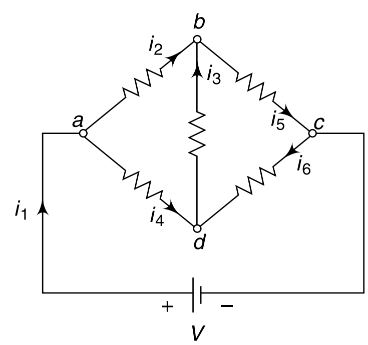



**Fig. 1.29** *Circuit of Ex. 1.16*

***Solution***

Applying KCL at node '*a*'

$$-i_1 + i_2 + i_4 = 0 \cdots (x)$$

or $$i_4 = i_1 - i_2 = 20 - 12 = 8 \text{ A}$$

Applying KCL at node '*b*'

$$-i_2 - i_3 + i_5 = 0 \cdots (y)$$

or $$i_3 = i_5 - i_2 = 8 - 12 = -4 \text{ A}$$

Applying KCL at node '*d*'

$$-i_4 + i_3 - i_6 = 0 \cdots (z)$$

or $$i_6 = i_3 - i_4 = -4 - 8 = -12 \text{ A}$$

The actual currents are now marked in Fig. 1.29(a).

We can interpret as follows:

$i_3 = -4$ A (from *d* to *b*)

i.e. $i_3 = 4$ A (from *b* to *d*)

$i_4 = 8$ A (from *a* to *d*)

$i_6 = -12$ A (from *c* to *d*)

or $i_6 = 12$ A (from *d* to *c*)

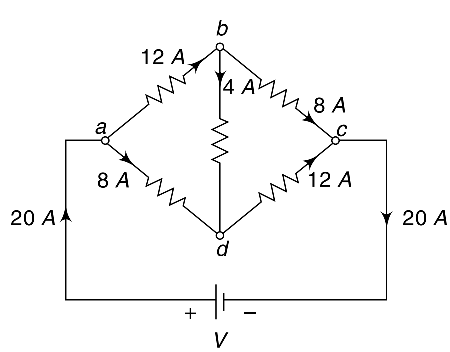



**Fig. 1.29(a)** *Actual current flows in circuit of Fig. 1.29*

**1.17** In Fig. 1.30, find $v$. Also find the magnitudes and direction of the unknown currents through 10 Ω, 2 Ω and 5 Ω resistors.

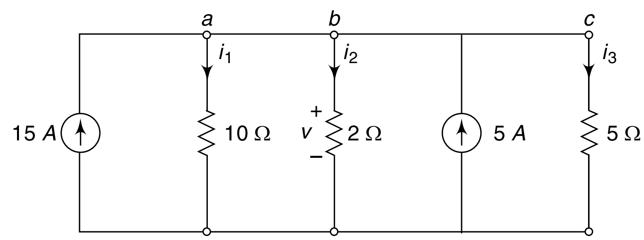



**Fig. 1.30** ***Circuit of Ex. 1.17***

***Solution***

Applying KCL at node '$a$', (Fig. 1.30),

$$-15 + i_1 + i_2 - 5 + i_3 = 0$$

or $$i_1 + i_2 + i_3 = 20$$

(i)

From Ohm's Law, $i_2 = \frac{v}{2}$; $i_1 = \frac{v}{10}$ and $i_3 = \frac{v}{5}$.

Then from equation (i), we have

$$\frac{v}{10} + \frac{v}{2} + \frac{v}{5} = 20$$

or $$v + 5v + 2v = 200$$

∴ $$v = 25 \text{ V}.$$

Hence $$i_1 = \frac{v}{10} = \frac{25}{10} = 2.5 \text{ A}$$

$$i_2 = \frac{25}{2} = 12.5 \text{ A}$$

$$i_3 = \frac{25}{5} = 5 \text{ A}.$$

**1.18** In the part of the electrical network, shown in Fig. 1.31, find $v_1$. Assume $i_2 = (10e^{-3t})$ A, $i_4 = 6(\sin t)$ A and $v_3 = (8e^{-3t})$ V.

***Solution***

Applying KCl at the node '0' in Fig. 1.31,

$$-i_1 - i_2 - i_3 + i_4 = 0$$

or $$i_1 + 10e^{-3t} + c\frac{dv_3}{dt} - 6 \sin t = 0$$

or $$i_1 + 10e^{-3t} + 2 \times \frac{d}{dt}(8e^{-3t}) - 6 \sin t = 0$$

or $$i_1 + 10e^{-3t} - 48e^{-3t} - 6 \sin t = 0$$

∴ $$i_1 = 38e^{-3t} + 6 \sin t$$

Now, $$v_1 = L\frac{di_1}{dt} = 4 \times \frac{d}{dt}(38e^{-3t} + 6 \sin t)$$

$$= 4\{-114e^{-3t} + 6 \cos t\}$$

∴ $$v_1 = (24 \cos t - 456\, e^{-3t}) \text{ V}$$

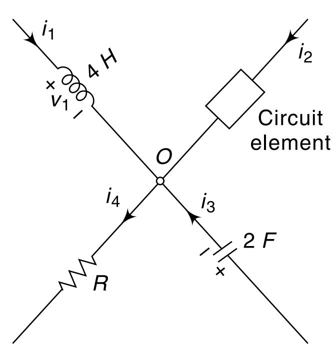



**Fig. 1.31** ***Circuit of Ex. 1.18***

**1.19** Find branch currents in the bridge circuit shown in Fig. 1.32.

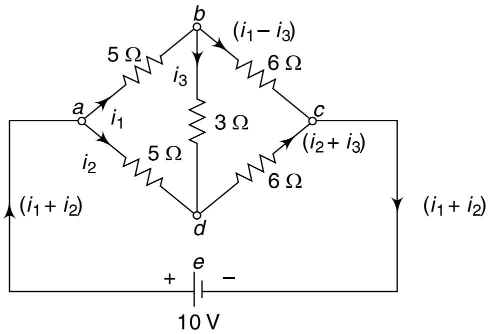



**Fig. 1.32** *Circuit of Ex. 1.19*

### *Solution*

We assume currents $i_1$, $i_2$ and $i_3$ in the directions as shown in Fig. 1.32.

Applying KVL in loop '*abda*', we find

$$5i_1 + 3i_3 - 5i_2 = 0. \quad \text{(i)}$$

Applying KVL in loop '*bcdb*', we find

$$6(i_1 - i_3) - 6(i_2 + i_3) - 3i_3 = 0$$

or $$6i_1 - 6i_2 - 15i_3 = 0. \quad \text{(ii)}$$

Applying KVL in loop '*adcea*', we find

$$5i_2 + 6(i_2 + i_3) - 10 = 0$$

or $$11i_2 + 6i_3 - 10 = 0. \quad \text{(iii)}$$

Solving equations (i), (ii) and (iii) we get

$$i_1 = i_2 = 0.91 \text{ A}; i_3 = 0.$$

**1.20** In the network of Fig. 1.33, find $v_1$ and $v_2$ using KVL.

### *Solution*

In loop '*abca*', from KVL we can write,

$$1 + v_2 - v_1 = 0$$

or $$v_1 - v_2 = 1.$$

(i)

In loop '*bcdb*', using KVL we find,

$$-v_2 + 1 + 4 = 0$$

or $$v_2 = 5 \text{ V}.$$

Substituting the value of $v_2$ in equation (i) we get

$$v_1 = 6 \text{ V}.$$

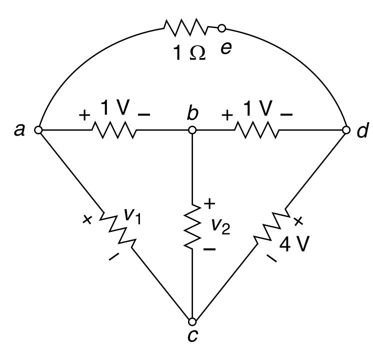



**Fig. 1.33** *Circuit of Ex. 1.20*

**1.21** Find current $i$ in the circuit shown in Fig. 1.34.

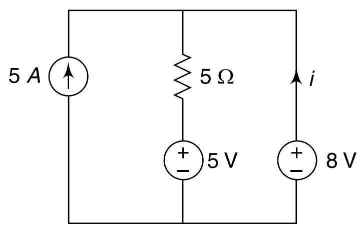



**Fig. 1.34** ***Circuit of Ex. 1.21***

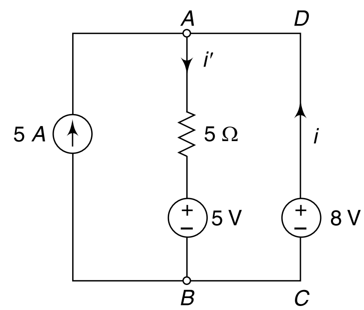



**Fig. 1.34(a)** ***Circuit of Ex. 1.21***

***Solution***

The assumed and given currents in various branches of the circuit shown in Fig. 1.34 are drawn in Fig. 1.34(a).

Using KCL at node '$A$',

$$-5 + i' - i = 0$$

or $$i' - i = 5. \qquad \text{(i)}$$

Applying KVL in loop '$ABCDA$',

$$5i' + 5 - 8 = 0$$

i.e. $$i' = \frac{3}{5} = 0.6 \text{ A}.$$

Substituting the value of $i'$ in (i), we have $i = 0.6 - 5 = -4.4$ A.

Thus $i(4.4\ A)$ flows from node $D$ to node $C$ in the actual circuit.

## 1.7 NODAL ANALYSIS

Nodal analysis is based on Kirchhoff's current law. This method has the advantage that a minimum number of equations are needed to determine the unknown quantities. Moreover, it is particularly suited for networks having many parallel branches and also when there are current sources in the network.

For the application of this method one of the nodes in the network is regarded as the *reference* or *datum* node or *zero potential* node. The number of simultaneous equations to be solved becomes $(n - 1)$, where $n$ is the number of independent nodes.

**Illustration**

Referring Fig. 1.35, we find that nodes '$A$' and '$B$' are independent nodes. Let node '$B$' be considered as reference node and the voltages at nodes '$A$' and '$B$' be $(V_A)$ and $(V_B)$ respectively. Obviously, $V_B = 0$.

Using Ohm's Law,

$$I_1 = \frac{E_1 - V_A}{R_1};$$

$$I_2 = \frac{E_2 - V_A}{R_2}$$

$$I_3 = \frac{V_A - V_B}{R_3} = \frac{V_A}{R_3}$$

Applying KCL at node $A$,

$$-I_1 - I_2 + I_3 = 0$$

i.e., $$-\frac{E_1 - V_A}{R_1} - \frac{E_2 - V_A}{R_2} + \frac{V_A}{R_3} = 0. \qquad (1.20)$$

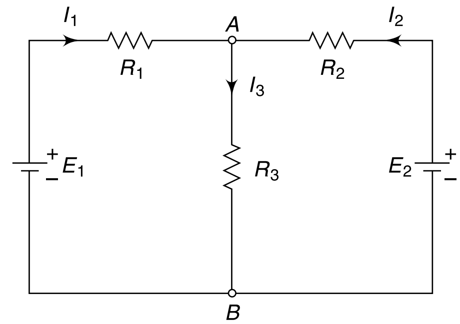



**Fig. 1.35** *Illustration for nodal method*

This equation represents the *nodal* form of KCL. *In nodal analysis we usually assume inward currents as negative while outward currents as positive.*

### 1.7.1 Nodal Analysis Procedure

1. Convert all voltage sources to current sources and redraw the circuit diagram.
2. Identify all nodes and choose a reference node. (Usually, the common node is the reference node.)
3. Write the equation for the currents flowing into and out of each node, with the exception of the reference node.
4. Solve the equation to determine the node voltage and the required branch currents.

**1.22** Find the voltage $v$ in the circuit shown in Fig. 1.36.

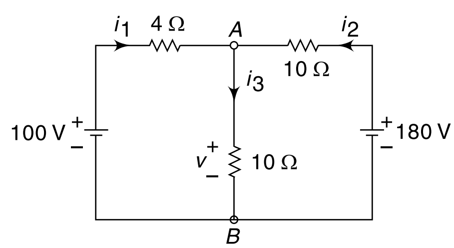



**Fig. 1.36** *Circuit of Ex. 1.22*

***Solution***

Considering '$B$' as reference node, $V_B = 0$. Let $V_A$ be the potential at node '$A$'.

Obviously, $V_A - V_B = v$, i.e., $V_A = v$.

Using nodal analysis at node '$A$', we get

$$\frac{V_A - 100}{4} + \frac{V_A}{10} + \frac{V_A - 180}{10} = 0$$

or $$\frac{v - 100}{4} + \frac{v}{10} + \frac{v - 180}{10} = 0$$

or $$v\left(\frac{1}{10}+\frac{1}{10}+\frac{1}{4}\right)=43$$

$\therefore$ $$v=\frac{43\times 10}{4.5}=95.55\text{ V.}$$

**1.23** Find the currents in different branches of the network shown in Fig. 1.37 using nodal analysis.

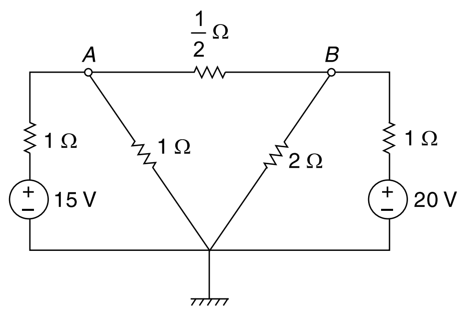



**Fig. 1.37** ***Circuit of Ex. 1.23***

Let $V_A$ and $V_B$ be the nodal voltages at nodes '*A*' and '*B*' in the given figure. The ground node is the reference node. Using nodal analysis, at node '*A*' we can write

$$\frac{V_A-15}{1}+\frac{V_A}{1}+\frac{V_A-V_B}{1/2}=0$$

or $$4V_A-2V_B=15. \qquad \text{(i)}$$

At node '*B*' we can write

$$\frac{V_B-V_A}{1/2}+\frac{V_B}{2}+\frac{V_B-20}{1}$$
$$=0$$

or $$3.5\,V_B-2\,V_A=20 \qquad \text{(ii)}$$

Solving equations (i) and (ii), we get

$$V_B=11\text{ V};\ V_A=9.25\text{ V.}$$

Hence, current through the respective resistors can be calculated as follows:

$$I_1=\frac{V_A-15}{1}=-5.75\text{ A}$$

$$I_2=\frac{V_A}{1}=9.25\text{ A}$$

$$I_3=\frac{V_A-V_B}{1/2}=-3.5\text{ A}$$

$$I_4=\frac{V_B}{2}=5.5\text{ A}$$

$$I_5=\frac{V_B-20}{1}=-9\text{ A.}$$

Figure 1.37(a) represents the circuit alongwith associated currents.

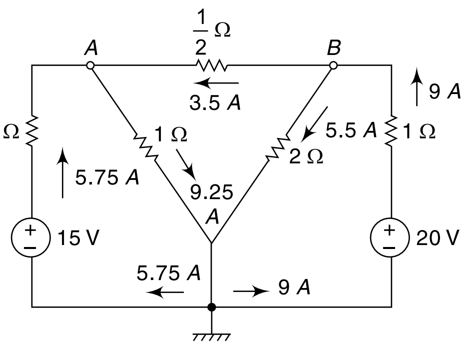



**Fig. 1.37(a)** ***Current values in branches of Fig. 1.37***

**1.24** Find the node voltages ($V_x$) and ($V_y$) using nodal analysis (Fig. 1.38).

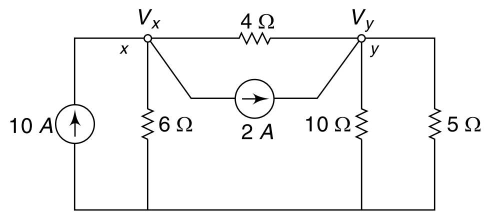



**Fig. 1.38** ***Circuit of Ex. 1.24***

***Solution***

At node '*x*', we have

$$-10 + \frac{V_x}{6} + 2 + \frac{V_x - V_y}{4} = 0$$

or $$V_x\left(\frac{1}{4}+\frac{1}{6}\right)\frac{V_y}{4} = 8$$

or $$5V_x - 3V_y = 96. \quad \text{(i)}$$

Applying nodal analysis at '*y*', we get

$$-2 + \frac{V_y - V_x}{4} + \frac{V_y}{10} + \frac{V_y}{5} = 0$$

or $$-\frac{V_x}{4} + V_y\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{10}\right) = 2$$

or $$5V_x - 11V_y = -40. \quad \text{(ii)}$$

Solving equations (i) and (ii), we get

$$V_x = 29.4 \text{ V}; \ V_y = 17 \text{ V}.$$

**1.25** Find current in the 15 Ω resistor using nodal method (Fig. 1.39).

***Solution***

Let us first designate the nodes '1' and '2' in Fig. 1.39 and assume nodal voltages to be ($V_1$) and ($V_2$) respectively

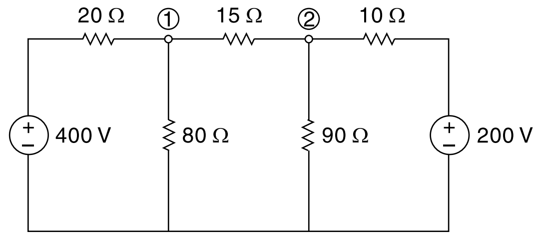



**Fig. 1.39** ***Circuit of Ex. 1.25***

At node '1',

$$\frac{V_1 - 400}{20} + \frac{V_1}{80} + \frac{V_1 - V_2}{15} = 0$$

or $$V_1\left(\frac{1}{20} + \frac{1}{80} + \frac{1}{15}\right) - \frac{V_2}{15} = 20$$

or $$\frac{31}{240} \cdot V_1 - \frac{1}{15} \cdot V_2 = 20 \qquad \text{(i)}$$

Similarly, using nodal analysis at node '2',

$$\frac{V_2 - 200}{10} + \frac{V_2}{90} + \frac{V_2 - V_1}{15} = 0$$

or $$-\frac{V_1}{15} + V_2\left(\frac{1}{10} + \frac{1}{15} + \frac{1}{90}\right) V_2 = 20$$

or $$-\frac{1}{15} \cdot V_1 + \frac{16}{90} \cdot V_2 = 20$$

Solving equations (i) and (ii) we get

$$V_1 = 264.88 \text{ V}; \quad V_2 = 211.33 \text{ V}$$

Hence, current in the 15 Ω resistor is obtained as

$$I_{15} = \frac{V_1 - V_2}{15}$$

$$= \frac{264.88 - 211.33}{15} = 3.57 \text{ A}$$

This current is directed from node '1' and node '2'.

**1.26** In Fig. 1.40, find "*v*" in the given circuit using nodal analysis.

***Solution***

Let us mark the junction of two resistors (1 Ω and 2 Ω) as node '*A*' and assume the voltage at this node to be ($V_A$). Applying nodal analysis at '*A*' we get

$$\frac{V_A}{2} + \frac{V_A + 5}{1 + 1} + \frac{V_A + 10}{1} = 0$$

or $$V_A\left(\frac{1}{2} + \frac{1}{2} + 1\right) + \frac{5}{2} + 10 = 0$$

or $$V_A = -6.25 \text{ V}.$$

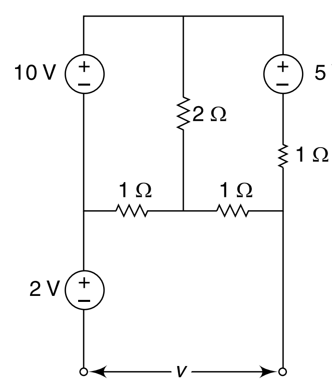



**Fig. 1.40** ***Circuit of Ex. 1.26***

We now find the currents passing through both side resistors of the node '$A$'. We redraw Fig. 1.40 as Fig. 1.40(a) and mark the corresponding resistors as $r_1$ and $r_2$. The current through $r_1$ is given by $i_{r_1} = (V_A + 10)/1 = 3.75$ A, directed outwards of node '$A$'. Similarly, current through $r_2$ is given by $i_{r_2} = \dfrac{V_A + 5}{1+1} = -0.625$ A, directed towards the node '$A$'.

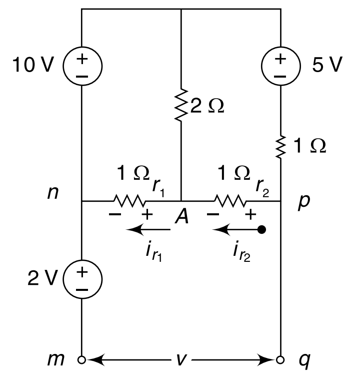



**Fig. 1.40(a)** *Figure 1.40, redrawn, for analysis*

$\therefore$ Voltage drop across $r_1$ is $(3.75 \times 1)$ i.e., 3.75 V while that across $r_2$ is $(-0.625 \times 1)$ i.e., $-0.625$ V. The corresponding polarities have been marked in Fig. 1.40(a).

Finally, in loop '*mnApq*' we can write, from KVL,

$$-v - 2 - 3.75 + 0.625 = 0$$

i.e. $$v = -5.125 \text{ V}.$$

(It means, polarity of '$m$' is actually negative while polarity of '$q$' is actually positive in Fig. 1.40(a)).

**1.27** Obtain the current through the 1 Ω resistor using node voltage method for the circuit shown in Fig. 1.41.

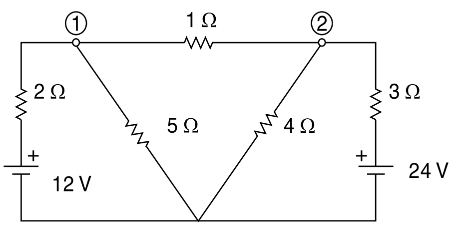



**Fig. 1.41** *Circuit of Ex. 1.27*

### *Solution*

Let us first mark the nodes '1' and '2' in Fig. 1.41 and assume corresponding nodal voltages to be $V_1$ and $V_2$.

At node '1', we have

$$\frac{V_1 - 12}{2} + \frac{V_1}{5} + \frac{V_1 - V_2}{1} = 0$$

or $$V_1\left(\frac{1}{5} + \frac{1}{2} + 1\right) - V_2 = 6$$

or $$17\,V_1 - 10\,V_2 = 60. \qquad \text{(i)}$$

At node '2', we have

$$\frac{V_2 - 24}{3} + \frac{V_2}{4} + \frac{V_2 - V_1}{1} = 0$$

or $$-V_1 + V_2\left(\frac{1}{3} + \frac{1}{4} + 1\right) = 8$$

or $$-V_1 + \frac{19}{12}V_2 = 8$$

or $$-12V_1 + 19V_2 = 96. \qquad \text{(ii)}$$

Solving (i) and (ii) we get, $V_1 = 10.35$ V; $V_2 = 11.6$ V

Hence the current through 1 Ω resistor is

$$I_1 = \frac{V_2 - V_1}{1} = \frac{11.6 - 10.35}{1}.$$

$$= 1.25 \text{ V, directed from node '2' and to node '1'.}$$

## 1.8 MESH ANALYSIS (OR LOOP ANALYSIS)

The *mesh* or *loop analysis* is based on Kirchhoff's voltage law. Here the currents in different meshes are assigned continuous paths so that they do not split at a junction into branch currents. In this method loop voltage equations are written by KVL in terms of unknown loop currents. Circuits with voltage sources are comparatively easier to be solved by this method.

**Illustration**

Figure 1.42 shows that two batteries having emf $E_1$ and $E_2$ are connected in a network containing five resistors. There are two loops and the respective loop currents are $I_1$ and $I_2$. Applying KVL in loop 1, we have

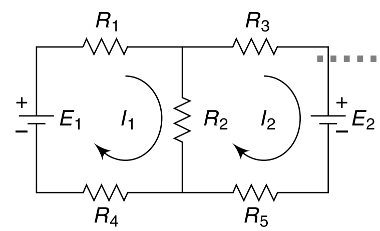



**Fig. 1.42** ***Illustration of mesh analysis***

$$-E_1 + I_1R_1 + (I_1 - I_2)R_2 + I_1R_4 = 0$$

or $E_1 = I_1(R_1 + R_2 + R_4) - I_2R_2$ (i)

Applying KVL in loop 2, we get

$$E_2 + I_2R_3 + (I_2 - I_1)R_2 + I_2R_5 = 0$$

or $E_2 = I_1R_2 - (R_2 + R_3 + R_5)I_2.$ (ii)

Solving equations ($x$) and ($y$), we can find the values of $I_1$ and $I_2$ and subsequently branch currents can be evaluated.

### 1.8.1 DC Circuit Analysis Procedure using Loop Equations

1. Convert all current sources to voltage sources.
2. Draw all loop currents in a clockwise direction and identify them.
3. Identify all resistor voltage drops as + to – in the direction of the loop current and assume these drops to be positive.
4. Identify all voltage sources according to their correct polarity.
5. Write the equations for the voltage drops around each loop in turn, by equating the sum of the voltage drops to zero.
6. Solve the equations to find the unknown currents and/or voltage drops.

**1.28** Calculate the current supplied by the battery in Fig. 1.43 using loop current method.

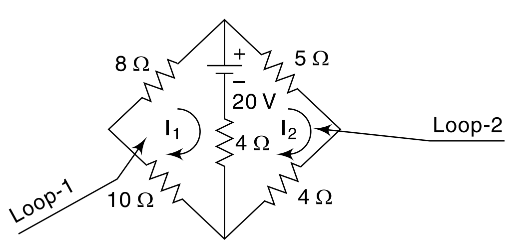



**Fig. 1.43** ***Circuit of Ex. 1.28***

Applying KVL in loop-1

$$8I_1 + 20 + (I_1 - I_2)\,4 + 10I_1 = 0$$

or $22I_1 - 4I_2 = -20$

or $11I_1 - 2I_2 = -10$

or $$I_2 = \frac{11I_1 + 10}{2} \qquad \text{(i)}$$

Applying KVL in loop-2,

$$5I_2 + 4I_2 + (I_2 - I_1)\,4 - 20 = 0$$

or $-4I_1 + 13I_2 = 20$ (ii)

Substituting the value of $I_2$ from equation (i) in equation (ii), we get

$$-4I_1 + 13 \times \frac{10 + 11I_1}{2} - 20 = 0$$

or $I_1 = -0.667$ A

Also, $$I_2 = \frac{10 + 11\,(-0.667)}{2} = 1.33 \text{ A.}$$

Hence the current supplied by the battery is obtained as $(I_2 - I_1)$, i.e, 1.33 + 0.667 = 1.997 A.

**1.29** Find the currents in 2 Ω, 3 Ω, 4 Ω, 5 Ω and 10 Ω resistances in the circuit shown in Fig. 1.44 using loop method.

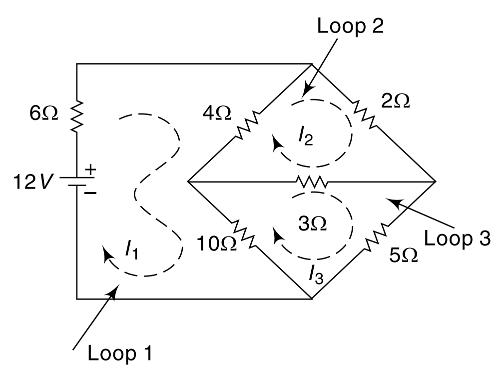



**Fig. 1.44** ***Circuit of Ex. 1.29***

***Solution***

Let us first mark the loop current in Fig. 1.44 as shown by dotted arrows.

For loop 1 we can write,

$-12 + 6I_1 + (I_1 - I_2)\,4 + (I_1 - I_3)\,10 = 0$

or $10I_1 - 2I_2 - 5I_3 = 6.$ (i)

Applying mesh method in loop 2, we have

$$2I_2 + (I_2 - I_3)\,3 + (I_2 - I_1)\,4 = 0$$

or $4I_1 - 9I_2 + 3I_3 = 0.$ (ii)

Applying mesh method in loop 3, we have

$$5I_3 + (I_3 - I_1)\,10 + (I_3 - I_2)\,3 = 0$$

or $10I_1 + 3I_2 - 18I_3 = 0.$ (iii)

Comparing equations (i) and (iii), we get

$$5I_2 - 13I_3 = -6$$

∴ $$I_2 = \frac{13I_3 - 6}{5} \qquad \text{(iv)}$$

Again, from equation (ii) we can write

$$I_1 = \frac{9I_2 - 3I_3}{4}.$$

Substituting this value of $I_1$ in equation (i), we get

$$10 \times \frac{9I_2 - 3I_3}{4} - 2I_2 - 5I_3 = 6.$$

Simplifying,

$$I_2 = \left(6 + \frac{50}{4} I_3\right) \frac{4}{82}. \qquad \text{(v)}$$

From equations (iv) and (v), we have

$$\frac{13I_3 - 6}{5} = \frac{4}{82}\left(6 + \frac{50}{4} I_3\right)$$

or $$82\,(13I_3 - 6) = 20\left(6 + \frac{50}{4} I_3\right)$$

or $$I_3 = \frac{612}{816} = 0.75 \text{ A}.$$

∴ From equation (iv) we now can write

$$I_2 = \frac{13 \times 0.75 - 6}{5} = 0.75 \text{ A}$$

Also from equation (i), we can write

$$10I_1 = 2 \times 0.75 + 5 \times 0.75 + 6 = 11.25$$

∴ $$I_1 = 1.125 \text{ A}.$$

Thus current in the 2 Ω and 5 Ω resistors is 0.75 A each;

current in the 4 Ω resistor is $(I_1 - I_2)$ i.e., 0.375 A,

current in the 10 Ω resistor is $(I_1 - I_3)$ i.e., 0.375 A,

and current in the 3 Ω resistor is $(I_2 - I_3)$ i.e., 0 A.

**1.30** From the mesh analysis find the current flow through a 50 V source in Fig. 1.45.

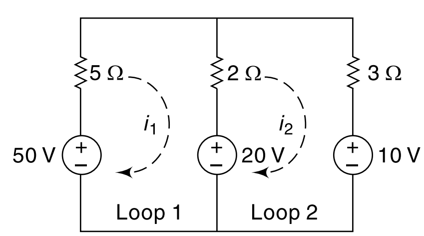



**Fig. 1.45** ***Circuit of Ex. 1.30***

### *Solution*

Let us designate the loop currents by dotted arrows in the network of Fig. 1.47.

In loop 1 we have

$$-50 + 5i_1 + (i_1 - i_2)\,2 + 20 = 0$$

or $$7i_1 - 2i_2 - 30 = 0 \qquad \text{(i)}$$

In loop 2 we can write,

$$3i_2 + 10 - 20 + (i_2 - i_1)\,2 = 0$$

or
$$i_1 = \frac{5}{2}i_2 - 5 \qquad \text{(ii)}$$

Substituting the value of $i_1$ from equation (ii) to equation (i), we get

$$7\left(\frac{5}{2}i_2 - 5\right) - 2i_2 - 30 = 0$$

Simplification yields,

$$i_2 = 4.19 \text{ A}$$

Thus from equation (ii) we get

$$i_1 = \frac{5}{2} \times 4.19 - 5 = 5.475 \text{ A.}$$

The current through the 50 V source is thus 5.475 A.

**1.31** Find the voltage drop between terminals ($y$) and ($d$) in the network of Fig. 1.46.

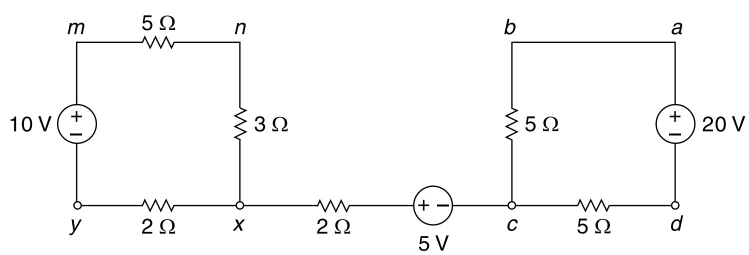



**Fig. 1.46** ***Circuit of Ex. 1.31***

***Solution***

The current supplied by the 10 V source in the loop-*mnxy* is obtained as

$$i_1 = \frac{10}{5 + 3 + 2} = 1 \text{ A.}$$

The current supplied by the 20 V source in the loop-*badc* is given by

$$i_2 = \frac{20}{5 + 5} = 2 \text{ A.}$$

The corresponding drops with polarities are shown in Fig. 1.46(a).

∴ $$V_{yd} = -V_{yx} + 5V + V_{cd} = -2 + 5 + 10 = 13 \text{ V.}$$

The drop across terminal ($y$) and ($d$) is 13 V.

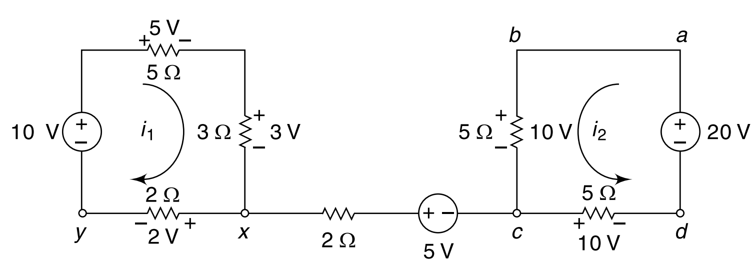



**Fig. 1.46(a)** ***Currents and voltages for the circuit of Ex. 1.31***

**1.32** Find the current through the resistors using mesh method for the network shown in Fig. 1.47.

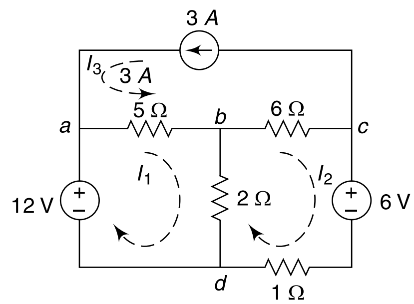



**Fig. 1.47** *Circuit of Ex. 1.32*

***Solution***

Let us first draw the loop currents in the network of Fig. 1.47. The loop currents are shown by dotted arrows. It may be noted that due to presence of current source of 3 A, the corresponding loop current $I_3$ is 3 A.

In the loop containing 12 V source, we have

$$-12 + (I_1 + 3)5 + (I_1 - I_2)2 = 0$$

or $$7I_1 - 2I_2 + 3 = 0. \quad \text{(i)}$$

Applying mesh analysis in the loop containing 6*V* source, we get

$$I_2\, 1 + (I_2 - I_1)\, 2 + (I_2 + 3)\, 6 + 6 = 0$$

or $$I_1 = \frac{9}{2} \cdot I_2 + 12. \quad \text{(ii)}$$

Substituting the value of $I_1$ from equation (ii) in (i), we get

$$7\left(\frac{9}{2} I_2 + 12\right) - 2I_2 + 3 = 0$$

or $$I_2 = -2.95 \text{ A}$$

Thus from equation (ii), we get

$$I_1 = \frac{9}{2} \times (-2.95) + 12 = -1.275 \text{ A}$$

We now can find currents in respective resistors:

Current through 5 Ω resistor $(= I_1 + I_3) = -1.275 + 3 = 1.725$ A.

[It may be noted that the current obtained through the 5 Ω resistor is directed from *a* to *b*].

Current through the 6 Ω resistor $(= I_2 + I_3) = -2.95 + 3 = 0.05$ A.

[This current is directed from *b* to *c*].

Current through the 2 Ω resistor $(= I_1 + I_2) = -1.275 + 2.95 = 1.675$ A.

[The current through the 2 Ω resistor is directed from *b* to *d*].

Finally, the current through the 1 Ω resistor ($I_2$) is (–2.95 *A*) and is directed from *d* to *c*.

**1.33** In the bridge network shown in Fig. 1.48 find the current through the galvanometer having 20 Ω series resistance. Use mesh analysis.

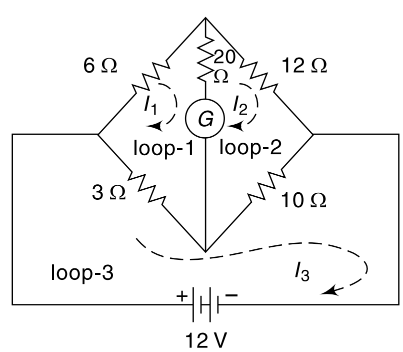



**Fig. 1.48** *Circuit of Ex. 1.33*

***Solution***

We first assign loops as *loop 1*, *loop 2* and *loop 3* with circulating currents $I_1$, $I_2$ and $I_3$ through these loops (Fig. 1.48).

In loop 1 we have

$$6\, I_1 + (I_1 - I_2)\, 20 + (I_1 - I_3)\, 3 = 0$$

or $$29\, I_1 - 20\, I_2 - 3\, I_3 = 0. \quad \text{(i)}$$

In loop 2 we have

$$12\,I_2 + (I_2 - I_3)\,10 + (I_2 - I_1)\,20 = 0$$

or $$-20\,I_1 + 42\,I_2 - 10I_3 = 0. \quad \text{(ii)}$$

Similarly, in loop 3 we can write

$$-12 + (I_3 - I_1)\,3 + (I_3 - I_2)\,10 = 0$$

or $$-3\,I_1 - 10\,I_2 + 13\,I_3 - 12 = 0. \quad \text{(iii)}$$

Let us now solve these three simultaneous equations.

From equation (i), $$I_1 = \frac{20I_2 + 3I_3}{29} \quad \text{(iv)}$$

and from equation (ii), $$I_1 = \frac{42I_2 - 10I_3}{20} \quad \text{(v)}$$

Comparing equation (iv) and (v) we get

$$\frac{20I_2 + 3I_3}{29} = \frac{42I_2 - 10I_3}{20}$$

or $$818\,I_2 - 350\,I_3 = 0 \quad \text{(vi)}$$

Again, from equation (iii) we find

$$I_1 = \frac{-\,10I_2 + 13I_3 - 12}{3} \quad \text{(vii)}$$

Comparing equation (v) with equation (vii) we get

$$\frac{-\,10I_2 + 13I_3 - 12}{3} = \frac{42I_2 - 10I_3}{20}$$

or $$326\,I_2 - 290\,I_3 = -240 \quad \text{(viii)}$$

From equation (vi) we find $I_2 = (350\,I_3/818)$; substitution of value of $I_2$ in equation (viii) yields

$$326 \times \frac{350}{818} \cdot I_3 - 290\,I_3 = -240$$

or $$I_3 = 1.59 \text{ A}.$$

From (vi), $I_2$ can be found as $I_2 = (350/818) \times 1.59$

i.e., $$I_2 = 0.68\,A$$

From (vii) we can find the value of $I_1$;

$$I_1 = \frac{-\,10 \times 0.68 + 13 \times 1.59 - 12}{3} = 0.633 \text{ A}.$$

The current $(I_2 - I_1)$ through the galvanometer is then obtained. Obviously,

$$(I_2 - I_1) = I_G = 0.68 - 0.633 = 0.047 \text{ A (directed upwards)}.$$

**1.34** Find current in all branches of the network shown in Fig. 1.49.

***Solution***

Let the current in the arm *AF* be *I* amps, as shown by dotted arrow. Using the concept of KCL, the currents at each of the branches have been identified in Fig. 1.51 in terms of the assumed current *I*. Next we apply the mesh analysis at the hexagonal network *AFEDCBA*. We have

$$0.02(I) + 0.01(I - 60) + 0.03(I) + 0.01(I - 120) + 0.01(I - 50) + 0.02(I - 80) = 0.$$

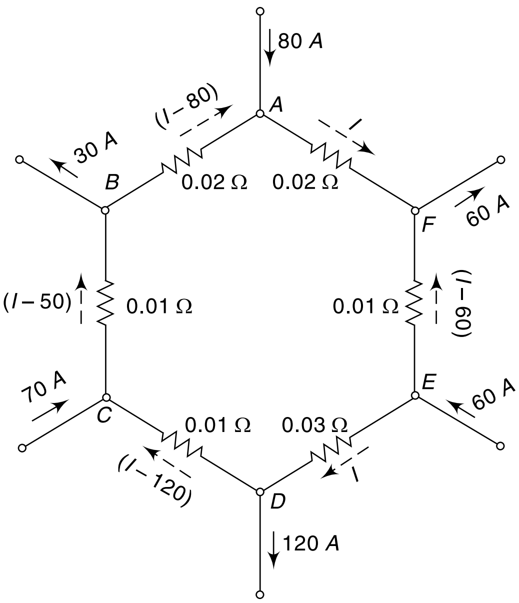



**Fig. 1.49** ***Circuit of Ex. 1.34***

Solving for $I$, we get $I = 39$ A.

Thus we can identify the branch currents as

current in $AF = I_{AF} = 39\ A\ (=I)$

current in $FE = (I - 60) = -21$ A

current in $ED = I_{ED} = I = 39$ A

current in $DC = I_{DC} = (I - 120) = -81$ A

current in $CB = I_{CB} = (I - 50) = -11$ A

current in $BA = I_{BA} = (I - 80) = -41$ A.

## 1.8.2 Mesh Analysis Using Matrix Form

Let us consider the network shown in Fig. 1.50; it contains three meshes. The three mesh currents are $I_1$, $I_2$ and $I_3$ and they are assumed to flow in a clockwise direction.

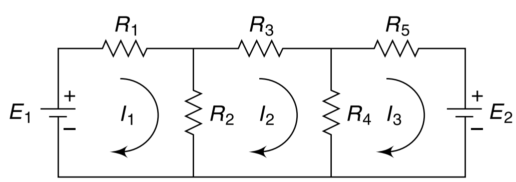



**Fig. 1.50** ***Concept of mesh analysis in matrix form***

Applying KVL to mesh 1,

$$-E_1 + (I_1 - I_2)R_2 + I_1R_1 = 0$$

or $$I_1R_1 + (I_1 - I_2)R_2 = E_1$$

or $$I_1(R_1 + R_2) - I_2R_2 = E_1$$

or $$I_1(R_1 + R_2) + I_2(-R_2) = E_1. \tag{1.21}$$

Applying KVL to mesh 2,

$$(I_2 - I_1)R_2 + I_2R_3 + (I_2 - I_3)R_4 = 0$$

or $$-I_1R_2 + I_2(R_2 + R_3 + R_4) - I_3R_4 = 0$$

or $$I_1(-R_2) + I_2(R_2 + R_3 + R_4) + I_3(-R_4) = 0 \tag{1.22}$$

Applying KVL to mesh 3,

$$E_2 + I_3R_5 + (I_3 - I_2)R_4 = 0$$

or $$-I_2R_4 + I_3(R_4 + R_5) = -E_2. \tag{1.23}$$

It should be noted that the signs of resistances in the above equations have been so arranged as to make the items containing self-resistances positive. The matrix equivalent of the above three equations is

$$\begin{bmatrix} R_1 + R_2 & -R_2 & 0 \\ -R_2 & R_2 + R_3 + R_4 & -R_4 \\ 0 & -R_4 & R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ 0 \\ -E_2 \end{bmatrix}$$

In general the resistance matrix $[R]$ can be written as

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{33} & R_{33} \end{bmatrix}$$

where $R_{11}$ = *self-resistance* of mesh 1 = $R_1 + R_2$

$R_{22}$ = *self-resistance* of mesh 2 = $R_2 + R_3 + R_4$

$R_{33}$ = *self-resistance* of mesh 3 = $R_4 + R_5$

$R_{12} = R_{21}$

= – [sum of all the resistances common to meshes 1 and 2]

$= -R_2$

$R_{23} = R_{32}$

= – [sum of all the resistances common to meshes 2 and 3]

$= -R_4$

$R_{13} = R_{31}$

= – [sum of all the resistances common to meshes 3 and 1]

= 0 (here).

[$R_{11}$, $R_{22}$, $R_{33}$ … are called *diagonal elements* of the resistance matrix while $R_{12}$, $R_{13}$, $R_{21}$, $R_{23}$, … are called *off-diagonal elements*.]

**1.35** Find the mesh currents in Fig. 1.51 using mesh current method.

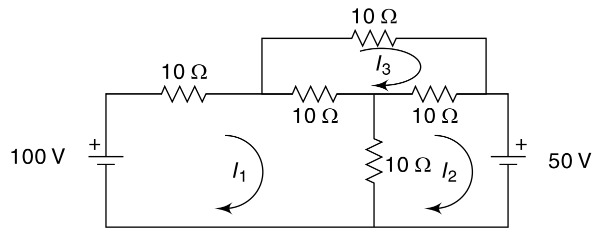



**Fig. 1.51** ***Circuit of Ex. 1.35***

***Solution***

Applying KVL in loop 1

$$-100 + 10(I_1 - I_2) + 10(I_1 - I_3) + 10I_1 = 0$$

or $$I_1(10 + 10 + 10) - 10I_2 - 10I_3 = 100$$

Applying KVL in loop 2

$$50 + 10(I_2 - I_3) + 10(I_2 - I_1) = 0$$

or $$-10I_1 + I_2(10 + 10) - 10I_3 = -50$$

Applying KVL in loop 3

$$10I_3 + 10(I_3 - I_1) + 10(I_3 - I_2) = 0$$

or $$-10I_1 - 10I_2 + I_3(10 + 10 + 10) = 0$$

The above equations in matrix form can be written as

$$\begin{bmatrix} 30 & -10 & -10 \\ -10 & 20 & -10 \\ -10 & -10 & 30 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ -50 \\ 0 \end{bmatrix}$$

Hence $$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30 & -10 & -10 \\ -10 & 20 & -10 \\ -10 & -10 & 30 \end{bmatrix}^{-1} \begin{bmatrix} 100 \\ -50 \\ 0 \end{bmatrix}$$

or $$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \frac{1}{8000} \begin{bmatrix} 500 & 400 & 300 \\ 400 & 800 & 400 \\ 300 & 400 & 500 \end{bmatrix} \begin{bmatrix} 100 \\ -50 \\ 0 \end{bmatrix}$$

Therefore, $$I_1 = \frac{500 \times 100 - 400 \times 50}{8000} \text{ A} = 3.75 \text{ A}$$

$$I_2 = \frac{400 \times 100 - 50 \times 800}{8000} \text{ A} = 0 \text{ A}$$

and $$I_3 = \frac{300 \times 100 - 400 \times 50}{8000} \text{ A} = 1.25 \text{ A}$$

**1.36** Find the ammeter current in Fig. 1.52 using mesh analysis.

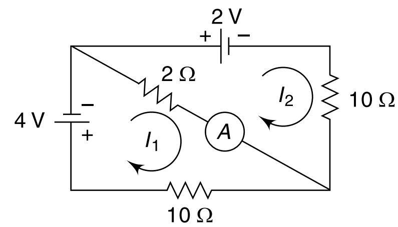



**Fig. 1.52** *Circuit of Ex. 1.36*

***Solution***

Applying mesh method in mesh 1

$$4 + 10I_1 + 2(I_1 - I_2) = 0$$

or $$12I_1 - 2I_2 = -4.$$

Applying mesh method in mesh 2

$$2 + 2(I_2 - I_1) + 10I_2 = 0$$

$$-2I_1 + 12I_2 = -2.$$

In the matrix form the above equations can be written as

$$\begin{bmatrix} 12 & -2 \\ -2 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$

or $$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 & -2 \\ -2 & 12 \end{bmatrix}^{-1} \begin{bmatrix} -4 \\ -2 \end{bmatrix} = \frac{1}{144 - 4} \begin{bmatrix} 12 & 2 \\ 2 & 12 \end{bmatrix} \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$

or $$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{140} \begin{bmatrix} -52 \\ -32 \end{bmatrix}$$

Hence $$I_1 = \frac{-52}{140} \text{ A} \quad \text{and} \quad I_2 = -\frac{32}{140} \text{ A}$$

Therefore, the current through ammeter is

$I_2 - I_1 = \frac{52}{140} - \frac{32}{140} = \frac{1}{7}$ A [in the direction of ($I_2$) as shown in Fig. 1.54].

## 1.9 STAR DELTA CONVERSION

Like series and parallel connections the resistances may be connected in *star* ($Y$) or *delta* ($\Delta$) connection as shown in Fig. 1.53(a) and Fig. 1.53(b).

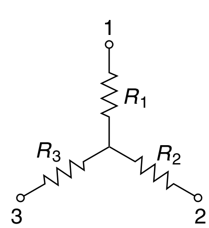



**Fig. 1.53(a)** *A star (or* **T***) connection*

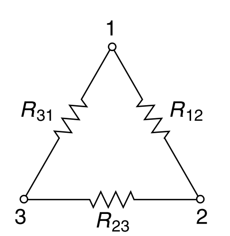



**Fig. 1.53(b)** *A delta (or mesh) connection*

Circuits shown in Fig. 1.53(a) and Fig. 1.53(b) are identical provided their respective resistances from terminals (12), (23) and (32) are equal.

In star connection,

Resistance between terminals 1 and 2 is $(R_1 + R_2)$

Resistance between terminals 2 and 3 is $(R_2 + R_3)$

Resistance between terminals 3 and 1 is $(R_3 + R_1)$.

Similarly in delta connection,

Resistance between terminals 1 and 2 is $[R_{12} \parallel (R_{23} + R_{31})]$

$$= \frac{R_{12}\,(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

Resistance between terminals 2 and 3 is $[R_{23} \parallel (R_{31} + R_{12})]$

$$= \frac{R_{23}\,(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}}$$

Resistance between terminals 3 and 1 is $[R_{31} \parallel (R_{12} + R_{23})]$

$$= \frac{R_{31}\,(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}}$$

Now, we equate the resistances in star and delta across appropriate terminals.

i.e. $$R_1 + R_2 = \frac{R_{12}\,(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \tag{1.24}$$

$$R_2 + R_3 = \frac{R_{23}\,(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \tag{1.25}$$

$$R_3 + R_1 = \frac{R_{31}\,(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \tag{1.26}$$

Subtracting equation (1.25) from equation (1.24) we get

$$R_1 - R_3 = \frac{R_{12}\,(R_{23} + R_{31}) - R_{23}\,(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}}$$

$$= \frac{R_{12}\,R_{31} - R_{23}\,R_{31}}{R_{12} + R_{23} + R_{31}} \tag{1.27}$$

Adding equations (1.26) and (1.27)

$$2R_1 = \frac{2\,R_{12}\,R_{31}}{R_{12} + R_{23} + R_{31}}$$

or $$R_1 = \frac{R_{12}\,R_{31}}{R_{12} + R_{23} + R_{31}}$$

In a similar way, $R_2 = \dfrac{R_{12}\,R_{23}}{R_{12} + R_{23} + R_{31}}$ and

$$R_3 = \frac{R_{31}\,R_{23}}{R_{12} + R_{23} + R_{31}}$$

Thus we see that if the resistances in delta connected resistance network are known, we can find the equivalent star network where

$$R_1 = \frac{R_{31}\,R_{12}}{R_{12} + R_{23} + R_{31}} \qquad 1.28(a)$$

$$R_2 = \frac{R_{12}\,R_{23}}{R_{12} + R_{23} + R_{31}} \qquad 1.28(b)$$

$$R_3 = \frac{R_{23}\,R_{31}}{R_{12} + R_{23} + R_{31}} \qquad 1.28(c)$$

$R_1$, $R_2$ and $R_3$ being equivalent resistances in the star network and $R_{12}$, $R_{23}$ and $R_{31}$ the resistances in the delta network.

Next, multiplying each equation [1.28(a), 1.28(b) and 1.28(c)] with another and adding

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{31}R_{12}^2R_{23} + R_{12}R_{23}^2R_{31} + R_{23}R_{31}^2R_{12}}{(R_{12} + R_{23} + R_{31})^2} \qquad (1.29)$$

Dividing equation (1.29) by ($R_1$), we get

$$R_2 + \frac{R_2R_3}{R_1} + R_3 = \frac{R_{12}R_{23}R_{31}(R_{12} + R_{23} + R_{31})}{R_1(R_{12} + R_{23} + R_{31})^2}$$

$$= \frac{R_{12}\,R_{23}\,R_{31}}{R_1(R_{12} + R_{23} + R_{31})^2} \qquad (1.30)$$

Substituting the value of $R_1$ from equation (1.28(a)) in equation (1.30) we get

$$R_2 + R_3 + \frac{R_2R_3}{R_1} = \frac{R_{12}R_{23}R_{31}(R_{12} + R_{23} + R_{31})}{R_{31}R_{12}(R_{12} + R_{23} + R_{31})}$$

$$= \frac{R_{12}R_{23}R_{31}}{R_{31} + R_{12}} = R_{23}$$

i.e.,

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}.$$

Similarly, dividing equation (1.29) by $R_2$ we get

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

and dividing equation (1.29) by ($R_3$) we get

$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}.$$

Thus we find $R_{12}$, $R_{23}$ and $R_{31}$, i.e. the equivalent delta network provided $R_1$, $R_2$ and $R_3$ of the star network are given.

The equations are

$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \quad [1.31(a)]$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \quad [1.31(b)]$$

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \quad [1.31(c)]$$

### 1.9.1 Delta-Star (Δ-Y) and Star-Delta (Y-Δ) Transformation Procedures

1. When starting with a Δ network, draw a *Y* network; when starting with a *Y* network, draw a Δ network.
2. Identify the three corresponding terminals on each network as 1, 2 and 3.
3. Identify the resistors on the Δ network as follows:

   Resistor between terminals 1 and 2 as ($R_{12}$)

   Resistor between terminals 1 and 3 as ($R_{13}$)

   Resistor between terminals 2 and 3 as ($R_{23}$).
4. Identify the resistors on the *Y* network as follows:

   Resistor connected to terminal 1 as ($R_1$)

   Resistor connected to terminal 2 as ($R_2$)

   Resistor connected to terminal 3 as ($R_3$).
5. For Δ to *Y* transformation, substitute the Δ network resistor values into equations 1.28(a), 1.28(b) and 1.28(c) to obtain the *Y* network resistor values.
6. For *Y* to Δ transformation, substitute the *Y* network resistor values into equations 1.31(a), 1.31(b) and 1.31(c) to obtain the Δ network resistor values.

   [A *Y* network is also called as *T*(*Te*) network while a Δ network may be called as a mesh or $\pi$(pi) network].

**1.37** Convert the $\pi$ network shown in Fig. 1.54 into equivalent *T* network.

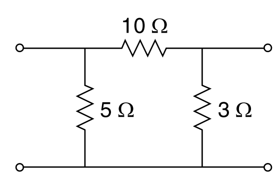



**Fig. 1.54** ***Circuit of Ex. 1.37***

***Solution***

The network in Fig. 1.56 can be redrawn as shown in Fig. 1.56(a), Here $R_1$, $R_2$ and $R_3$ in star combination represent the equivalent of the given delta network.

$$R_1 = \frac{R_{12} \times R_{31}}{R_{12} + R_{23} + R_{31}} = \frac{10 \times 5}{10 + 3 + 5} = 2.78\ \Omega$$

$$R_2 = \frac{R_{23} \times R_{12}}{R_{12} + R_{23} + R_{31}} = \frac{3 \times 10}{10 + 3 + 5} = 1.67\ \Omega$$

$$R_3 = \frac{R_{23} \times R_{31}}{R_{12} + R_{23} + R_{31}} = \frac{3 \times 5}{10 + 3 + 5} = 0.83\ \Omega$$

Thus we have obtained the equivalent star (or $T$) resistances given by

$R_1 = 2.78\ \Omega$; $R_2 = 1.67\ \Omega$; $R_3 = 0.83\ \Omega$.

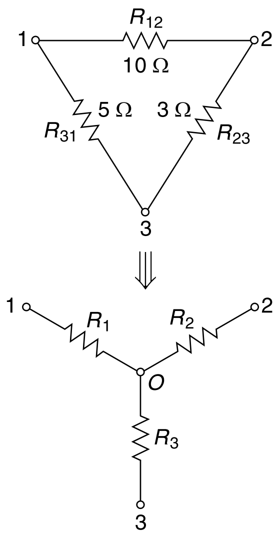



**Fig. 1.54(a)** *Δ-Y conversion of the given network*

**1.38** Find the input resistance ($R$) of the network shown in Fig. 1.55.

***Solution***

Converting the upper delta network of Fig. 1.55 into a star network [Fig. 1.55(a)] we obtain the arm impedances of the equivalent star network as

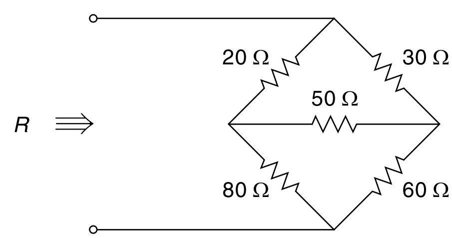



**Fig. 1.55** *Circuit of Ex. 1.38*

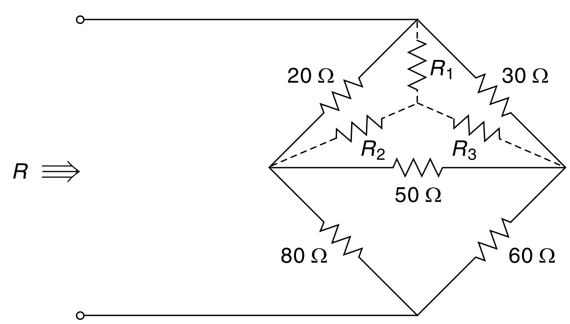



**Fig. 1.55(a)** *Conversion of upper delta network to equivalent star for the network of Ex. 1.40*

$$R_1 = \frac{20 \times 30}{20 + 30 + 50} = 6\ \Omega$$

$$R_2 = \frac{20 \times 50}{20 + 30 + 50} = 10\ \Omega$$

$$R_3 = \frac{30 \times 50}{20 + 30 + 50} = 15\ \Omega.$$

Next we further reorient the network as shown in Fig. 1.55(b).

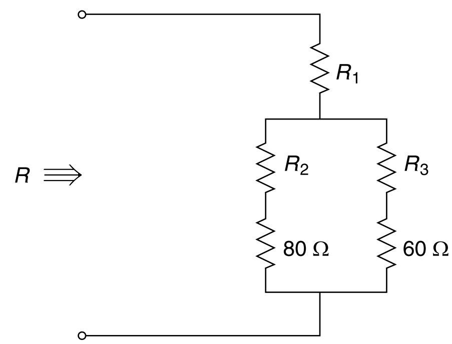



**Fig. 1.55(b)** ***Simplified equivalent of network of Fig. 1.55(a)***

Here,
$$R = 6 + \frac{(10 + 80) \times (15 + 60)}{(10 + 80) + (15 + 60)}$$
$$= 6 + \frac{90 \times 75}{90 + 75}$$
$$= 46.9\ \Omega.$$

Thus, the equivalent resistance of the network given in Fig. 1.55 is 46.9 Ω.

**1.39** Using star-delta conversion, find the equivalent resistance between terminals *A* and *B* in the network shown in Fig. 1.56.

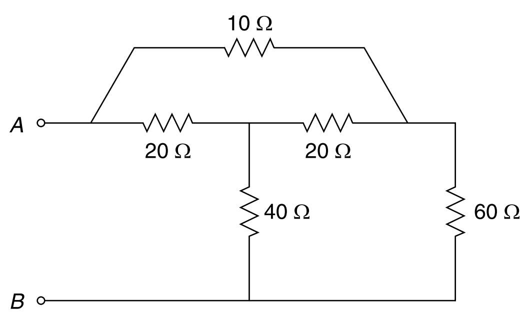



**Fig. 1.56** ***Circuit of Ex. 1.39***

### *Solution*

Let us first convert the star connected network using 20 Ω, 20 Ω and 40 Ω resistors to an equivalent delta network [Ref. Fig. 1.56(a)].

Here,
$$R_1 = \frac{20 \times 40 + 20 \times 40 + 20 \times 20}{20} = 100\ \Omega$$
$$R_2 = \frac{20 \times 40 + 20 \times 40 + 20 \times 20}{20} = 100\ \Omega$$
$$R_3 = \frac{20 \times 40 + 20 \times 40 + 20 \times 20}{40} = 50\ \Omega.$$

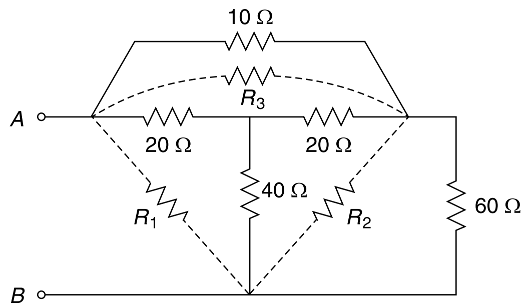



**Fig. 1.56(a)** ***Equivalent Δ network of a part of circuit of Fig. 1.56***

Figure 1.56(b) is the final form of the given network as reduced in Fig. 1.56(a).

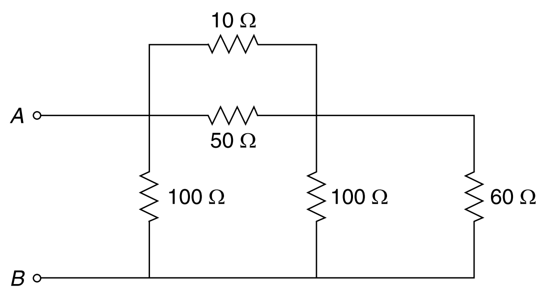



**Fig. 1.56(b)** ***Simplified equivalent of network of Ex. 1.39***

Thus the equivalent resistance between terminals *A* and *B* is obtained as

$$R_{eq} = 100 \,||\, \frac{50 \times 10}{50 + 10} + \frac{100 \times 60}{100 + 60}$$

$$= 100 \,||\, \left(\frac{50}{6} + \frac{600}{16}\right)$$

$$= 100 \,||\, 45.83$$

$$= \frac{100 \times 45.83}{100 + 45.83} = 31.43\ \Omega.$$

The equivalent resistance across terminals *A* and *B* is thus 31.43 Ω.

**1.40** Find the resistance across terminals *AB* for the circuit shown in Fig. 1.57.

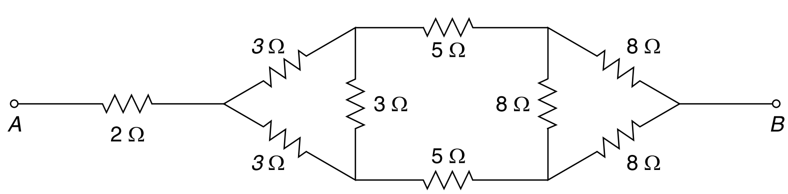



**Fig. 1.57** ***Circuit of Ex. 1.40***

***Solution***

We convert the two delta networks formed in the given circuit to equivalent star networks as shown in Fig. 1.57(a).

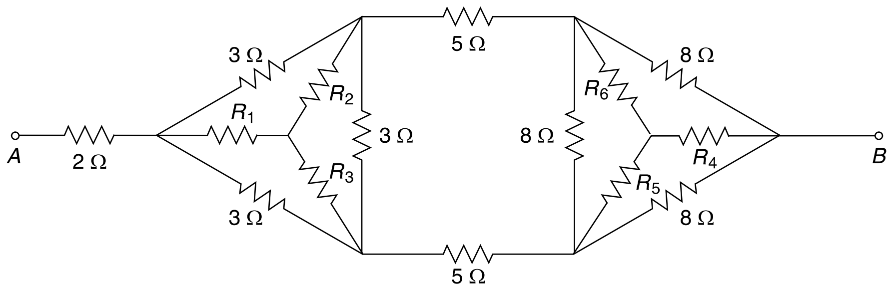



**Fig. 1.57(a)** ***Formation of equivalent stars for the given network in Ex. 1.40***

We find, $R_1 = R_2 = R_3 = \dfrac{3 \times 3}{3 + 3 + 3} = 1\ \Omega$

and $R_4 = R_5 = R_6 = \dfrac{8 \times 8}{8 + 8 + 8} = 2.67\ \Omega.$

The equivalent resistance between terminals *CD* (Fig. 1.57(b)) can be obtained by redrawing Fig. 1.57(a) as Fig. 1.57(b).

$$R_{CD} = (R_2 + 5 + R_6) \,||\, (R_3 + 5 + R_5)$$
$$= (1 + 5 + 2.67) \,||\, (1 + 5 + 2.67) = 4.335\ \Omega$$

Hence the resistance between terminals *AB* of the given network is

$$R = 2 + R_1 + 4.335 + R_4 = 2 + 1 + 4.335 + 2.67$$
$$= 10\ \Omega.$$

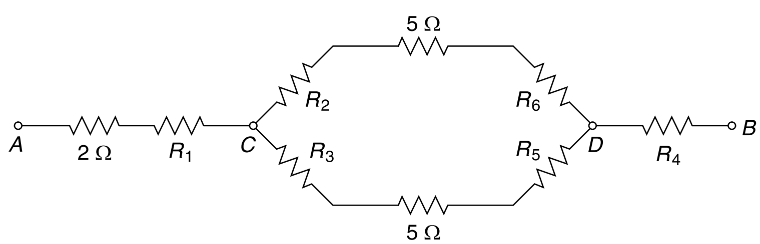



**Fig. 1.57(b)** ***Equivalent network of the circuit shown in Fig. 1.57(a)***

**1.41** Determine the resistance between points *A* and *B* in the network shown in Fig. 1.58.

***Solution***

Figure 1.60(a) is drawn to represents a star equivalent to the part of the given network containing resistaces 3 Ω, 2 Ω and 5 Ω.

In Fig. 1.60(a),

$$R_1 = \frac{3 \times 2}{3 + 2 + 5} = 0.6\ \Omega$$

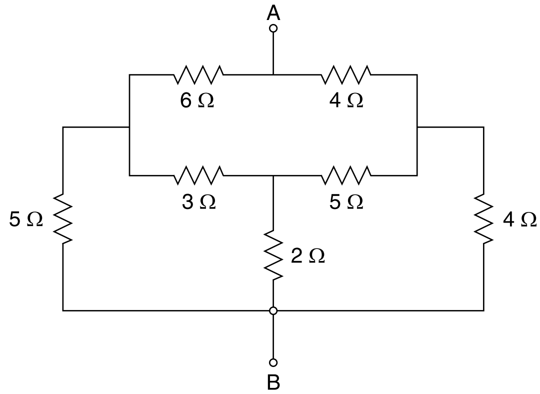



**Fig. 1.58** ***Circuit of Ex. 1.41***

$$R_2 = \frac{2 \times 5}{5 + 2 + 3} = 1\ \Omega$$

$$R_3 = \frac{3 \times 5}{3 + 5 + 2} = 1.5\ \Omega.$$

We redraw Fig. 1.58(a) as Fig. 1.58(b) and the circuit is further reduced to Fig. 1.58(c). We draw a star equivalent for the delta connected resistances 5.6 Ω, 4 Ω and 1 Ω in Fig. 1.58(c).

Here,
$$R_4 = \frac{1 \times 4}{1 + 4 + 5.6} = 0.377\ \Omega$$

$$R_5 = \frac{1 \times 5.6}{1 + 4 + 5.6} = 0.528\ \Omega$$

$$R_6 = \frac{4 \times 5.6}{1 + 4 + 5.6} = 2.11\ \Omega.$$

The final configuration of the given network is shown in Fig. 1.58(d). The resistance between terminals *AB* is then obtained as

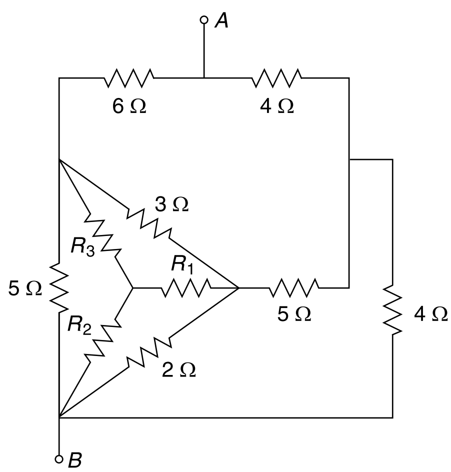



**Fig. 1.58(a)** ***Formation of star equivalent for a portion of network shown in Fig. 1.58***

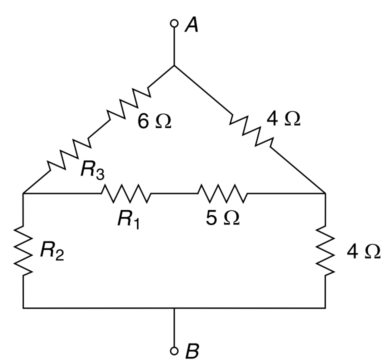



**Fig. 1.58(b)** ***Reduction of circuit shown in Fig. 1.58(a)***

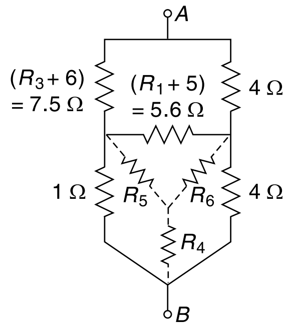



**Fig. 1.58(c)** ***Further simplification of circuit shown in Fig. 1.58(c)***

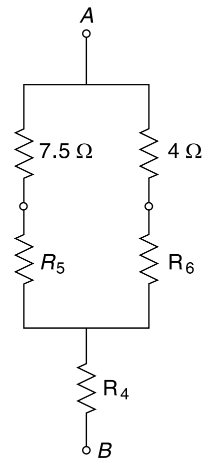



**Fig. 1.58(d)** ***Final simplified equivalent circuit of Ex. 1.41***

$$R_{AB} = \{(7.5 + 0.528) \parallel (4 + 2.11)\} + 0.377 = \frac{8.028 \times 6.11}{8.028 + 6.11} + 0.377 = 3.84\ \Omega.$$

The equivalent resistance across *AB* is then 3.84 Ω.

## 1.10 VOLTAGE SOURCES AND CURRENT SOURCES

A network can sometimes be simplified by converting *voltage sources* to *current sources* and vice versa.

Voltage sources can be represented by an ideal voltage cell in series with the internal resistance of the cell or battery. The ideal cell is assumed to be a constant voltage source and the output current produces a voltage drop across the internal resistance. Figure 1.59 shows such a constant voltage source, with voltage *E*, source resistance $R_S$ and an external load resistance $R_L$. Using the voltage divider rule in Art. 2.45 the output voltage developed across $R_L$ can be determined.

$$V_L = E\frac{R_L}{R_S + R_L} \tag{1.32}$$

If $R_S$ $R_L$, then $V_L \cong E$

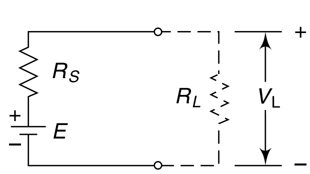



**Fig. 1.59** ***A constant voltage source with a load resistor***

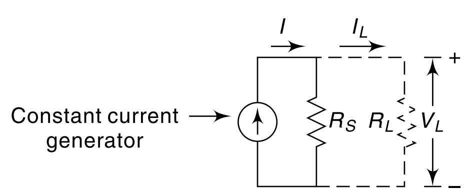



**Fig. 1.60** ***A constant current source with a load resistor***

When the load resistance is very much larger than the source resistance, the constant voltage source is assumed to have zero source resistance and all of the source voltage is assumed to be applied to the load.

Certain electronic devices can produce a current that tends to remain constant regardless of how the load resistance varies. Hence it is possible to have a constant current source. The circuit of constant current source is shown in Fig. 1.60 with its source resistance $R_S$ and a load resistance $R_L$.

Here $R_S$ is in parallel with the current source. Hence some current flows through $R_S$ and remaining through $R_L$. Using the current divider rule as shown in Art 1.4.3, the output current (or load current) from a constant current source can be determined in terms of $R_L$ and $R_S$:

$$I_L = I \frac{R_S}{R_S + R_L} \tag{1.33}$$

If $R_S$ $R_L$ then $I_L \cong I$.

Figure 1.61(a) and Fig. 1.61(b) show how a voltage source can be converted into an equivalent current source that will produce the same current level in a given load resistor.

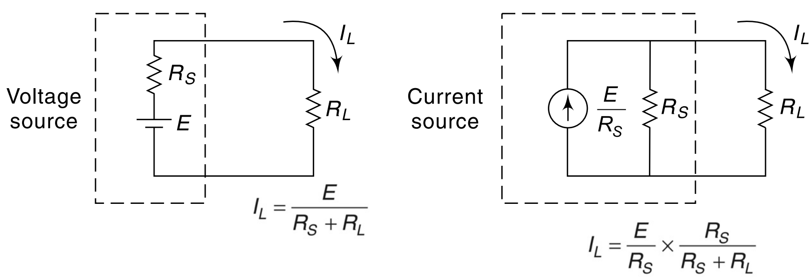



**Fig. 1.61(a) & (b)** ***Conversion of a constant voltage source to an equivalent constant current source***

When the load resistance is very much smaller than the source resistance, a constant current source is assumed to have an infinite source resistance, and all of the source current is assumed to flow through the load.

## 1.10.1 Source Conversion

According to source conversion technique a given voltage source with a series resistance can be converted into an equivalent current source with a parallel resistance (as explained in Art. 1.10). Similarly, a current source with a parallel resistance can be converted into a voltage source with a series resistance.

Here we explain again the conversion of the constant voltage source shown in Fig. 1.62 into an equivalent constant current source.

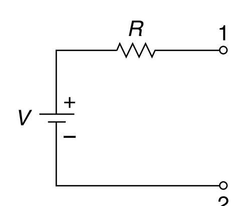



**Fig. 1.62** ***A constant voltage source with series resistor***

The current supplied by the constant voltage source when a short circuit is placed across terminals 1 & 2 is $I = V/R$.

A constant current source supplying this current $I$ and having the same resistance $R$ connected in parallel with it represents the equivalent current source as shown in Fig. 1.63.

Similarly, a constant current source of $I$ and a parallel resistance $R$ can be converted into a constant voltage source of voltage $V$ (= $IR$) and a resistance $R$ in series with it.

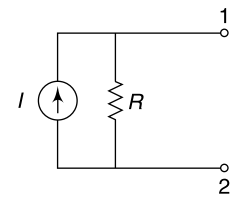



**Fig. 1.63** *Equivalent constant current source of the voltage source (V)*

**1.42** Convert the constant voltage source shown in Fig. 1.64 into equivalent current source.

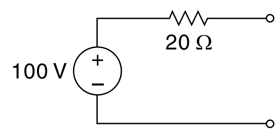



**Fig. 1.64** *Circuit of Ex. 1.42*

*Solution*

The current supplied by the 100 V source when a short circuit is placed across the output terminals is $I = 100/20 = 5$ A. So the value of the equivalent constant current source is 5 A; the equivalent circuit with current source is shown in Fig. 1.65.

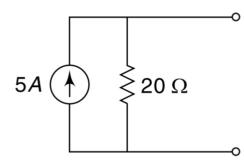



**Fig. 1.65** *Equivalent current source of the 100 V voltage source*

**1.43** Convert the constant current source of Fig. 1.66 into equivalent voltage source.

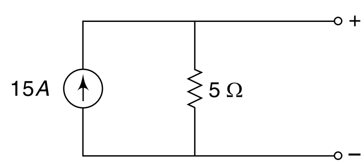



**Fig. 1.66** *Circuit of Ex. 1.43*

*Solution*

The value of the equivalent constant voltage source is given as $V$ (= $IR$) = $15 \times 5 = 75$ V.

The equivalent network with the voltage source is shown in Fig. 1.66(a).

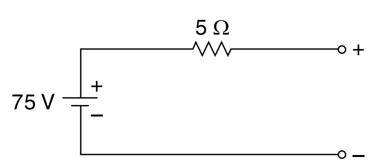



**Fig. 1.66(a)** *Equivalent voltage source of 15 A current source*

**1.44** Use source transformation technique to find the current through the 2 Ω resistor in Fig. 1.67.

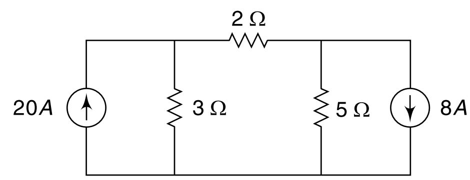



**Fig. 1.67** ***Circuit of Ex. 1.44***

***Solution***

Converting the two sources into equivalent voltage source, the network shown in Fig. 1.68 is obtained.

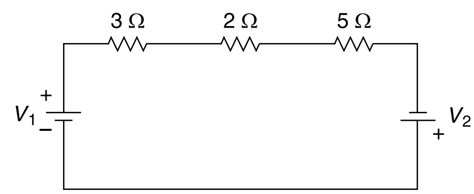



**Fig. 1.68** ***Current sources of Fig. 1.67 converted to voltage sources***

The values of $V_1$ and $V_2$ are $20 \times 3 = 60$ V and $8 \times 5 = 40$ V respectively.

As the voltage sources are series connected hence they deliver current in the same direction. Hence the current through 2 Ω resistor is

$$\frac{60 + 40}{3 + 2 + 5} = 10 \text{ A.}$$

**1.45.** By using source conversion technique find the value of voltage across $R_L$ where $R_L = 4\ \Omega$ in Fig. 1.69.

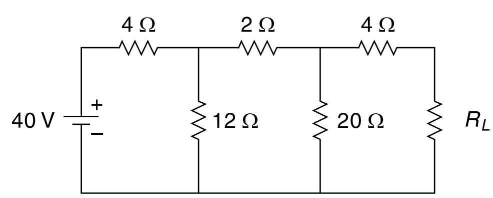



**Fig. 1.69** ***Circuit of Ex. 1.45***

***Solution***

Converting 40 V source into equivalent current source, in Fig. 1.69(a) the value of the current source is 40/4 = 10 A.

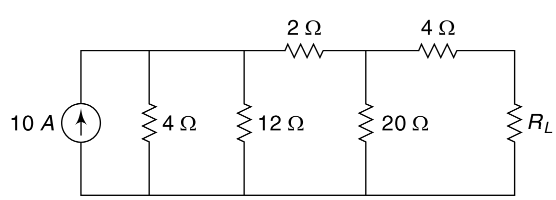



**Fig. 1.69(a)** ***Conversion of 40 V voltage source to equivalent current source***

The combination of the parallel resistances of 4 Ω and 12 Ω is (12 × 4)/(12 + 4) = 3 Ω (the network is shown in Fig. 1.69(b)).

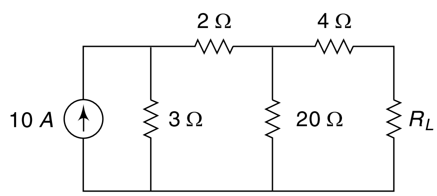



**Fig. 1.69(b)** *Reduction of network shown in Fig. 1.69(a)*

Converting 10 A current source into equivalent voltage source, the value of the voltage source is 10 × 3 = 30 V, the equivalent circuit is shown in Fig. 1.69(c).

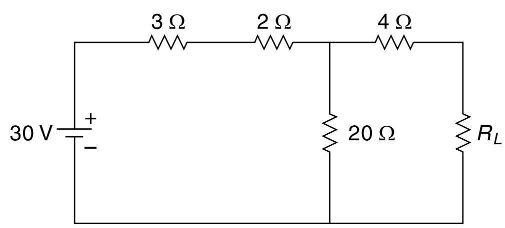



**Fig. 1.69(c)** *Conversion of 10 A current source to equivalent voltage source*

Again converting the voltage source into current source, the network in Fig. 1.69(d) is obtained where the parallel combination of 5 Ω and 20 Ω is $\frac{5 \times 20}{5 + 20} = 4\ \Omega$.

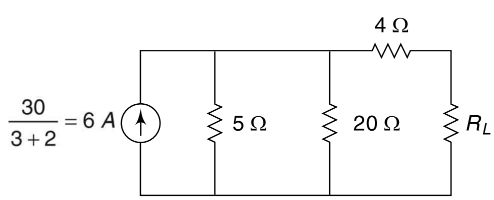



**Fig. 1.69(d)** *Conversion of 30 V voltage source to an equivalent current source*

Further converting the current source into voltage source (Fig. 1.69 (e)) we get current through $R_L$ as $\frac{24}{4+4+4} = 2$ A and the voltage across $R_L$ is 4 × 2 = 8 V.

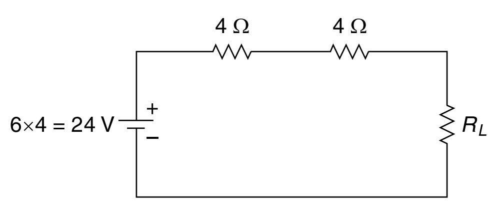



**Fig. 1.69(e)** *Conversion of 6 A current source to equivalent voltage source*

**1.46** Using source conversion technique find the current $I$ in Fig. 1.70.

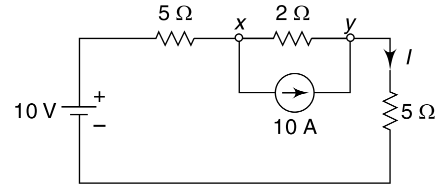



**Fig. 1.70** *Circuit of Ex. 1.46*

***Solution***

The current source is connected in parallel with a 2 Ω resistor; so the value of the equivalent voltage source is obtained as, $V = 10 \times 2 = 20$ V (as shown in Fig. 1.71)

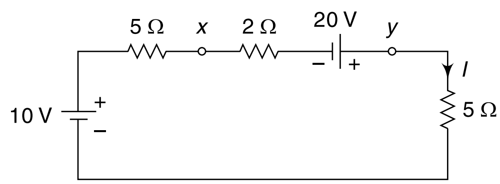



**Fig. 1.71** *Conversion of 10 A current source to equivalent voltage source*

The current delivered by a 10 A source would flow from $y$ to $x$ in 20 Ω resistor. The polarity of the 20 V source is shown in Fig. 1.71.

Therefore, $I = \dfrac{20 + 10}{5 + 2 + 5} = 2.5$ A.

**1.47** Convert the circuit of Fig. 1.72 into a single voltage source in series with a single resistor.

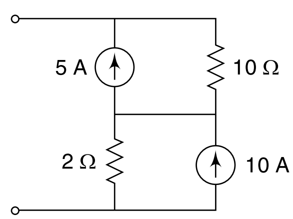



**Fig. 1.72** *Circuit of Ex. 1.47*

***Solution***

Figure 1.72(a) represents the conversion of 5 A source into equivalent voltage source.

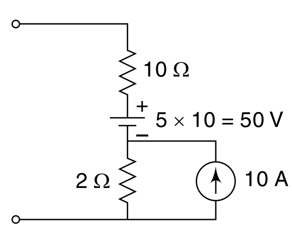



**Fig. 1.72(a)** *Conversion of 5 A current source to equivalent voltage source*

Fig. 1.72(b) represents conversion of 10 A current source into equivalent voltage source.

The net voltage of the single voltage source is thus (50 + 20) V = 70 V and the net resistance is (10 + 2) Ω = 12 Ω.

The equivalent circuit is shown in Fig. 1.72(c).

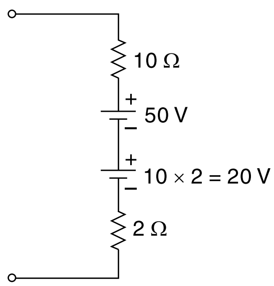



**Fig. 1.72(b)** ***Conversion of 10 A current source to equivalent voltage source***

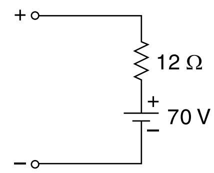



**Fig. 1.72(c)** ***Equivalent circuit of network shown in Fig. 1.72***

## 1.10.2 Independent and Dependent Sources

The voltage or current sources which do not depend on any other quantity in the circuit (i.e the strength of voltage or current in the sources), and do not change for any change in the connected network, are called *independent sources*. Independent sources are represented by circles. An independent voltage source and an independent current source is shown in Fig. 1.72.1(a) and 1.72.1(b)

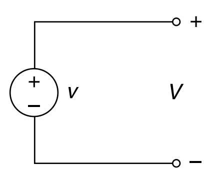



**Fig. 1.72-1(a)** ***Independent voltage source***

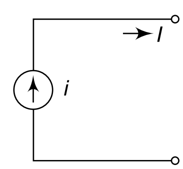



**Fig. 1.72-1(b)** ***Independent current source***

A *dependent voltage or current source* is one which depend on some other quantity in the circuit (may be either voltage or current) i.e the strength of voltage or current changes in the source for any change in the connected network. Dependent sources are represented by diamond-shaped symbol. There are four possible dependent sources:

Voltage dependent voltage source, as shown in Fig. 1.72.1(c).

Voltage dependent current source, as shown in Fig. 1.72.1(d).

Current dependent current source, as shown in Fig. 1.72.1(e).

Current dependent voltage source as shown in Fig. 1.72.1(f).

In the above figures *a*, *b*, *c* and *d* are the constants of proportionality *a* and *c* has no units, unit of *b* is siemens and unit of *d* is ohms.

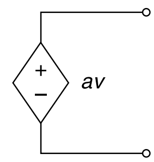



**Fig. 1.72-1(c)** ***Voltage dependent voltage source***

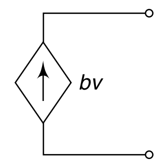



**Fig. 1.72-1(d)** ***Voltage dependent current source***

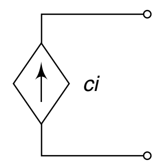



**Fig. 1.72-1(e)** ***Current dependent current source***

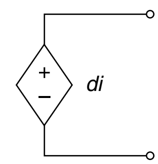



**Fig. 1.72-1(f)** ***Current dependent voltage source***

Some examples of independent sources are battery, dc (or ac) generator. Dependent sources are parts of models which are used to represent electrical properties of electronic devices such as operational amplifiers and transistors etc.

## 1.11 SUPERPOSITION THEOREM

*Statement: In a linear bilateral network containing several sources, the current through or voltage across any branch in the network equals the algebraic sum of the currents or voltages of each individual source considered separately with all other sources made inoperative, i.e., replaced by resistances equal to their internal resistances.*

It may be noted here that while removing the voltage source it should be replaced by its internal resistance (if any) or by a short circuit and while removing the current source it should be replaced by an open circuit. Superposition theorem is applicable only to linear networks (both ac and dc) where current is linearly related to voltage as per Ohm's law.

**Illustration**

Let us find the current $I$ as shown in Fig. 1.73 applying superposition theorem.

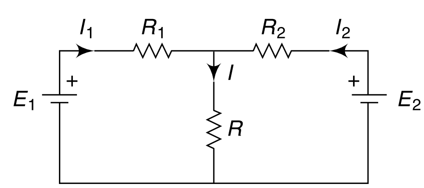



**Fig. 1.73** ***A simple resistive network with two voltage sources***

Considering the voltage source $E_1$ acting alone and removing the other voltage source $E_2$ after replacing it by its internal resistance (if any) otherwise short circuiting the source, the current through $R_1$ is [Fig. 1.73(a)]

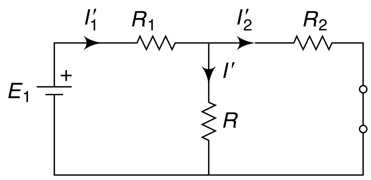



**Fig. 1.73(a)** ***Source $E_1$ retained, $E_2$ deactivated***

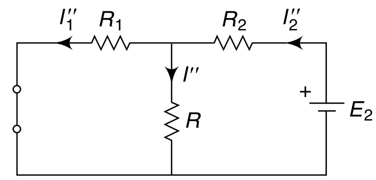



**Fig. 1.73(b)** ***Source $E_2$ retained, $E_1$ deactivated***

$$I_1' = \frac{E_1}{R_1 + \dfrac{RR_2}{R + R_2}} \tag{1.34}$$

Hence current through $R$ is

$$I' = I_1' \times \frac{R_2}{R + R_2} = \frac{E_1 R_2}{RR_1 + R_1 R_2 + R_2 R} \tag{1.35}$$

Similarly considering the voltage source $E_2$ acting alone removing the source $E_1$ and replacing it by a short circuit [Fig. 1.73(b)], the current through $R_1$, $R_2$ and $R$ being $I_1''$, $I_2''$ and $I''$ repectively, we find for $E_2$ acting alone

$$I_2'' = \frac{E_2}{R_2 + \dfrac{RR_1}{R + R_1}} \tag{1.36}$$

$$I'' = I_2'' = \frac{R_1}{R + R_1} = \frac{E_2 R_1}{RR_2 + R_2 R_1 + RR_1} \tag{1.37}$$

Therefore, if there are two sources connected through a network, the resultant current flowing through $R$ is

$$I = I' + I'' = \frac{E_1 R_2 + E_2 R_1}{RR_2 + R_2 R_1 + RR_1} \tag{1.38}$$

### 1.11.1 Procedure for Applying Superposition Theorem

1. Select one source and replace all other sources by their internal impedances.
2. Determine the level and direction of the current that flows through the desired branch as a result of the single source acting alone.
3. Repeat steps 1 and 2 using each source in turn until the branch current components have been calculated for all sources.
4. Algebrically sum the component currents to obtain the actual branch current(s).

**1.48** Compute the current in the 10 Ω resistor as shown in Fig. 1.74 using Superposition theorem.

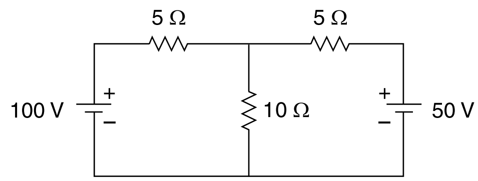



**Fig. 1.74** ***Circuit of Ex. 1.48***

### *Solution*

Considering the 100 V source acting alone, the direction of currents supplied by the source has been shown in Fig. 1.74(a).

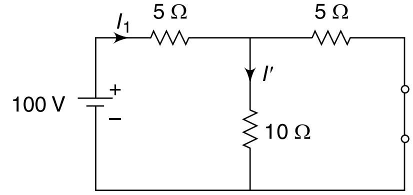



**Fig. 1.74(a)** ***Source 100 V only considered***

Here
$$I_1 \frac{100}{5+\frac{10\times 5}{10+5}} = \frac{1500}{125}\text{ A}$$

Hence current through 10 Ω resistor

$$I' = I_1 \times \frac{5}{5+10} = 4\text{ A}$$

Considering a 50 V source acting alone the direction of currents supplied by the source are shown in Fig. 1.76(b).

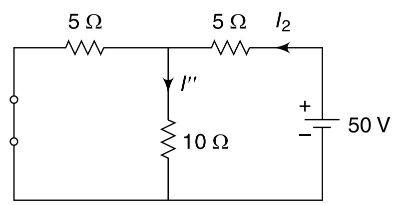



**Fig. 1.74(b)** ***Source 50 V only considered***

Here,
$$I_2 = \frac{50}{5+\frac{10\times 5}{5+10}} \frac{750}{125}\text{ A}$$

Hence current through the 10 Ω resistor is

$$I'' = I_2 \times \frac{5}{5+10} = 2\text{ A}$$

When both the sources are acting simultaneouly, the current through 10 Ω resistor (according to Superposition theorem) is given by ($I' + I''$) i.e., (4 A + 2 A = 6 A).

**1.49** Find the current in the 50 Ω resistor in Fig. 1.75 using Superposition theorem.

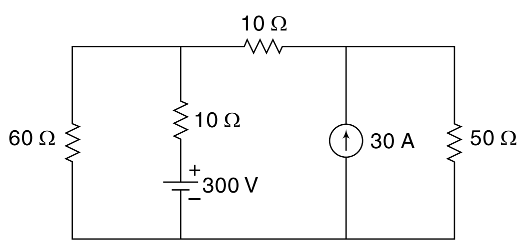



**Fig. 1.75** ***Circuit of Ex. 1.49***

### *Solution*

Considering the voltage source acting alone and removing the current source (the corresponding figure being shown in Fig. 1.75(a)) the total current supplied by the voltage source is

$$I_1 = \frac{300}{10 + \dfrac{60\,(10 + 50)}{60 + (10 + 50)}} = \frac{15}{2}\text{ A}$$

Hence the current through the 50 Ω resistor due to the voltage source acting alone is

$$I' = \frac{15}{2} \times \frac{60}{60 + 10 + 50} = \frac{15}{4}\text{ A} = 3.75\text{ A (from } a \text{ to } b)$$

Next, removing the voltage source and considering the current source acting alone (the corresponding networks being shown in Fig. 1.75(b) and Fig. 1.75(c)), the current through the 50 Ω resistor is

$$I'' = 30 \times \frac{10 + 60/7}{50 + 10 + 60/7} = 8.124\text{ A (from } a \text{ to } b)$$

[The combined resistance of the 60 Ω and 10 Ω is parallel is $\dfrac{60 \times 10}{60 + 10} = \dfrac{60}{7}$ Ω]

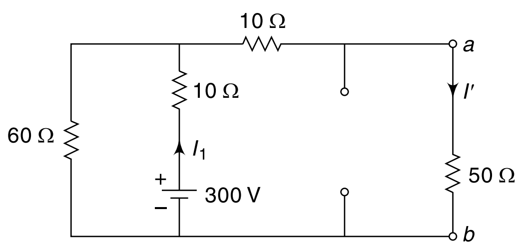



**Fig. 1.75(a)** ***Voltage source is acting only***

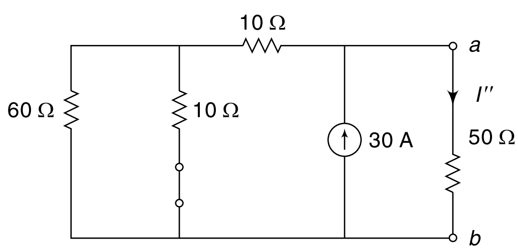



**Fig. 1.75(b)** ***Current source is acting only***

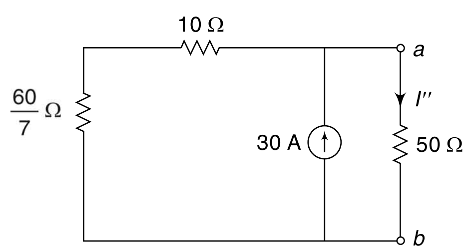



**Fig. 1.75(c)** ***Simplified circuit of network shown in Fig. 1.75(b)***

According to the Superposition theorem when both the sources are acting simultaneously, the current through the 50 Ω resistor is

$$I' + I'' = (3.75 + 8.124)\text{ A} = 11.874\text{ A (from } a \text{ to } b)$$

**1.50** Obtain $I$ using the Superposition theorem for the network shown in Fig. 1.76.

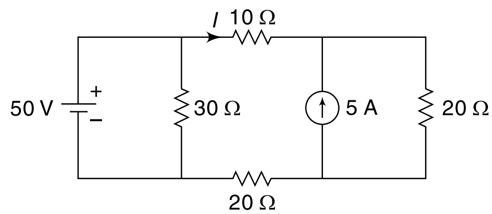



**Fig. 1.76** ***Circuit of Ex. 1.50***

***Solution***

Considering the voltage source acting alone [Fig. 1.76(a)] the current supplied by the source is

$$I_1 = \frac{50}{\dfrac{30\,(10+20+20)}{30+(10+20+20)}} = \frac{8}{3}\text{ A}$$

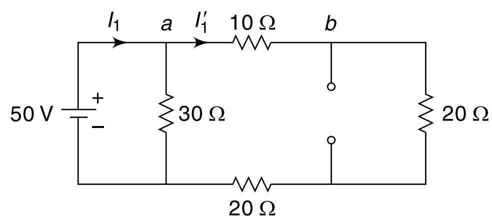



**Fig. 1.76(a)** ***Voltage source is acting alone***

Hence the current through the 10 Ω resistor is

$$I_1' = \frac{8}{3} \times \frac{30}{30+10+20+20} = 1\text{ A (from } a \text{ to } b)$$

Removing the voltage source and considering the current source acting alone [Fig. 1.76(b)] the current through the 30 Ω resistor is zero as there is a short circuit path in parallel with it. Hence the network of Fig. 1.76(b) reduces to that in Fig. 1.76(c). The current through the 10 Ω resistor is then given by

$$I'' = 5 \times \frac{20}{20+20+10} = 2\text{ A} \quad \text{(from } b \text{ to } a) \text{ (or } -2\text{ A from } a \text{ to } b)$$

Therefore according to the Superposition theorem when both the sources are acting simultaneously the current

$$I = I' + I'' = 1 - 2 = -1\text{ A (from } a \text{ to } b)$$

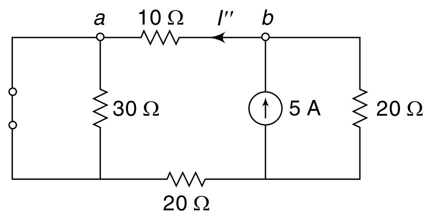



**Fig. 1.76(b)** *Current source is acting alone*

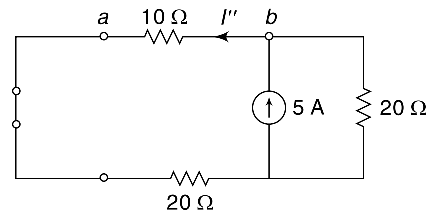



**Fig. 1.76(c)** *Simplified circuit of Fig. 1.78(b)*

**1.51** Find the voltage across 20 Ω resistor using the Superposition theorem in Fig. 1.77.

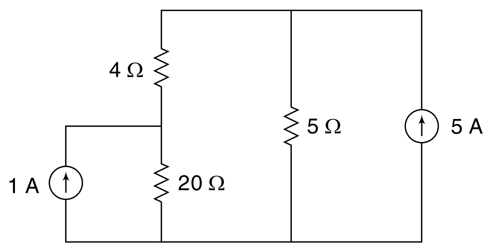



**Fig. 1.77** *Circuit of Ex. 1.51*

***Solution***

When 1 A current source is acting alone (the corresponding figure being shown in Fig. 1.77(a), the current through the 20 Ω resistor under this condition is obtained as $1 \times \dfrac{4+5}{20+4+5} = \dfrac{9}{29}$ A (from *a* to *b*)

Hence voltage across 20 resistor is $\dfrac{9}{29} \times 20 = \dfrac{180}{29}$ V $(= V'_{ab})$

Fig. 1.77(b) shows the network when 1 A source is deactivated and 5 A source acts alone.

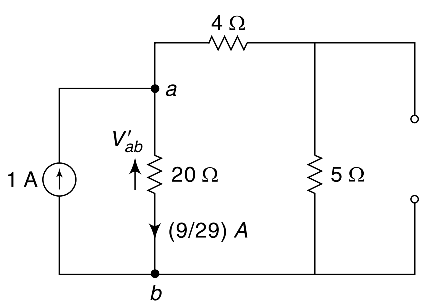



**Fig. 1.77(a)** *Current source (1 A) acting alone*

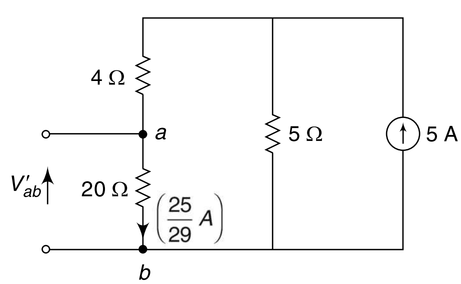



**Fig. 1.77(b)** *Current source (5 A) is acting alone*

The current through 20 Ω resistor under this condition is

$$5 \times \frac{5}{5+4+20}\ \text{A} = \frac{5 \times 5}{29}\ \text{A} = \frac{25}{29}\ \text{A (from } a \text{ to } b)$$

The voltage across the 20 Ω resistor is then $V'_{ab} = \frac{25}{29} \times 20\ \text{V} = \frac{500}{29}\ \text{V}$

According to the Superposition theorem the voltage across 20 Ω resistor ($V_{ab}$) when both sources are acting simultaneously is

$$V_{ab} = V'_{ab} + V''_{ab} = \frac{180}{29} + \frac{500}{29} = \frac{680}{29}\ \text{V} = 23.45\ \text{V}$$

**1.52** Find the current through 40 Ω resistor using Superposition theorem in Fig. 1.78.

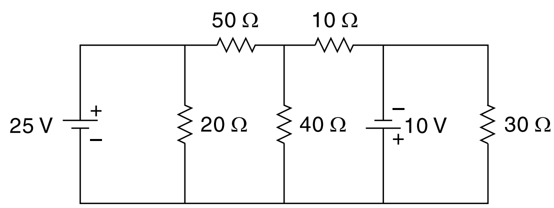



**Fig. 1.78** ***Circuit of Ex. 1.52***

***Solution***

Let us consider that the 25 V source is acting alone and the other source is deactivated. The corresponding figures are shown in Fig. 1.78(a) and Fig. 1.78(b).

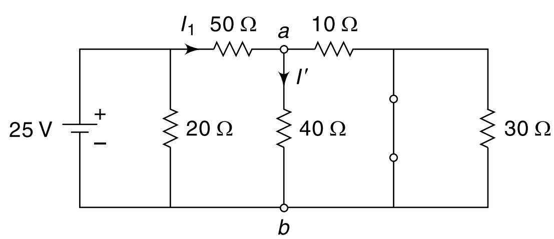



**Fig. 1.78(a)** ***25 V source is acting alone***

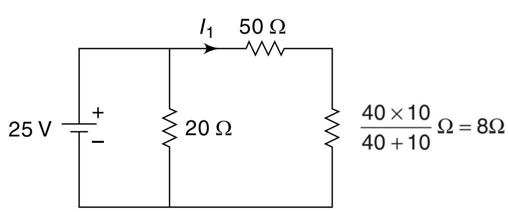



**Fig. 1.78(b)** ***Simplified circuit of Fig. 1.78(a)***

The current through 50 Ω resistor is

$$I_1 = \frac{25}{\frac{20 \times 58}{20 + 58}} \times \frac{20}{20 + 50 + 8} = 0.431\ \text{A}$$

Hence the current through 40 Ω resistor due to 25 V source alone is

$$I' = 0.431 \times \frac{10}{40 + 10} \text{ A} = 0.0862 \text{ A} \qquad \text{(from } a \text{ to } b \text{ in Fig. 1.78(a))}$$

Next consider the 10 V source acting alone deactivating the 25 V source.

The current through the 10 Ω resistor [Fig. 1.78(c) and Fig. 1.78(d)] is

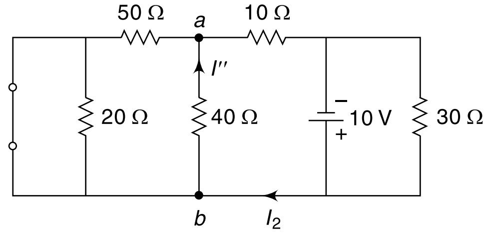



**Fig. 1.78(c)** ***10 V source is acting alone***

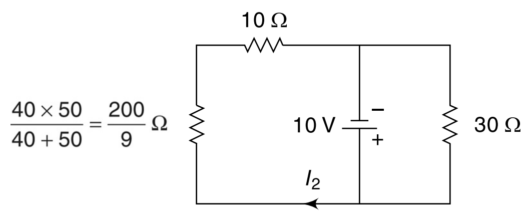



**Fig. 1.78(d)** ***Simplified circuit of Fig. 1.78(c)***

$$I_2 = \frac{10}{\dfrac{30 \times (200/9 + 10)}{30 + 200/9 + 10}} \times \frac{30}{30 + 200/9 + 10} = 0.31 \text{ A}$$

Hence the current through the 40 Ω resistor is

$$I'' = 0.31 \times \frac{50}{50 + 40} = 0.172 \text{ A (from } b \text{ to } a)$$

Using the Superposition theorem the current through the 40 Ω resistor is

$$I'' - I' = 0.172 - 0.086 = 0.086 \text{ A (from } b \text{ to } a)$$

**1.53** Utilising the Superposition theorem find the current through the 20 Ω resistor for the network shown in Fig. 1.79.

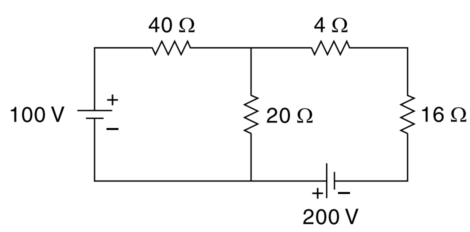



**Fig. 1.79** ***Circuit of Ex. 1.53***

***Solution***

Considering the 10 $V$ source acting alone [Fig. 1.79(a)] the current through the 20 Ω resistor is

$$I_1 = \frac{100}{40 + \dfrac{20 \times (4+16)}{20+4+16}} \times \frac{4+16}{4+16+20}\,\text{A}$$

$$= 1\text{ A (from } a \text{ to } b)$$

Considering the 200 V source acting alone [Fig. 1.79(b)] the current through the 20 Ω resistor is

$$I_2 = \frac{200}{16+4+(40\times 20)/(40+20)} \times \frac{40}{40+20} = 4\text{ A (from } b \text{ to } a)$$

Then, according to the Superposition theorem, the current through 20 Ω resistor is $(I_2 - I_1) = 4 - 1 = 3$ A (from $b$ to $a$).

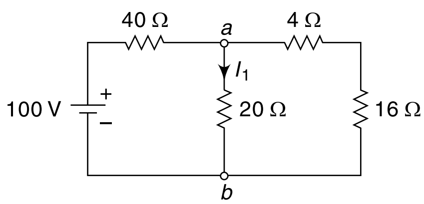



**Fig. 1.79(a)** ***100 V source is acting alone***

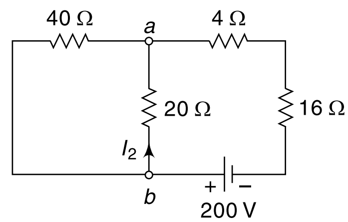



**Fig. 1.79(b)** ***200 V source is acting alone***

**1.54** Find the current through 1 Ω resistor applying the Superposition theorem in Fig. 1.80.

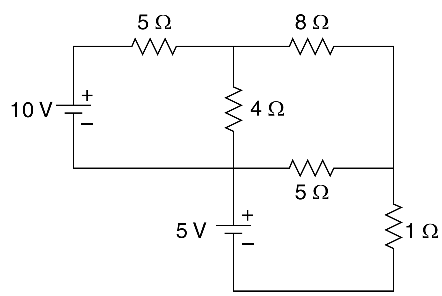



**Fig. 1.80** ***Circuit of Ex. 1.54***

***Solution***

Consider 10 V source acting alone (the corresponding figures are shown in Fig. 1.82(a) and Fig. 1.80(b)).

$$I_1 = \frac{10}{5 + \dfrac{4 \times (8 + (5/6))}{4 + 8 + (5/6)}} \times \frac{4}{4 + 8 + (5/6)}$$

$$= 0.4\text{ A}$$

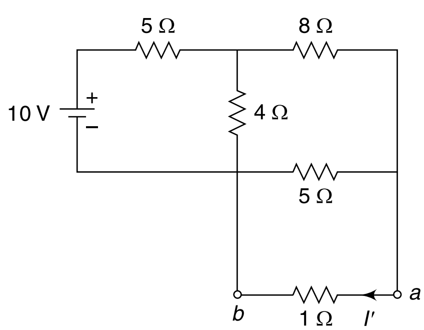



**Fig. 1.80(a)** ***10 V source is acting alone***

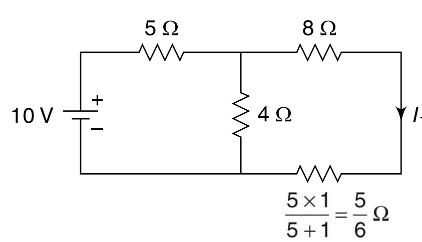



**Fig. 1.80(b)** ***Simplified circuit of Fig. 1.80(a)***

Therefore current through the 1 Ω resistor is

$$I' = 0.4 \times \frac{5}{5+1} = 0.33 \text{ (from } a \text{ to } b)$$

Next considering the 5 V source acting alone (corresponding figures are shown in Fig 1.80(c) and Fig. 1.80(d)

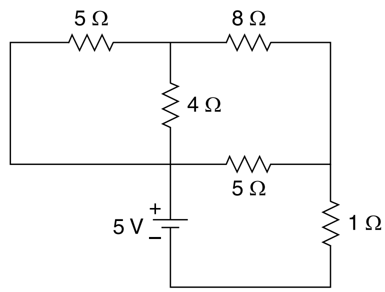



**Fig. 1.80(c)** ***5 V source is acting alone***

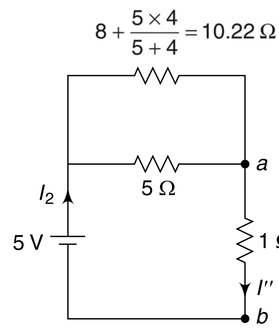



**Fig. 1.80(d)** ***Simplified circuit of Fig. 1.80(c)***

The current supplied by 5 V source is

$$I_2 = \frac{5}{1 + \frac{5 \times 10.22}{5 + 10.22}} \text{ A} = 1.147 \text{ A}$$

The current through the 1 Ω resistor due to the 5 V source acting alone is then

$$I'' = I_2 = 1.147 \text{ (from } a \text{ to } b)$$

Hence according to Superposition theorem the current through the 1 Ω resistor is obtained as

$$I' + I'' = 0.333 + 1.147 = 1.48 \text{ A}$$

**1.55** Find the current through resistance ($R_L$) for the network shown in Fig. 1.81 using the Superposition theorem

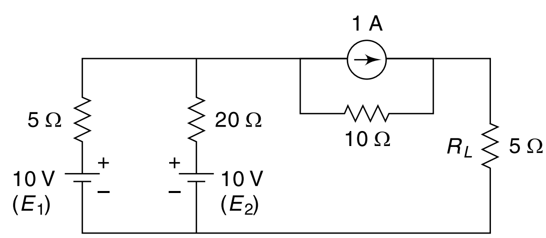



**Fig. 1.81** ***Circuit of Ex. 1.55***

***Solution***

Considering the 10 V source ($E_1$) acting alone the current through $R_L$ [Fig. 1.81(a)] is

$$I_1 = \frac{10}{5 + \frac{20 \times 15}{20 + 15}} \times \frac{20}{20 + 15} = 0.42 \text{ A (from } a \text{ to } b)$$

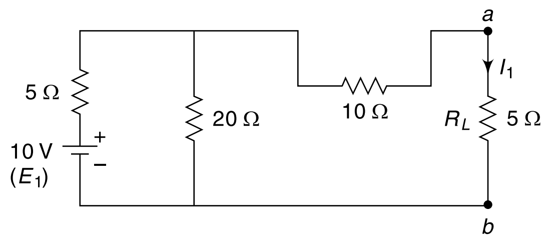



**Fig. 1.81(a)** ***10 V source ($E_1$) is acting alone***

Next, considering the other 10 V source ($E_2$) acting alone the current through $R_L$ [Fig. 1.81(b)] is

$$I_2 = \frac{10}{20 + \frac{5 \times 15}{5 + 15}} \times \frac{5}{5 + 15} = 0.105 \text{ A (from } a \text{ to } b)$$

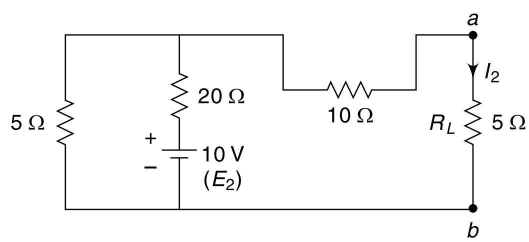



**Fig. 1.81(b)** ***Another 10 V source ($E_2$) is acting alone***

Considering the current source (1A) acting alone the current through $R_L$ [Fig 1.81(c) and Fig. 1.83(d)] is

$$I_3 = 1 \times \frac{10}{10 + 5 + 4} = \frac{10}{19} \text{ A} = 0.5263 \text{ A (from } a \text{ to } b).$$

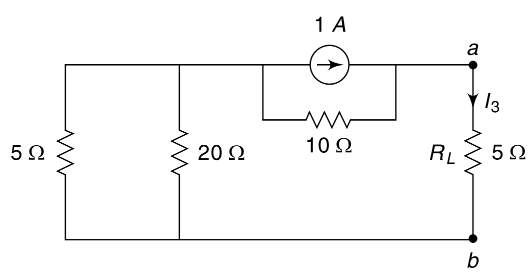



**Fig. 1.81(c)** ***1 A current source is acting alone***

Hence, according to the Superposition theorem the current through $R_L$, when all the sources are acting simultaneously, is obtained as $I_1 + I_2 + I_3 = 1.0513$ A.

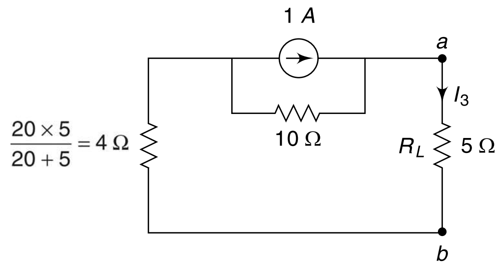



**Fig. 1.81(d)** *Simplified circuit of Fig. 1.81(c)*

## 1.12 THEVENIN'S THEOREM

*Statement*: The current flowing through a load resistance $R_L$ connected across any two terminals $A$ and $B$ of a linear, bilateral network is given by $\frac{V_{oc}}{R_i + R_L}$, where $V_{oc}$ is the open circuit voltage (i.e. voltage across terminals $AB$ when $R_L$ is removed) and $R_i$ is the internal resistance of the network as viewed back into the open circuited network from terminals $AB$ deactivating all the independent sources.

The following are the limitations of this theorem:

(i) Thevenin's theorem can not be applied to a network which contains non-linear impedances.

(ii) This theorem can not calculate the power consumed internally in the circuit or the efficiency of the circuit.

Thevenin's theorem can be explained with the help of the following simple example. The steps are as follows:

### *Step I*

$R_L$ is to be removed from the circuit terminals $a$ and $b$ for the network shown in Fig. 1.82.

### *Step II*

The open circuit voltage ($V_{oc}$) which appears across terminals $a$ and $b$ in Fig. 1.82(a) is calculated as

$$V_{oc}\,(= \text{Voltage across } R_3) = \frac{E}{r + R_1 + R_3} \times R_3$$

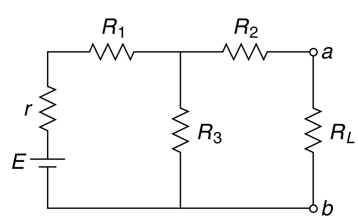



**Fig. 1.82** *Circuit to explain Thevenin's theorem*

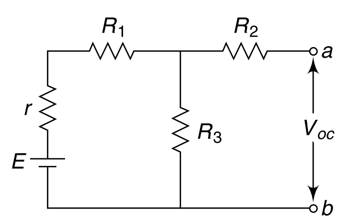



**Fig. 1.82(a)** *$R_L$ removed from circuit of Fig. 1.82*

$V_{o/c}$ is called the "Thevenin's voltage" ($V_{\text{Th}}$)

Hence, $$V_{\text{Th}} = \frac{ER_3}{r + R_1 + R_3} \quad (1.39)$$

***Step III***

Removing the battery from the circuit leaving the internal resistance ($r$) of the battery behind it [Fig. 1.82(b)] when viewed from terminals $a$ and $b$, the internal resistance of the circuit is given by

$$R_i = R_2 + \frac{R_3\,(R_1 + r)}{R_3 + R_1 + r}$$

This resistance $R_i$ is called *Thevenin's equivalent resistance* ($R_{\text{Th}}$)

$$\therefore \quad R_i = R_{\text{Th}} = R_2 + \frac{R_3\,(R_1 + r)}{R_3 + R_1 + r} \quad (1.40)$$

***Step IV***

Thevenin's equivalent circuit is drawn as shown in Fig. 1.82(c) and $R_L$ is reconnected acrosss terminals $a$ and $b$. The current through $R_L$ is

$$I_{\text{Th}} = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

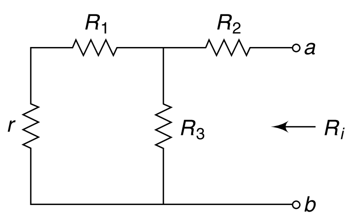



**Fig. 1.82(b)** ***Internal resistance $R_i$ of the given network***

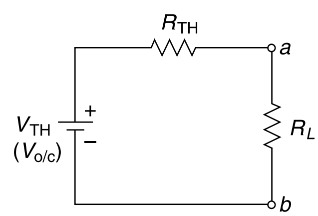



**Fig. 1.82(c)** ***Thevenin's equivalent circuit***

### *Different methods of finding $R_{Th}$*

(a) *For independent sources:* Deactivate the sources, i.e for independent current source deactivate it by open circuiting its terminals and for voltage source deactivate it by shorting it. Then find the internal resistance of the network looking through the load terminals kept open circuited. In case these independent sources are non-ideal, the internal resistance will remain connected across the deactivated source terminals.

(b) *For dependent sources in addition or in absence of independent source:*

**First Method**

(i) Find open circuit voltage $V_{oc}$ across the open circuited load terminals. Next short circuit the load terminals and find the short circuit current ($I_{sc}$) through the shorted terminals.

The Thevenin's equivalent resistance is then obtained as

$$R_{\text{Th}} = \frac{V_{oc}}{I_{sc}}$$

**Second Method**

(ii) Remove the load resistance and apply a dc voltage $V_{dc}$ at the open circuited load terminals. Keep the other independent sources deactivated. A dc current $I_{dc}$ will flow in the circuit from the load terminals.

The Thevenin's equivalent resistance is then

$$R_{\text{Th}} = \frac{V_{dc}}{I_{dc}}$$

## 1.12.1 Thevenizing Procedure

1. Calculate the open circuit voltage ($V_{\text{Th}}$) across the network terminals.
2. Redraw the network with each independent source replaced by its internal resistance. This is called "deactivation of the sources".
3. Calculate the resistance ($R_{\text{Th}}$) of the redrawn network as seen from the output terminals.

**1.56** Using Thevenin's theorem find the current through the 15 Ω resistor in Fig. 1.83.

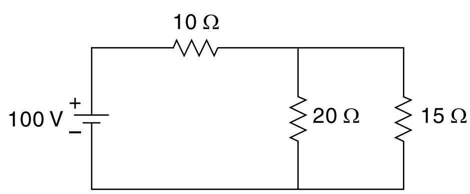



**Fig. 1.83** ***Circuit of Ex. 1.56***

***Solution***

Removing the 15 Ω resistor the open circuit voltage across *a* and *b* [Fig. 1.83(a)] is $V_{oc}$ $= \frac{100}{10 + 20} \times 20 \text{ V} = \frac{200}{3} \text{ V}$.

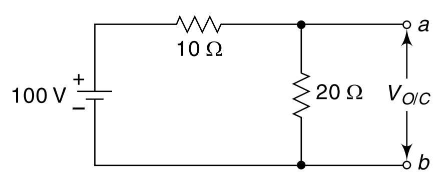



**Fig. 1.83(a)** ***Finding ($V_{o/c}$)***

The Thevenin's equivalent voltage is

$$V_{\text{Th}} \ (= V_{oc}) = \frac{200}{3} \text{ V}$$

Next removing the source, the internal resistance of the network as viewed from the open circuited terminals [Fig. 1.83(b)] is $r_i = \frac{10 \times 20}{20 + 10} =$ i.e. Thevenin's equivalent resistance is $R_{\text{Th}}$ $= R_i = (20/3)$ Ω. Thevenin's equivalent circuit is shown in Fig. 1.83(c). The current through the 15 Ω resistor (according to Thevenin's theorem) is then given by

$$I_{15} = \frac{(200/3)}{(20/3) + 15} \text{ A} = 200 \text{ A} = 3.077 \text{ A}$$

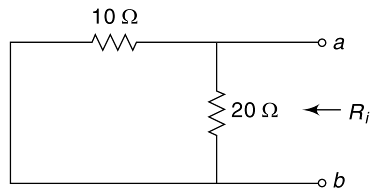



**Fig. 1.83(b)** ***Finding of $R_i$ ($R_{th}$)***

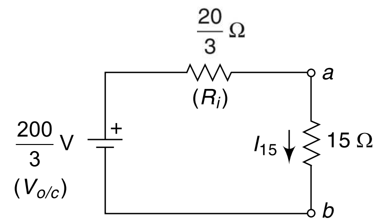



**Fig. 1.83(c)** ***Thevenin's equivalent circuit (Ex. 1.56)***

**1.57** Find the current through the 2 Ω resistor using Thevenin's theorem [Fig. 1.84].

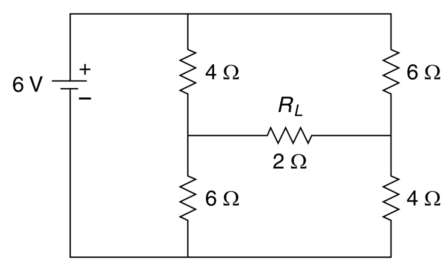



**Fig. 1.84** ***Circuit of Ex. 1.57***

***Solution***

The circuit is redrawn in Fig. 1.84(a) with terminals of $R_L$ open circuited.

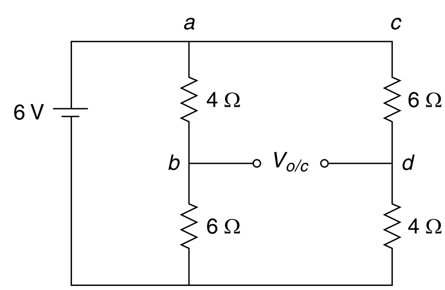



**Fig. 1.84(a)** ***Finding of ($V_{o/c}$)***

Thevenin's equivalent voltage is

$$V_{Th} = V_{bd} = V_{cd} - V_{ab}$$
$$= 6 \times \frac{6}{6+4} - 6 \times \frac{4}{6+4}$$
$$= 3.6 - 1.4$$
$$= 1.2 \text{ V } [V_b \text{ is higher potential}].$$

Deactivating the voltage source, Thevenin's equivalent resistance is shown in Fig. 1.84(b) and Fig. 1.84(c).

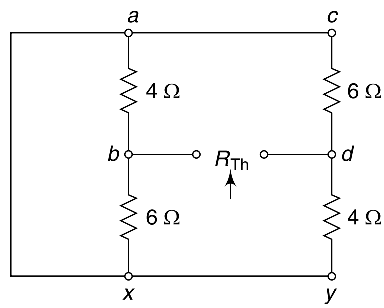



**Fig. 1.84(b)** *Finding of ($R_{th}$)*

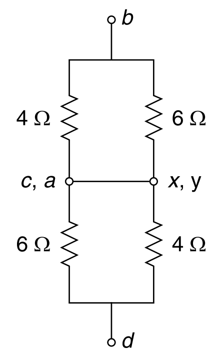



**Fig. 1.84(c)** *Reduced equivalent network to find $R_{th}$*

$$R_{Th} = (4 \parallel 6) + (6 \parallel 4) = 2 \times \frac{4 \times 6}{4 + 6}\ \Omega = 4.8\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 1.86(d)

The current through ($R_L$) is then $I_L = \frac{1.2}{4.8 + 2}$ A $= \frac{1.2}{6.8} = 0.176$ A

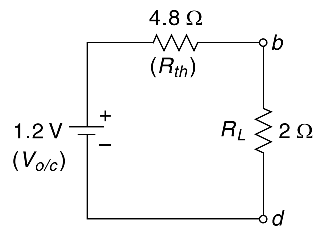



**Fig. 1.84(d)** *Thevenin's equivalent circuit of Ex. 1.57*

**1.58** Find the current through 10 Ω resistor in Fig. 1.85 using Thevenin's theorem.

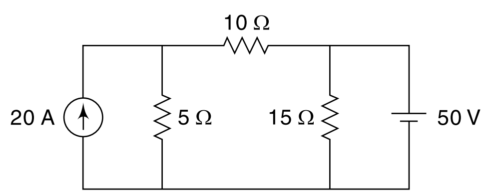



**Fig. 1.85** *Circuit of Ex. 1.58*

### *Solution*

Removing the load resistance of 10 Ω from its terminals, the open circuit voltage across terminals *a* and *b* (as shown in Fig. 1.85(a)) can be found out.

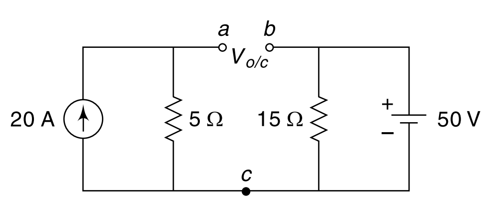



**Fig. 1.85(a)** *Finding of $V_{o/c}$*

The voltage across the 15 Ω resistor is due to the current supplied by the voltage source only.

∴ Voltage across the 15 Ω resistor is 50 V.

Hence $V_{bc} = 50$ V; Also, $V_{ac} = 20 \text{ A} \times 5\ \Omega = 100$ V.

Therefore voltage across open circuit terminals $a$ and $b$ is

$$V_{o/c} = V_{ab} = V_{ac} - V_{bc} = (100 - 50) \text{ V} = 50 \text{ V}$$

i.e $V_{\text{Th}} = 50 \text{ V} (= V_{o/c})$.

Deactivating all the sources as shown in Fig. 1.85(b), the internal resistance of the network as viewed from the open circuited terminals is,

$$R_{\text{Th}} = 5\ \Omega \text{ (as 15 Ω resistor is short circuited)}$$

Thevenin's equivalent circuit is shown in Fig. 1.85(c). The current through 10 Ω resistor is

$$I_{10} = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{50}{5 + 10} \text{ A} = 3.33 \text{ A}.$$

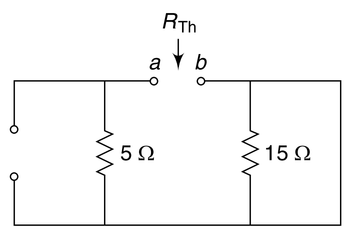



**Fig. 1.85(b)** ***Finding of $R_{Th}$***

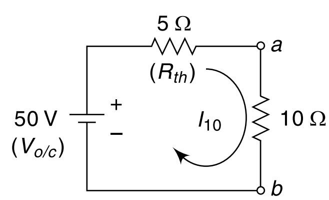



**Fig. 1.85(c)** ***Thevenin's equivalent circuit of Ex. 1.58***

**1.59** Find the current through 15 Ω resistor for the network shown in Fig. 1.86 using Thevenin's theorem.

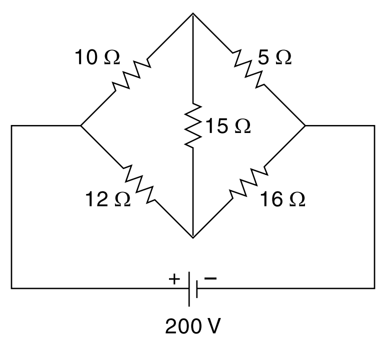



**Fig. 1.86** ***Circuit of Ex. 1.59***

***Solution***

Removing 15 Ω resistor the open circuit voltage across its terminals is found out in the network of Fig. 1.86(a).

The current through 10 Ω resistor is obtained as $\frac{200}{10+5}$ A $= \frac{200}{15}$ A.

Voltage across the 10 Ω resistor is given by

$$V_{xa} = 10 \times \frac{200}{15} = \frac{2000}{15}.$$

Current through the 12 Ω resistor is found as $\frac{200}{12+16}$ A $= \frac{200}{28}$ A.

Voltage across the 12 Ω resistor is obtained as $V_{xb} = 12 \times \frac{200}{28} \text{ V} = \frac{2400}{28}$ V.

$$V_{ab} = V_{xb} - V_{xa}$$

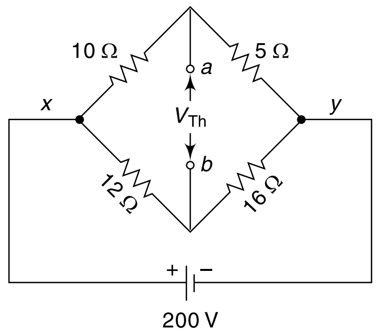



**Fig. 1.86(a)** *Finding of* $V_{Th}$

$$= \frac{2400}{28} - \frac{2000}{15} = 85.71 \text{ V} - 133.33 \text{ V}.$$

$$= -47.62 \text{ V}.$$

Hence $b$ is at higher potential with respect to $a$. Therefore $V_{\text{Th}} = V_{ba} = 47.62$ V.

Deactivating the voltage source, Thevenin's equivalent resistance can be obtained as shown in Fig. 1.86(b) and Fig. 1.86(c).

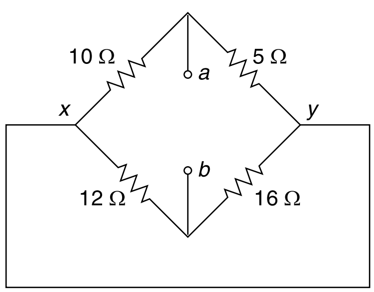



**Fig. 1.86(b)** *Finding of* $R_i$

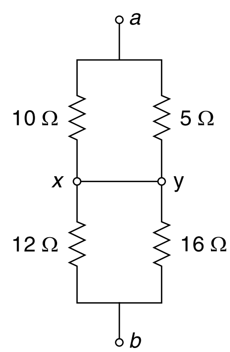



**Fig. 1.86(c)** *Reduced network to find* $R_i$

The resistance between $a$ and $b$ is then found as

$$R_{\text{Th}} = (10 \parallel 5) + (12 \parallel 16)$$

$$= \frac{10 \times 5}{10 + 5} = \frac{12 \times 16}{12 + 16}$$

$$= 10.19\ \Omega\ (= R_i)$$

∴ Current through 15 Ω resistor [Fig. 1.86d] is

$$I_{15} = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{47.62}{10.19 + 15} = 1.89 \text{ A}$$

[flowing from terminal $b$ to terminal $a$]

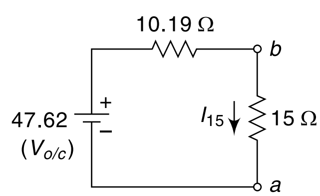



**Fig. 1.86(d)** ***Thevenin's equivalent circuit of Ex. 1.59***

**1.60** Find Thevenin's equivalent circuit of the network (shown in Fig. 1.87) across terminals *x-y*.

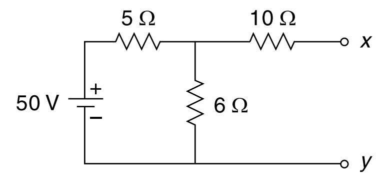



**Fig. 1.87** *Circuit of Ex. 1.60*

***Solution***

The voltage across the open circuited terminals is same as the voltage across the 6 Ω resistor.

$\therefore \quad V_{Th}$ = Voltage across the 6 Ω resistor

$$= \frac{50}{5+6} \times 6 = \frac{300}{11} = 27.27 \text{ V}.$$

Removing the source, Thevenin's equivalent resistance $R_{Th}$ (Fig. 1.87(a)) is $10 + \frac{5 \times 6}{5+6}$ $= 10 + \frac{30}{11} = 12.72\ \Omega$.

Thevenin's equivalent circuit is shown in Fig. 1.87(b).

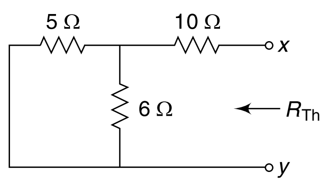



**Fig. 1.87(a)** *Finding of ($R_{Th}$)*

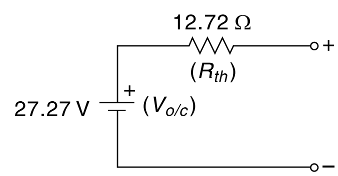



**Fig. 1.87(b)** *Thevenin's equivalent circuit of Ex. 1.60*

**1.61** Find Thevenin's equivalent circuit of the network shown in Fig. 1.88 across terminals *a-b*.

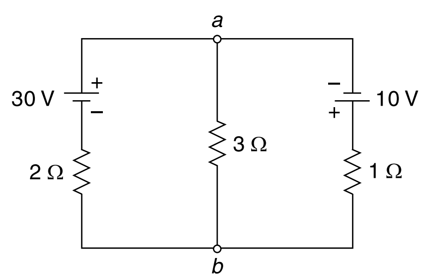



**Fig. 1.88** *Circuit of Ex. 1.61*

***Solution***

Removing the 3 Ω resistor, the circuit is redrawn as shown in Fig. 1.88(a). From Fig. 1.88(a) the circulating current is,

$$I = \frac{30 + 10}{2 + 1} \text{ A} = 13.33 \text{ A}.$$

Applying KVL in loop *abyx*,

$$V_{ab} = -13.33 \times 2 + 30 = 3.34 \text{ V}$$

$$\therefore \quad V_{o/c} \ (= V_{Th}) = 3.34 \text{ V}.$$

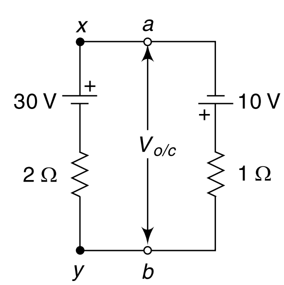



**Fig. 1.88(a)** *Finding of ($V_{o/c}$)*

Next deactivating the sources, Thevenin's equivalent resistance [Fig. 1.88(b)] is given by

$$R_{\text{Th}} = \frac{2 \times 1}{2 + 1} = 0.667\ \Omega.$$

Thevenin's equivalent circuit is then drawn in Fig. 1.88(c).

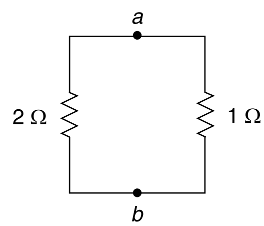



**Fig. 1.88(b)** ***Finding of $R_{Th}$***

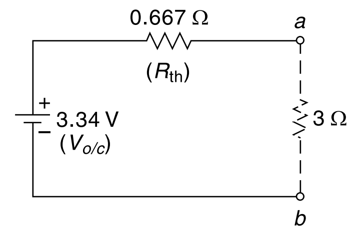



**Fig. 1.88(c)** ***Thevenin's equivalent circuit of Ex. 1.61***

**1.62** Find the current through the 5 Ω resistor using Thevenin's theorem in Fig. 1.89.

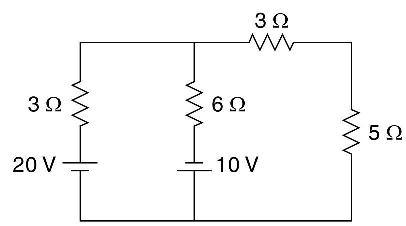



**Fig. 1.89** ***Circuit of Ex. 1.62***

***Solution***

Removing the 5 Ω resistor [Fig. 1.89(a)] the current circulating in the loop *abyx* is

$$I = \frac{20 + 10}{3 + 6}\text{ A} = \frac{10}{3}\text{ A}$$

(in the clockwise direction)

Let $V_{\text{Th}}$ = voltage across branch *xy* = voltage across branch *ab*

Here, $V_{xy} = -10 + 6 \times \frac{10}{3} = 10\text{ V} = V_{\text{Th}}.$

$\left[\text{Otherwise } V_{ab} = 20 - 3 \times \frac{10}{3} = 10\text{ V} = V_{\text{Th}}.\right]$

Deactivating all the sources,

$$R_{\text{Th}} = 3 + (6 \parallel 3) = 3 + \frac{6 \times 3}{6 + 3} = 5\ \Omega.$$

From Fig. 1.89(b) the current through 5 Ω resistor is $\frac{10}{5 + 5} = 1$ A

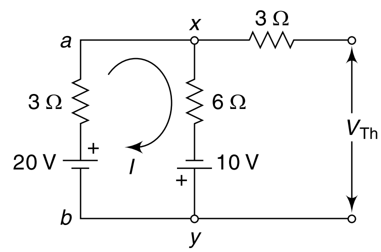



**Fig. 1.89(a)** ***Finding of ($V_{Th}$)***

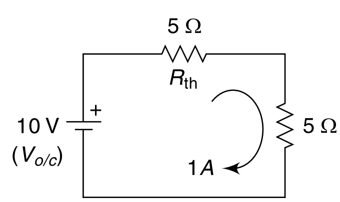



**Fig. 1.89(b)** ***Thevenin's equivalent circuit of Ex. 1.62***

**1.63** Find the Thevenin's equivalent circuit of Fig. 1.90, across $R_L$.

***Solution***

$R_L$ is removed and the terminals are open circuited as shown in Fig. 1.90(a). The current supplied by the 24 V source circulates through the 3 Ω and 6 Ω resistor only while the current due to the current source circulated through the 4 Ω only when the circuit is open circuited at *a* and *b*.

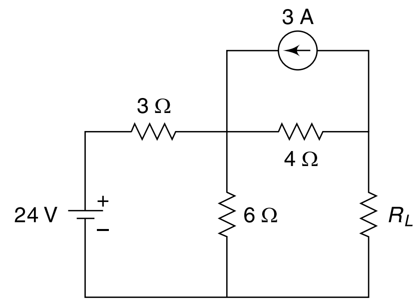



**Fig. 1.90** *Circuit of Ex. 1.63*

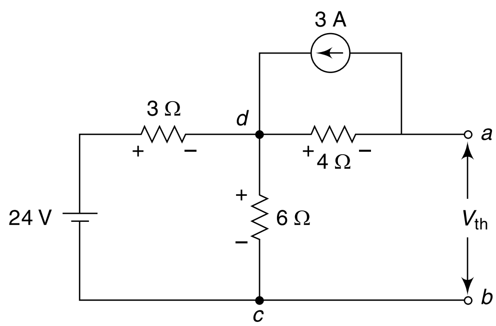



**Fig. 1.90(a)** *Finding of ($V_{Th}$)*

Voltage across *dc* is

$$V_{dc} = \frac{24 \times 6}{3 + 6} \text{ V} = 16 \text{ V}$$

Voltage across *da* is $V_{da} = 3 \times 4$ V = 12 V.

Applying KVL in the loop *abcd*

$$V_{ab} = 16 - 12 = 4 \text{ V i.e } V_{\text{Th}} = 4 \text{ V}$$

Next, all the sources in the network is deactivated [Fig. 1.90(b)].

$$\therefore \quad R_{\text{Th}} = 4 + \frac{3 \times 6}{3 + 6} = 6\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 1.90c.

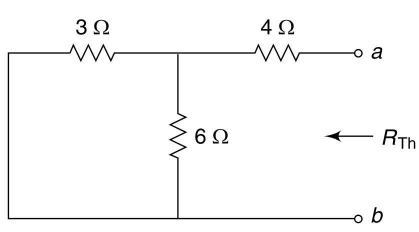



**Fig. 1.90(b)** *Finding of ($R_{Th}$)*

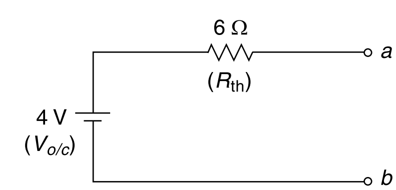



**Fig. 1.90(c)** *Thevenin's equivalent circuit of Ex. 1.63*

**1.64** Find the current in the ammeter of the 2 Ω resistance as shown in Fig. 1.91 using Thevenin's theorem.

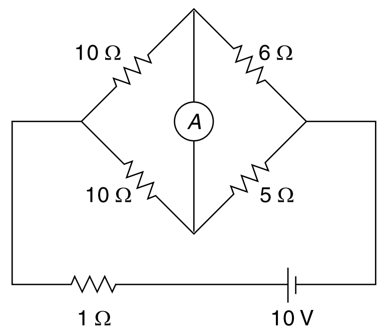



**Fig. 1.91** ***Circuit of Ex. 1.64***

***Solution***

The ammeter is removed and the circuit is shown in Fig. 1.91(a).

The total current delivered by 10 *V* source is

$$I = \frac{10}{1 + \frac{(10+6)\times(10+5)}{(10+6)+(10+5)}}$$

$$= \frac{10}{1 + \frac{16\times 15}{31}}$$

$$= 1.144 \text{ A}$$

$$\therefore \qquad I_1 = 1.144 \times \frac{10+5}{15+16} = 0.55 \text{ A}$$

and $$I_2 = 1.144 \times \frac{10+6}{15+16} = 0.59 \text{ A.}$$

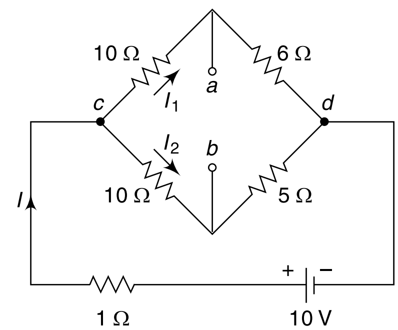



**Fig. 1.91(a)** ***Finding of ($V_{Th}$)***

Voltage across open circuited terminals *a* and *b* is

$$V_{\text{Th}} = V_{ab} = V_{cb} - V_{ca}$$
$$= 0.59 \times 10 - 0.55 \times 10 = 0.4 \text{ V.}$$

Deactivating the voltage source, the corresponding figure is drawn in Fig. 1.91(b). In this figure using delta star conversion values of $R_1$, $R_2$ and $R_3$ can be found out.

$$R_1 = \frac{10\times 6}{10+6+1}\ \Omega = \frac{60}{17}\ \Omega$$

$$R_2 = \frac{6\times 1}{10\ +6+1}\ \Omega = \frac{6}{17}\ \Omega$$

$$R_3 = \frac{1\times 10}{10+6+1}\ \Omega = \frac{10}{17}\ \Omega$$

Resistance between *a* and *b* as shown in Fig. 1.91(c) is given by

$$R_{\text{Th}} = \frac{60}{17} + \left[\left(\frac{10}{17}+10\right)\middle\|\left(\frac{6}{17}+5\right)\right]$$

$$= 3.53 + \frac{10.59\times 5.35}{10.59+5.35}$$

$$= 7.084\ \Omega$$

The current through the 2 Ω resistor is

$$I_{2\Omega} = \frac{V_{\text{Th}}}{R_{\text{Th}}+2} = \frac{0.4}{7.0841+2} = 0.044 \text{ A (directed from } a \text{ to } b)$$

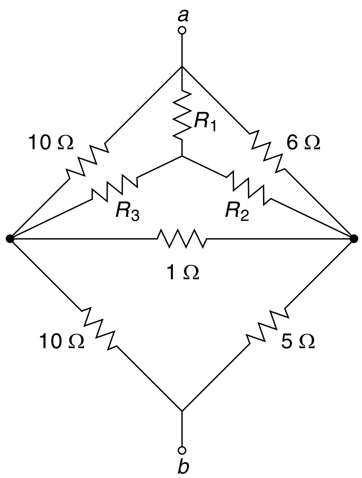



**Fig. 1.91(b)** ***Network reduction to find ($R_{Th}$)***

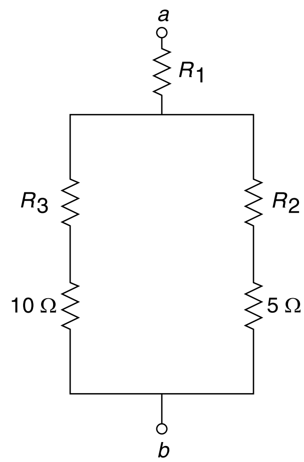



**Fig. 1.91(c)** ***Final network reduction to find ($R_{Th}$)***

**1.65** Find the current through the 5 Ω resistor in the network of Fig. 1.92 using Thevenin's theorem.

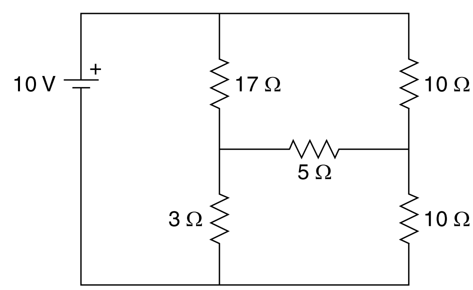



**Fig. 1.92** ***Circuit of Ex. 1.65***

***Solution***

The 5 Ω resistor is first removed. The circuit configuration is shown in Fig. 1.92(a).

The current through 17 Ω resistor is $\frac{10}{17 + 3}$ A = 0.5 A.

The current through pair of 10 Ω resistors is $\frac{10}{10 + 10}$ A = 0.5 A.

Voltage across 17 Ω resistor is $V_{ca} = 17 \times 0.5 = 8.5$ V

Voltage across 10 Ω resistor is $V_{cb} = 10 \times 0.5 = 5$ V

Hence $V_{ab} = V_{cb} - V_{ca} = 5 - 8.5 = -3.5$ V

or $V_{ba} = 3.5$ V (i.e *b* is positive terminal)

i.e. $V_{\text{Th}} = 3.5$ V

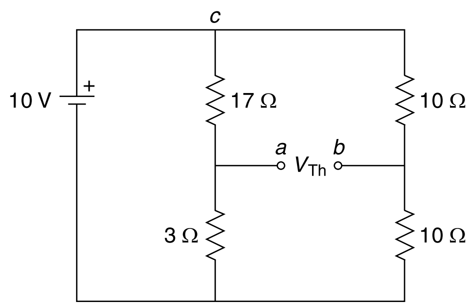



**Fig. 1.92(a)** *Finding of $V_{Th}$*

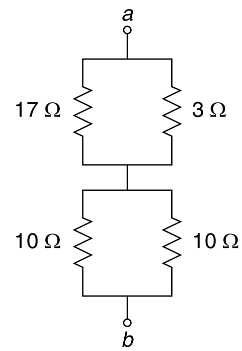



**Fig. 1.92(b)** *Finding of $R_{Th}$*

For finding $R_{Th}$, the circuit is redrawn in Fig. 1.92(b) deactivating the source;

$$R_{Th} = \frac{17 \times 3}{17 + 3} + \frac{10 \times 10}{10 + 10} = 7.55\ \Omega$$

Current through the 5 Ω resistor [Fig. 1.92(c)] is obtained as

$$I_{5\Omega} = \frac{V_{Th}}{R_{Th} + 5} = \frac{3.5}{7.55 + 5}\ \text{A} = 0.279\ \text{A}$$

(flowing from *b* to *a*).

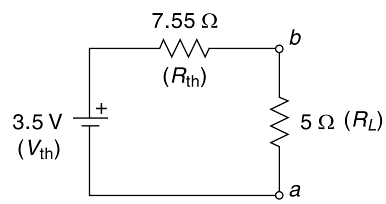



**Fig. 1.92(c)**

**1.66** Find the current in the 5 Ω resistor (using Thevenin's theorem) in Fig. 1.93.

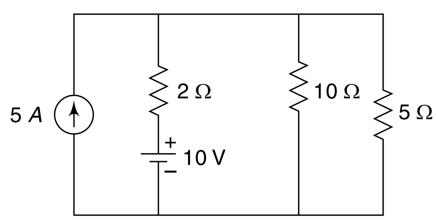



**Fig. 1.93** *Circuit of Ex. 1.66*

### *Solution*

Let us first remove the 5 Ω resistor. The circuit configuration is shown in Fig. 1.93(a).

Applying the Superposition theorem, we consider one source at a time. Considering 5*A* source alone and removing the voltage source, the current through the 10 Ω resistor is $5 \times \frac{2}{2 + 10} = \frac{10}{12}$ A.

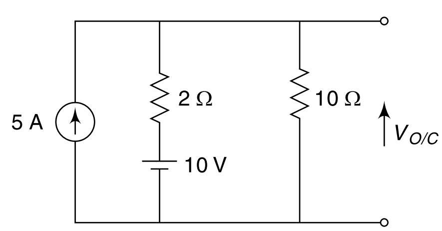



**Fig. 1.93(a)** *Finding of $V_{Th}$ ($V_{O/C}$)*

Considering the voltage source acting alone and removing the current source, the current through the 10 Ω resistor is

$$\frac{10}{2+10} = \frac{10}{12}\text{ A.}$$

Both the currents are directed in the same direction through 10 Ω resistor. So net current through 10 Ω resistor is $\frac{10}{12} + \frac{10}{12} = \frac{20}{12}$ A and voltage across 10 Ω resistor is $\frac{20}{12} \times 10 =$ V.

As 10 Ω is connected in parallel with the open circuited terminals hence,

$$V_{\text{Th}} = \text{Voltage acros 10 } \Omega \text{ resistor}$$

$$= \frac{200}{12} = \frac{50}{3} = 16.67\text{ V.}$$

Removing all the sources, $R_{\text{Th}}$ is found out and is shown in Fig. 1.93(b).

$$R_{\text{Th}} = \frac{10 \times 2}{10 + 2} = \frac{20}{2} = \frac{5}{3} = 1.67\ \Omega$$

So the current through the 5 Ω resistor is obtained as

$$I_{5\Omega} = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{16.67}{1.67 + 5} = 2.5\text{ A}$$

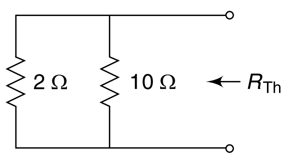



**Fig. 1.93(b)** *Finding of $R_{Th}$*

**1.67** Find the power loss in 10 Ω resistor using Thevenin's theorem (Fig. 1.94).

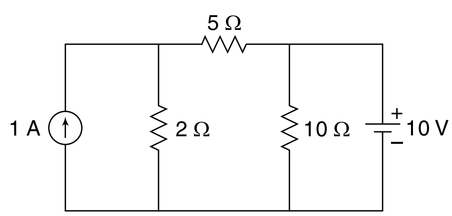



**Fig. 1.94** *Circuit of Ex. 1.67*

***Solution***

Removing 10 Ω resistor the circuit configuration is shown in Fig. 1.94(a).

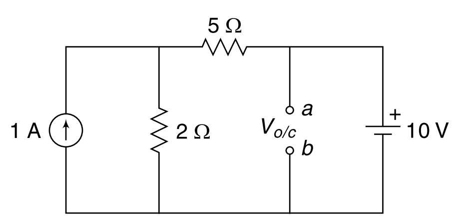



**Fig. 1.94(a)** *Finding of $V_{O/C}$*

As the open circuit voltage ($V_{o/c}$) across terminals *a* and *b* is in parallel with the 10 V source hence the open circuit voltage is becoming 10 V (or $V_{o/c} = V_{\text{Th}} = 10$ V)

Removing all the sources $R_{\text{Th}}$ is found out from Fig. 1.94(b). However there is a short circuit path across *ab*, so $R_{\text{Th}} = 0\ \Omega$.

Current through the 10 Ω resistor according to Thevenin's theorem is $\frac{V_{Th}}{R_{Th} + R_L} = \frac{10}{0 + 10}$ A = 1 A

Therefore power loss in 10 Ω resistor is

$$I^2R = 1^2 \times 10 \text{ W}$$
$$= 10 \text{ W}.$$

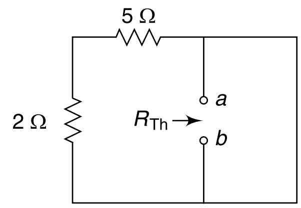



**Fig. 1.94(b)** ***Finding of $R_{Th}$***

**1.68** Find Thevenin's equivalent circuit of the network across $R_L$ in Fig. 1.95.

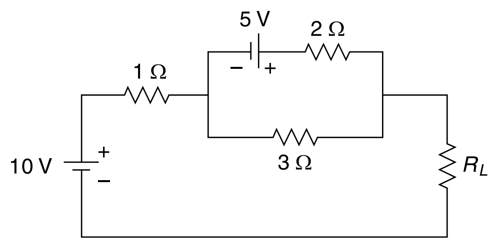



**Fig. 1.95** ***Circuit of Ex. 1.68***

***Solution***

$R_L$ is removed first and the corresponding figure is shown in Fig. 1.95(a). Under this condition 10 V source can not deliver any current. Current due to 5 V source circulates through 2 Ω and 3 Ω resistor

$$\therefore \qquad I = \frac{5}{2+3} \text{ A} = 1 \text{ A}$$

Voltage across 3 Ω resistor is 3 × 1 V = 3 V

Applying KVL in loop *a b c d e f a* of Fig. 1.97(a)

$$V_{ab} = 10 + 3 = 13 \text{ V}$$

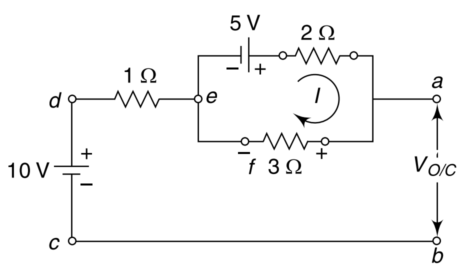



**Fig. 1.95(a)** ***Finding of $V_{O/C}$***

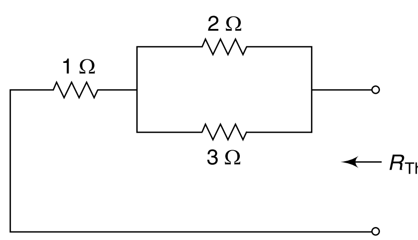



**Fig. 1.95(b)** ***Finding of $R_{Th}$***

Deactivating the sources $R_{Th}$ is found out [Fig. 1.95(b)].

$$\therefore \qquad R_{Th} = 1 + \frac{3 \times 2}{3 + 2}$$
$$= 2.2 \ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 1.95(c).

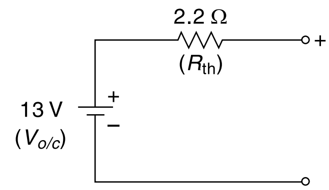



**Fig. 1.95(c)** *Thevenin's equivalent circuit of Ex. 1.68*

## 1.13 NORTON'S THEOREM

According to this theorem, *any two-terminal active network containing voltage sources and resistances when viewed from its output terminals is equivalent to a constant current source and an internal (parallel) resistance. The constant current source (known as Norton's equivalent current source) is of the magnitude of the short circuit current at the terminals. The internal resistance is the equivalent resistance of the network looking back into the terminals with all the sources replaced by their internal resistances.*

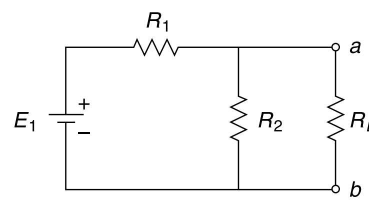



**Fig. 1.96** *Circuit to explain Norton's theorem*

A network is shown in Fig. 1.96 to explain Norton's theorem. Let us find out the current through $R_L$ using Norton's theorem.

The steps are as follows:

***Step I***

Remove ($R_L$) and short circuit the terminals *a* and *b* [Fig. 1.96(a)]. The current through the short circuited path is $I_{sc} = E/R_1$ ($= I_N$), where $I_N$ is the Norton's equivalent current.

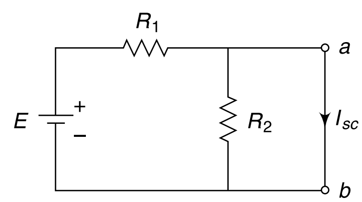



**Fig. 1.96(a)** *Developing Norton's current source*

***Step II***

For finding internal resistance $R_i$ of the network, terminals *a* and *b* is open circuited and the source is deactivated [Fig. 1.96(b)].

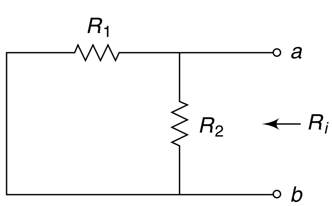



**Fig. 1.96(b)** *Finding of internal resistance*

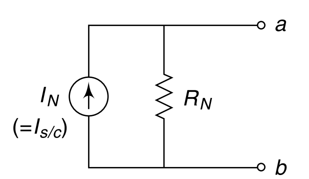



**Fig. 1.96(c)** *Norton's equivalent circuit*

$R_i = \dfrac{R_1 R_2}{R_1 + R_2}$ ($= R_N$), where $R_N$ is called the Norton's equivalent resistance.

***Step III***

Norton's equivalent circuit is shown in Fig. 1.96(c). It contains Norton's current source $I_N$ and a parallel resistance equal to internal resistance of the circuit $R_i$.

***Step IV***

Connect $R_L$ across terminals *a* and *b* and find current $I_L$ through $R_L$

$$I_L = I_N \times \frac{R_N}{R_N + R_L} \tag{1.41}$$

### 1.13.1 Nortonizing Procedure

1. Calculate the short-circut current $I_N$ at the network terminals.
2. Redraw the network with each source replaced by its internal resistance.
3. Calculate the resistance $R_N$ of the redrawn network as seen from the output terminals.
4. Draw Norton's equivalent circuit.

**1.69** Find the current through $R_L$ in Fig. 1.97 using Norton's theorem.

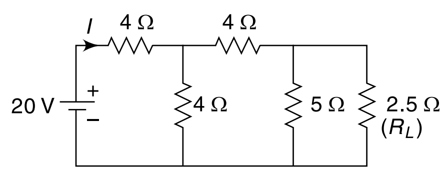



**Fig. 1.97** *Circuit of Ex. 1.69*

***Solution***

$R_L$ is removed and the terminals are short circuited as shown in Fig. 1.97(a). The battery circuit current is

$$I_f = \frac{20}{4 + \frac{4 \times 4}{4 + 4}} = \frac{20}{4 + 2} = 3.34 \text{ A}$$

$\therefore$ Norton's equivalent current $I_N = 1.67$ A $\left[\therefore I_N = 3.34 \times \frac{4}{4+4} = 1.67 \text{ A}\right]$

Norton's equivalent resistance $R_N$ is found from Fig. 1.97(b).

$$R_N = \left(\frac{4 \times 4}{4 + 4} + 4\right) \Big\| 5 = \frac{6 \times 5}{6 + 5} = 2.73\ \Omega.$$

Norton's equivalent circuit is drawn in Fig. 1.97(c).

Replacing $R_L$ across the open circuited terminals as shown in Fig. 1.97(d), the current through $R_L$ is

$$I_L = 1.67 \times \frac{2.73}{2.73 + 2.5} = 0.87 \text{ A}$$

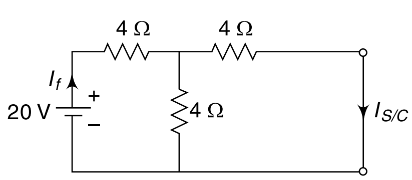



**Fig. 1.97(a)** *Finding of $I_{S/C}$*

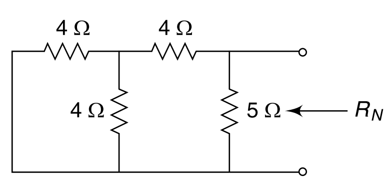



**Fig. 1.97(b)** *Finding of internal resistance $R_N$*

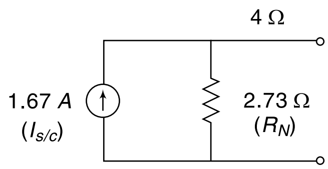



Fig. 1.97(c) *Norton's equivalent circuit*

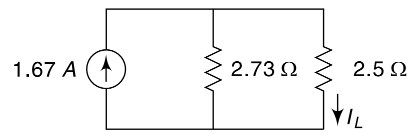



Fig. 1.97(d) *Finding of $I_L$*

**1.70** Find the current through 10 Ω resistor in Fig. 1.98 using Norton's theorem.

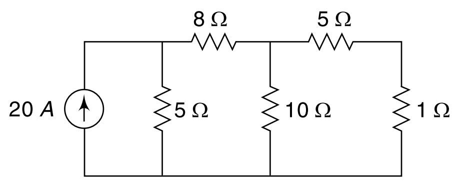



**Fig. 1.98** *Circuit of Ex. 1.70*

### *Solution*

The 10 Ω resistor is removed and the terminals are short circuited as shown in Fig. 1.98(a).

The current through the short circuited path is

$$I_{sc} = 20 \times \frac{5}{5+8} \text{ A} = 7.69 \text{ A}$$

Hence Norton's equivalent current $I_N = 7.69\text{A}$

Norton's equivalent resistance as seen from the open circuited terminals of the network (Fig. 1.98(b)), is obtained as

$$R_N = (8+5) \parallel (5+1) = \frac{13 \times 6}{13+6}\ \Omega = 4.1\ \Omega$$

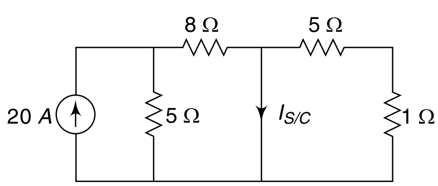



**Fig. 1.98(a)** *Finding of $I_{S/C}$*

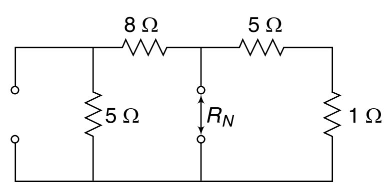



**Fig. 1.98(b)** *Finding of $R_N$*

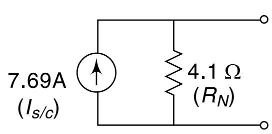



**Fig. 1.98(c)** *Norton's equivalent circuit of Ex. 1.70*

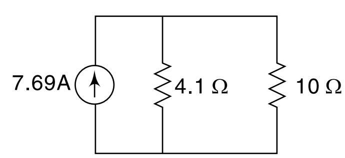



**Fig. 1.98(d)** *Current $I_L$ through the 10 Ω resistor*

Norton's equivalent circuit is shown in Fig. 1.98(c).

Therefore the current through the 10 Ω resistor is [Fig. 1.98(d)]

$$I_L = 7.69 \times \frac{4.1}{4.1 + 10} \text{ A} = 2.236 \text{ A}$$

**1.71** Find current in 6 Ω resistor using Norton's theorem for the network shown in Fig. 1.99.

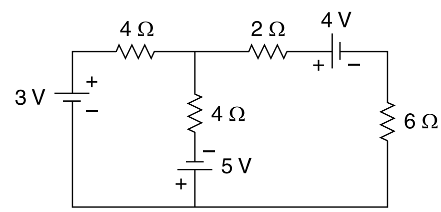



**Fig. 1.99** *Circuit of Ex. 1.71*

***Solution***

The load resistance 6 Ω is short-circuited as shown in Fig. 1.99(a).

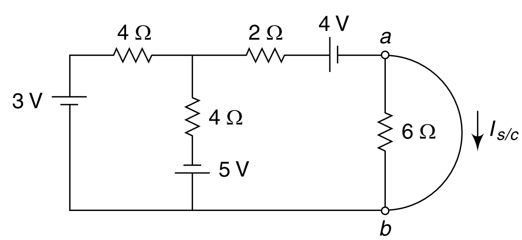



**Fig. 1.99(a)** *Determination of* $I_{S/C}$

The current through the short circuited path *ab* due to the 3 V source acting alone is

$$I_{sc_1} = \frac{3}{4 + \frac{2 \times 4}{2 + 4}} \times \frac{4}{4 + 2} = 0.375 \text{ A} \quad \text{(from } a \text{ to } b\text{)}.$$

The current through the short-circuited path *ab* due to the 5 V source acting alone is

$$I_{sc_2} = \frac{5}{4 + \frac{4 \times 2}{4 + 2}} \times \frac{4}{4 + 2} = \frac{5 \times 4}{24 + 8} = 0.625 \text{ A} \quad \text{(from } b \text{ to } a\text{)}.$$

The current through the short-circuited path *ab* due to the 4 V source acting alone is

$$I_{sc_3} = \frac{4}{2 + \frac{4 \times 4}{4 + 4}} = 1 \text{ A} \quad \text{(from } b \text{ to } a\text{)}.$$

Applying the superposition theorem when all the sources are acting simultaneously the short circuit current is obtained as

$$I_{sc} = (1 + 0.625 - 0.375) \text{ A} = 1.25 \text{ A} \quad \text{(from } b \text{ to } a\text{)}$$

Hence Norton's equivalent current is $I_N = 1.25$ A.

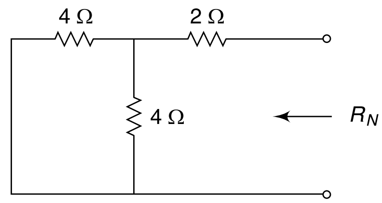



Fig. 1.99(b) *Finding of $R_N$*

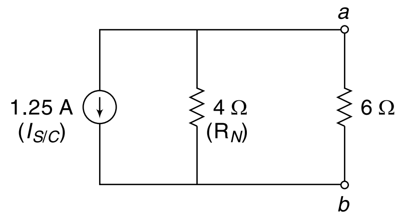



Fig. 1.99(c) *Norton's equivalent circuit*

Norton's equivalent resistance [Fig. 1.99(b)] is obtained as

$$R_N = 2 + \frac{4 \times 4}{4 + 4} = 4\ \Omega$$

The current through 6 Ω resistor is $I = 1.25 \times \dfrac{4}{4+6} = 0.5$ A (from $b$ to $a$).

**1.72** Find current through 5 Ω resistor in the circuit of Fig. 1.100 using Norton's theorem.

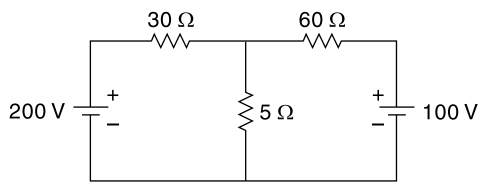



Fig. 1.100 *Circuit of Ex. 1.72*

***Solution***

5 Ω resistor is short circuited as shown in Fig. 1.100 (a). The current through the short-circuited path $I_{sc} = \dfrac{200}{30} + \dfrac{100}{60} = 8.33$ A from $a$ to $b$.

Hence $I_N = 8.33$ A

The Norton's equivalent resistance is obtained by removing all sources and looking from open circuited terminals $a$ and $b$ in Fig. 1.100 (b) is

$$R_N = \frac{30 \times 60}{30 + 60} = 20\ \Omega$$

Therefore, current through the 5 Ω resistor [Fig. 1.100(c)] is $I = 8.33 \times \dfrac{20}{20+5} = 6.664$ A from $a$ to $b$.

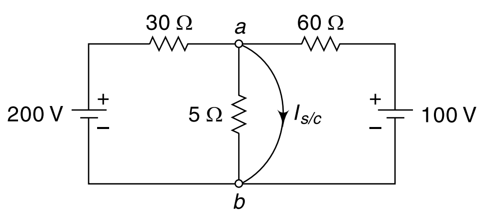



Fig. 1.100(a) *Determination of ($I_{S/C}$)*

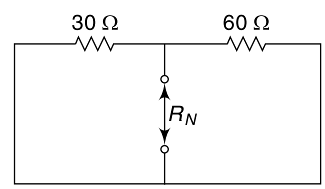



Fig. 1.100(b) *Determination of ($R_N$)*

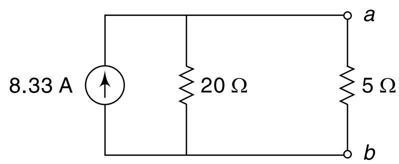



**Fig. 1.100(c)** ***Norton's equivalent circuit of Ex. 1.74***

**1.73** Find the current through $R_L$ in Fig. 1.101 using Norton's theorem.

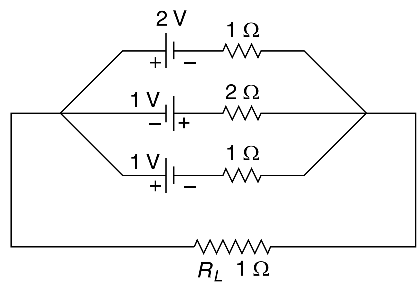



**Fig. 1.101** ***Circuit of Ex. 1.73***

***Solution***

Removing $R_L$ and short circuiting its terminals the network is redrawn in Fig. 1.101(a).

The current through the short circuited path is obtained as

$$I_{sc} = \frac{1}{2} - \frac{2}{1} - \frac{1}{1} = 0.5 - 2 - 1$$

$$= -2.5 \text{ A from } a \text{ to } b \text{ or } 2.5 \text{ A from } b \text{ to } a$$

i.e. $I_N = 2.5$ A.

Removing the sources and open circuiting the short circuited path [as shown in Fig. 1.101(b)], we get

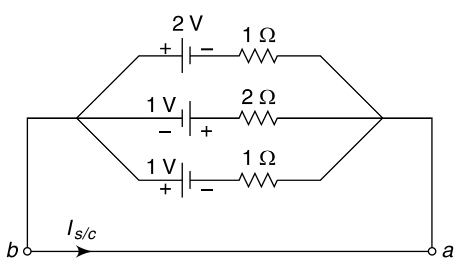



**Fig. 1.101(a)** ***Determination of $I_{S/C}$***

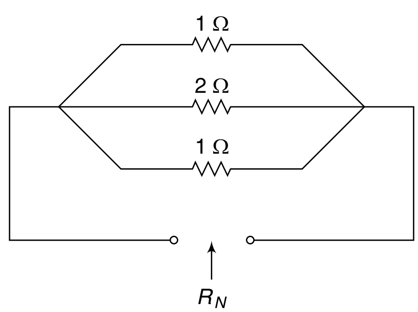



**Fig. 1.101(b)** ***Determination of $R_N$***

$$R_N = \frac{1 \times 2 \times 1}{1 \times 2 + 2 \times 1 + 1 \times 1}\ \Omega = 0.4\ \Omega$$

The current through $R_L$ [Fig 1.101(c)] is ($I$)

$$= 2.5 \times \frac{0.4}{1 + 0.4} \text{ A} = 0.714 \text{ A from } b \text{ to } a.$$

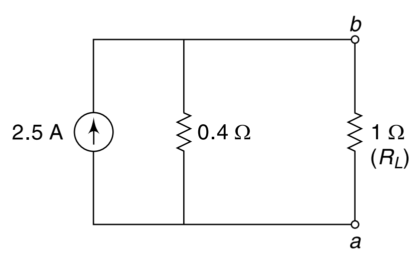



**Fig. 1.101(c)** ***Norton's equivalent circuit of Ex. 1.73***

**1.74** Find current through the 20 Ω resistor in Fig. 1.102 using Norton's theorem.

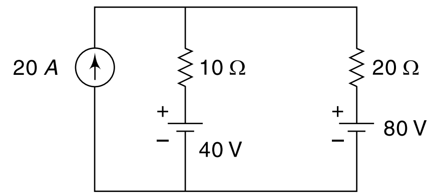



**Fig. 1.102** *Circuit of Ex. 1.74*

***Solution***

Short circuiting 20 Ω resistor [Fig. 1.102(a)] the current through the short circuited path due to 20 A source acting alone is $I_{sc_1}$ = 20 A from *a* to *b*.

Considering the 40 V source acting alone the current through the short circuited path is $I_{sc_2} = \frac{40}{10}$ A = 4 A (from *a* to *b*). Considering the 80 V source acting alone the current through the short circuited path $I_{sc_3} = \frac{80}{10}$ A = 8 A (from *b* to *a*).

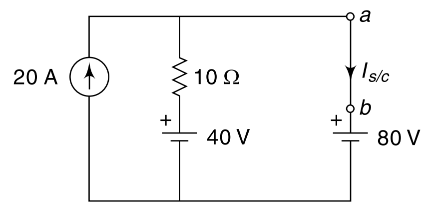



**Fig. 1.102(a)** *Determination of ($I_{S/C}$)*

Applying the Supperposition theorem the net current through the short circuited path $I_{sc}$ = (20 + 4 – 8) A = 16 A (from *a* to *b*).

Thus, Norton's equivalent current is $I_N$ = 16 A. Next, removing all the sources, $R_N$ is found out from Fig. 1.102(b) as $R_N$ = 10 Ω.

From Fig. 1.102(c) the current through the 20 Ω resistor can be found out as

$$I = 16 \times \frac{10}{10 + 20} \text{ A} = 5.33 \text{ A (from } a \text{ to } b).$$

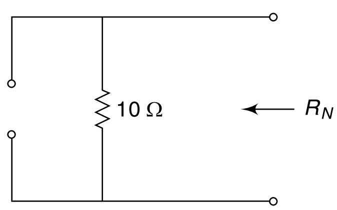



**Fig. 1.102(b)** *Determination of ($R_N$)*

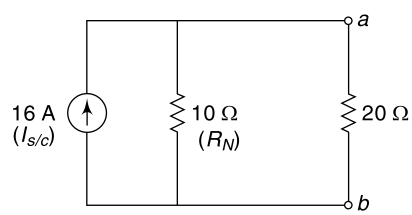



**Fig. 1.102(c)** *Norton's equivalent circuit of Ex. 1.74*

**1.75** Find the current through the 2 Ω resistor in Fig. 1.103 using Norton's theorem.

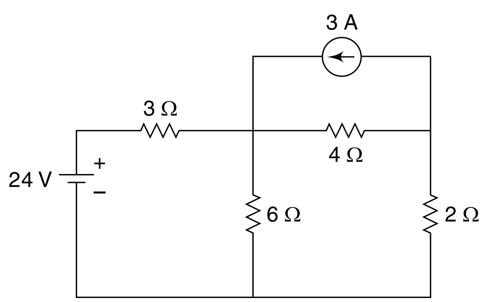



**Fig. 1.103** *Circuit of Ex. 1.75*

***Solution***

Short circuiting the 2 Ω resistor [as shown in Fig. 1.103(a)], and with 25 V source acting alone, the short circuit current through *ab* is $I_{sc_1} = \dfrac{24}{3 + \dfrac{6 \times 4}{6 + 4}} \times \dfrac{6}{6 + 4} = 2.67$ A from *a* to *b*.

Next with 3 A source acting alone, the current through *ab* is $I_{sc_2} = 3 \times \dfrac{4}{4 + \dfrac{3 \times 6}{3 + 6}} = 2$ A from *b* to *a*.

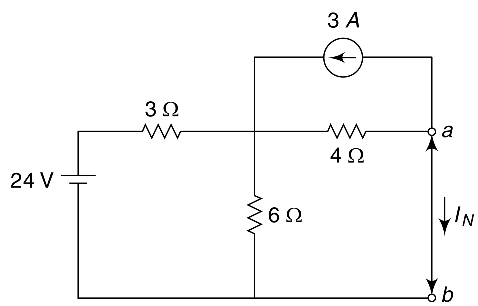



**Fig. 1.103(a)** ***Determination of $I_N$***

∴ the current through *ab* is

$$I_N = I_{sc_1} - I_{sc_2} = 2.67\text{ A} - 2\text{ A} = 0.67\text{ A} \qquad \text{from } a \text{ to } b.$$

Norton's equivalent resistance [Fig. 1.103(b)] is

$$R_N = 4 + \frac{3 \times 6}{3 + 6} = 6\ \Omega$$

The current through the 2 Ω resistor [Fig. 1.103(c)] is $I = 0.67 \times \dfrac{6}{6 + 2} = 0.5$ A from *a* to *b*.

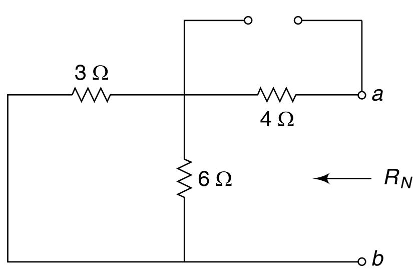



**Fig. 1.103(b)** ***Determination of $R_N$***

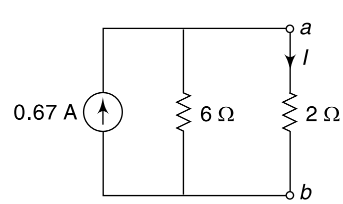



**Fig. 1.103(c)** ***Norton's equivalent circuit***

**1.76** Find Norton's equivalent circuit for the network shown in Fig. 1.104.

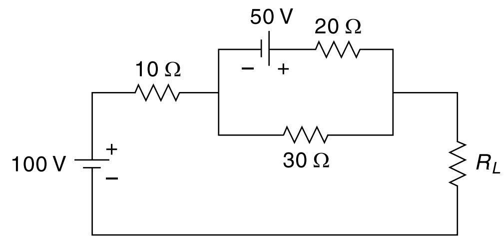



**Fig. 1.104** *Circuit of Ex. 1.76*

***Solution***

Remove $R_L$ and short circuit the terminals [as shown in Fig. 1.104(a)].

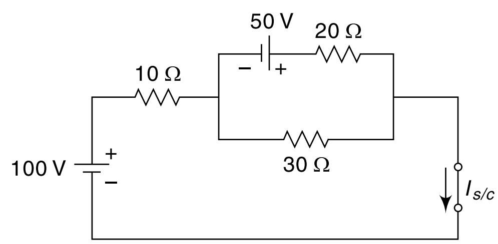



**Fig. 1.104(a)** *Determination of $I_{S/C}$*

The short circuit current is

$$I_{sc} = \frac{100}{10 + \dfrac{20 \times 30}{20 + 30}} + \frac{50}{20 + \dfrac{30 \times 10}{30 + 10}} \times \frac{30}{30 + 10}$$

$$= \frac{100}{10 + 12} + \frac{50 \times 30}{800 + 300}$$

$= 5.9$ A from $a$ to $b$ i.e. $I_N = 5.9$ A.

Norton's equivalent resistance looking back from the open circuited terminals [Fig. 1.104(b)] is $(R_N) = 10 + \dfrac{20 \times 30}{20 + 30} = 22\ \Omega$.

Norton's equivalent circuit is shown in Fig. 1.104(c).

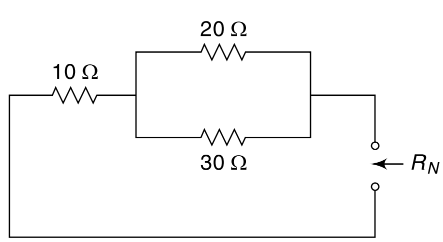



**Fig. 1.104(b)** *Finding of $R_N$*

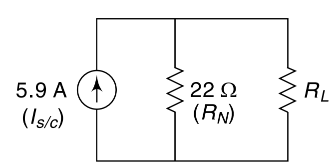



**Fig. 1.104(c)** ***Norton's equivalent circuit of Ex. 1.76***

**1.77** Find the current through the 1 Ω resistor in the network shown in Fig. 1.105 using Norton's theorem.

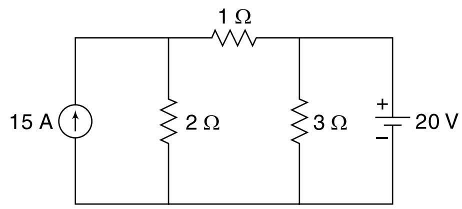



**Fig. 1.105** *Circuit of Ex. 1.77*

***Solution***

1 Ω resistor is removed and terminals are short circuited as shown in Fig. 1.105(a).

The current through the short circuited path is

$$I_{sc} = 15 - \frac{20}{\frac{2 \times 3}{2 + 3}} \times \frac{3}{3 + 2}$$

$$= 15 - \frac{20 \times 3}{6}$$

$$= 5 \text{ A (from } a \text{ to } b)$$

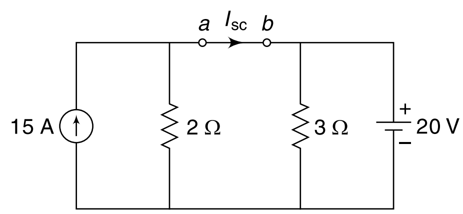



**Fig. 1.105(a)** ***Determination of*** $I_{S/C}$

Removing all the sources and open circuiting terminals *a* and *b* [Fig. 1.105(b)], $R_N = 2\ \Omega$

Thus the current through 1 Ω resistor [Fig. 1.105(c)] is

$$I = 5 \times \frac{2}{2 + 1} \text{ A} = 3.33 \text{ A}$$

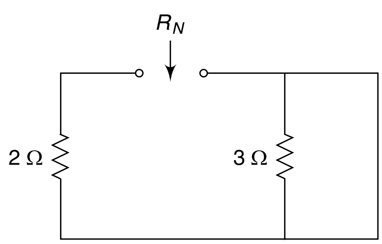



**Fig. 1.105(b)** ***Determination of*** $R_N$

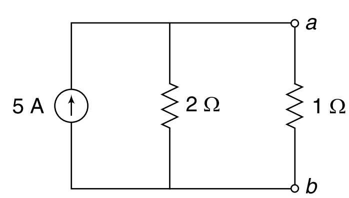



**Fig. 1.105(c)** ***Norton's equivalent circuit***

**1.78** Find the current through 8 Ω resistor using Norton's theorem in the network of Fig. 1.106.

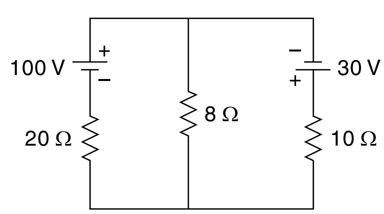



**Fig. 1.106** *Circuit of Ex. 1.78*

### *Solution*

Short circuiting the 8 Ω resistor as shown in Fig. 1.106(a), the current through the short circuited path is $I_N = \frac{100}{20} - \frac{30}{10} = 2$ A (from *a* to *b*).

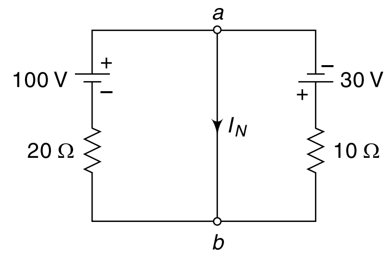



**Fig. 1.106(a)** ***Determination of*** $I_N$

Open circuiting *ab* and removing the sources the Norton's equivalent resistance [Fig. 1.106(b)] is

$$R_N = \frac{20 \times 10}{20 + 10}\ \Omega = 6.67\ \Omega$$

The current through the 8 Ω resistor [from Fig. 1.106(c)] is $(I) = 2 \times \frac{6.67}{6.67 + 8} = 0.9$ A.

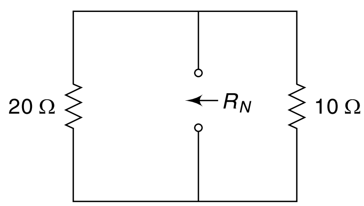



**Fig. 1.106(b)** ***Determination of*** $R_N$

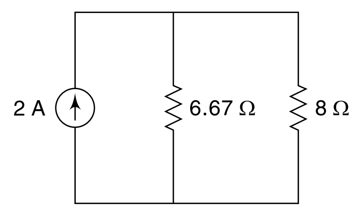



**Fig. 1.106(c)** ***Norton's equivalent circuit of Ex. 1.78***

**1.79** Find the current through the 20 Ω resistor in Fig. 1.107 using Norton's theorem.

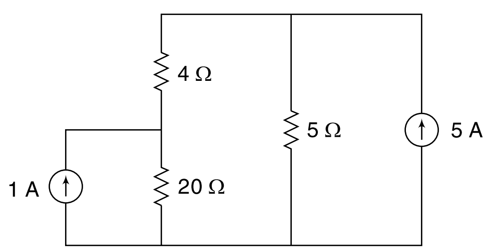



**Fig. 1.107** ***Circuit of Ex. 1.79***

### *Solution*

The 20 Ω resistor is short circuited and the circuit is redrawn in Fig. 1.107(a). The current through the short-circuited path due to 1 A current source only is $I_{sc_1} = 1$ A (from *a* to *b*).

The current through the short-circuited path due to 5 A source only is $I_{sc_2} = 5 \times \frac{5}{5+4}$ A $= \frac{25}{9}$ A (from *a* to *b*). Norton's equivalent current is then

$$I_N\ (= I_{sc}) = I_{sc_1} + I_{sc_2} = 1 + \frac{25}{9} = 3.78\ \text{A}.$$

Next, removing the sources and open-circuiting terminals *a* and *b* [as shown in Fig. 1.107(b)] $R_N$ is obtained as $(R_N) = 4 + 5 = 9\ \Omega$.

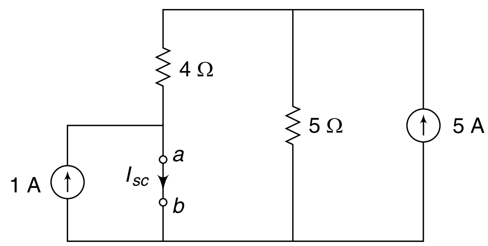



**Fig. 1.107(a)** *Determination of $I_{S/C}$*

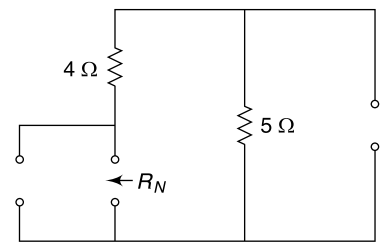



**Fig. 1.107(b)** *Finding of $R_N$*

The current $I$ through the 20 Ω resistor is obtained from Fig. 1.107(c), where

$$I = 3.78 \times \frac{9}{9+20} = 1.173 \text{ A (from } a \text{ to } b)$$

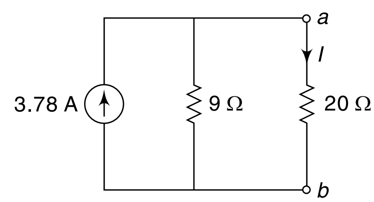



**Fig. 1.107(c)** *Norton's equivalent circuit of Ex. 1.79*

## 1.14 EQUIVALENCE OF THEVENIN'S AND NORTON'S THEOREMS

Figure 1.108 shows the equivalency of Thevenin's and Norton's theorems. It can be proved that the equivalent circuits given by Thevenin's and Norton's theorem yield exactly the same current and same voltage in the load impedance and they are

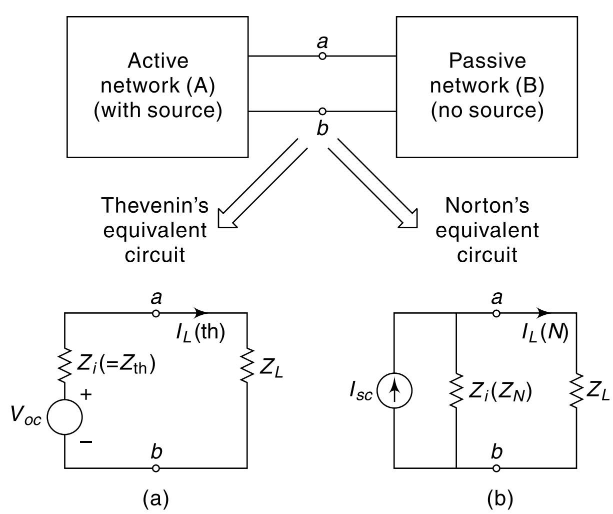



**Fig. 1.108** *Equivalence of Thevenin's and Norton's circuits*

effectively identical to one another. In any particular problem, either theorem can therefore be used. In most cases Thevenin's theorem is the easier to apply, although when the network impedance is high compared with the load impedance, the Norton's theorem concept may simplify calculations.

From Fig. 1.108(a) by applying Thevenin's theorem the load current is given by

$$I_{L(\text{Th})} = \frac{V_{oc}}{Z_i + Z_L} \tag{1.42}$$

where $V_{oc}$ = Open circuit voltage (Thevenin's equivalent voltage source)

$Z_i$ = Thevenin's equivalent impedance (or resistance for circuit), and

$Z_L$ = Load impedance of the load network.

On short circuiting the terminals *a* and *b* of the Thevenin's equivalent,

$$I_{sc} = \frac{V_{oc}}{Z_i} \tag{1.43}$$

or

$$V_{oc} = I_{sc} \times Z_i \tag{1.44}$$

However from Norton's equivalent circuit [Fig. 1.108(b)], the load current is given by

$$I_{L(N)} = \frac{I_{sc} \times Z_i}{Z_i + Z_L} \tag{1.45}$$

Substituting the equation (1.44) in equation (1.45),

$$I_{L(N)} = \frac{V_{oc}}{Z_i + Z_L} \tag{1.46}$$

Comparing equation (1.42) and equation (1.46)

$$I_{L(\text{Th})} \equiv I_{L(N)} \tag{1.47}$$

Thus for any passive network, being connected to an active network, one can have equivalent representation of Norton's equivalent or Thevenin's equivalent circuit (i.e. both the theorems are equivalent to each other). For easy understanding, a simple example is shown in the circuit of Fig. 1.109(a).

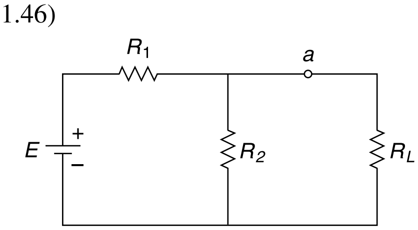



**Fig. 1.109(a)** ***Circuit for illustrating equivalence of Thevenin's and Norton's theorems***

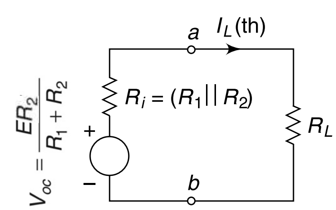



**Fig. 1.109(b)** ***Thevenin's equivalent circuit***

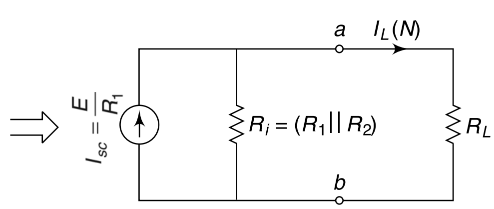



**Fig. 1.109(c)** ***Norton's equivalent circuit***

From Fig. 1.109(b) the load current is

$$I_{L(\text{Th})} = \frac{\dfrac{ER_2}{R_1 + R_2}}{\dfrac{R_1\, R_2}{R_1 + R_2} + R_L} = \frac{ER_2}{R_1\, R_2 + R_1\, R_L + R_2\, R_L} \tag{1.48}$$

$\Big[\because$ In Fig. 1.111(a), removing $R_L$ the equivalent resistance $R_i$ looking back to the network from $a - b$, is $\left\{\dfrac{R_1 R_2}{R_1 + R_2}\right\}$ and $V_{oc}$ is then $\left\{\left(\dfrac{E}{R_1 + R_2}\right) \times R_2\right\}\Big]$.

On the other hand, from Fig. [1.109(c)] the load current is given by

$$I_{L(N)} = \frac{\dfrac{E}{R_1} \times \dfrac{R_1\, R_2}{R_1 + R_2}}{\dfrac{R_1\, R_2}{R_1 + R_2} + R_L} = \frac{\dfrac{ER_2}{R_1 + R_2}}{\dfrac{R_1\, R_2 + R_1\, R_L + R_2\, R_L}{R_1 + R_2}}$$

$$= \frac{ER_2}{R_1\, R_2 + R_1\, R_L + R_2\, R_L}$$

[$\because$ Removing $R_2$ from $a - b$ terminal and applying short circuit at $a - b$, current through the terminals $a - b$ is ($I_{sc}$) i.e. $\left(\dfrac{E}{R_1}\right)$ while the internal resistance of the network is $\left\{R_i = \dfrac{R_1\, R_2}{R_1 + R_2}\right\}$].

$$\therefore \qquad I_{L(\text{Th})} = I_{L(N)} = \frac{ER_2}{R_1\, R_2 + R_1\, R_L + R_2\, R_L} \tag{1.49}$$

## 1.15 MAXIMUM POWER TRANSFER THEOREM

As applied to dc networks this theorem may be stated as follows: *A resistive load abstracts maximum power from a network when the load resistance equals the internal resistance of the network as viewed from the output terminals, with all energy sources removed, leaving behind their internal resistances.*

This theorem is aplicable to all branches of electrical engineering including analysis of communication networks. However the overall efficency of a network supplying maximum power to any branch is only 50%; hence application of this theorem to power transmission and distribution networks is limited because in that case, the final target is high efficiency and not maximum power transfer. But in electronics and communication network as the purpose is to receive or transmit maximum power, even at low efficiency, the problem of maximum power transfer is of crucial importance in the operation of communication lines and antennas.

**Illustration**

Figure 1.110 shows a simple resistive network in which a load resistance $R_L$ is connected across terminals $a$ and $b$ of the network. The network consists of a generator emf($E$) and internal resistance $r$ along with a series resistance $R$. The internal resistance of the network as viewed from the terminals $a$ and $b$ is $(R_i) = r + R$.

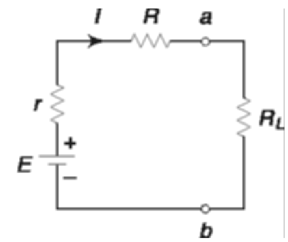



**Fig. 1.110** ***Circuit for illustrating maximum power transfer theorem***

According to maximum power transfer theorem $R_L$ will abstract maximum power from the network when

$$R_i = R_L \text{ or } R_L = (r + R).$$

### 1.15.1 Proof of Maximum Power Transfer Theorem

Let us assume that current $I$ flows through $R_L$ in the circuit shown in Fig. 1.110. Obviously, $I = \dfrac{E}{R_i + R_L}$

Power across the load $(P_L) = I^2 R_L = \dfrac{E^2}{(R_i + R_L)^2} \cdot R_L = \dfrac{E^2 R_L}{(R_i + R_L)^2}$ (1.50)

For $P_L$ to be maximum,

$$\frac{dP_L}{dR_L} = 0$$

Differentiating Eq. (1.50),

$$\frac{dP_L}{dR_L} = E^2 \left[ \frac{(R_i + R_L)^2 - 2R_L (R_i + R_L)}{R_i + R_L)^4} \right] = 0$$

or $\quad R_i + R_L = 2R_L$

or $\quad R_L = R_i = r + R$

Thus for maximum power transfer, $R_L = R_i$.

The maximum power is $(P_{L\max}) = I^2 R_L = \dfrac{E^2}{(R_L + R_L)^2} \times R_L = \dfrac{E^2}{4R_L}$

The power delivered by the source is $(EI) = \dfrac{E^2}{(R_L + R_L)} = \dfrac{E^2}{2R_L}$.

So the effeciency under maximum power transfer condition is $\dfrac{E^2/4R_L}{E^2/2R_L} = \dfrac{1}{2}$ (or 50%).

**1.80** Calculate the value of $R_L$ which will abstract maximum power from the circuit shown in Fig. 1.111 Also find the maximum power.

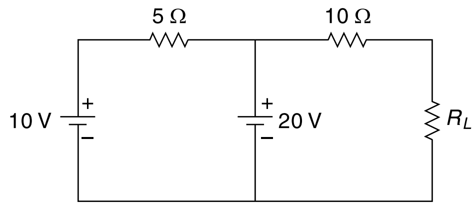



**Fig. 1.111** *Circuit of Ex. 1.80*

***Solution***

Removing all the sources and open circuiting the terminals of $R_L$ [Fig 1.111(a)] the internal resistance $R_i$ of the network is found out as 10 Ω.

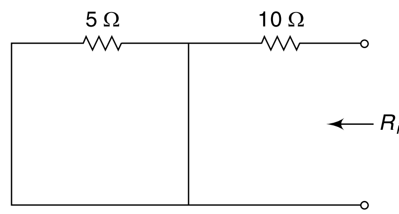



**Fig. 1.111(a)** *Determination of ($R_i$)*

i.e., $R_i = 10\ \Omega$

∴ For maximum power transfer

$$R_L = R_i = 10\ \Omega$$

Again for $R_L = 10\ \Omega$, the total current through $R_L$ due to both sources is given by

$$I = \frac{20}{\dfrac{5 \times (10 + 10)}{5 + 10 + 10}} \times \frac{5}{5 + 10 + 10} = 1\text{ A}$$

[The current due to 10 V source circulates through 5 Ω resistor and 20 V source only]

The maximum power across load is

$$I^2R_L = (1)^2 \times 10 = 10\text{ W}$$

**1.81** Calculate the value of $R_L$ which will absorb maximum power from the circuit shown in Fig. 1.112. Also calculate the value of this maximum power.

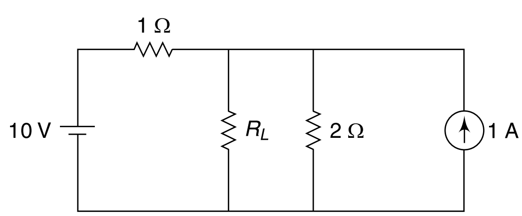



**Fig. 1.112** *Circuit of Ex. 1.81*

***Solution***

Let $R$ be removed and internal resistance of the network is calculated looking from the open circuited terminals after removing all the sources as shown in Fig. 1.112(a).

Here $$R_i = \frac{1 \times 2}{1 + 2}\ \Omega = \frac{2}{3}\ \Omega$$

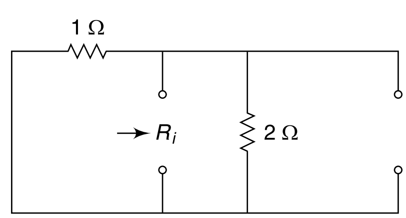



**Fig. 1.112(a)** *Determination of ($R_i$)*

i.e. $R = R_i = \frac{2}{3}\ \Omega = 0.667\ \Omega$ [for maximum power transfer]

The current through $R$ due to both the sources acting simultaneously is given by

$$I = \frac{10}{1 + \frac{0.667 \times 2}{0.667 + 2}} \times \frac{2}{2 + 0.667} + 1 \times \frac{\frac{2 \times 1}{2 + 1}}{0.667 + \frac{2 \times 1}{2 + 1}}$$

$$= 4.998 + 0.5 = 5.5 \text{ A}$$

The value of the maximum power is $(5.5)^2 \times 0.667$ W = 20 W.

**1.82** Obtain the maximum power transferred to $R_L$ in the circuit of Fig. 1.113 and also the value of $R_L$.

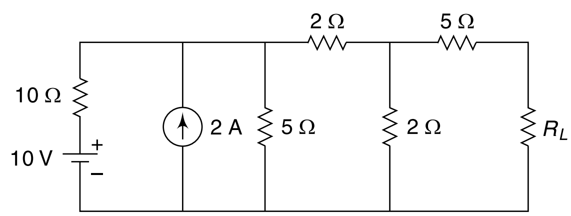



**Fig. 1.113** *Circuit of Ex. 1.82*

### *Solution*

$R_L$ is removed and its terminals are open circuited. Deactivating the sources the internal resistance $R_i$ of the network can be found out from Fig. 1.113(a).

$$R_i = \left[\left(\frac{10 \times 5}{10 + 5} + 2\right) \| \ 2\right] + 5 = 6.45\ \Omega$$

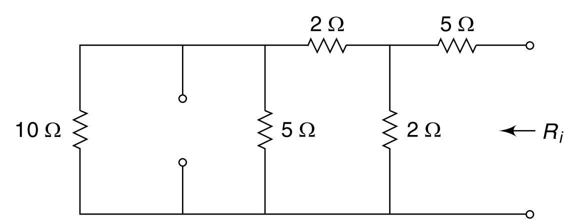



**Fig. 1.113(a)** *Finding of ($R_i$)*

Thus, according to the maximum power transfer theorem the value of $R_L$ is 6.45 Ω for maximum power transfer.

Next, considering the 10 V source acting alone in the network the total current supplied by the 10 V source [Fig. 1.113(b)] is

$$I = \frac{10}{10 + \frac{5 \times 3.7}{5 + 3.7}} = 0.82 \text{ A}.$$

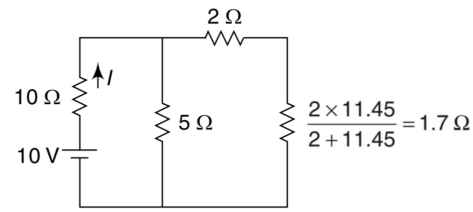



**Fig. 1.113(b)** *Current (I) for 10 V source only*

∴ Current through $R_L$ due to the 10 V source only is

$$I_1 = 0.82 \times \frac{5}{5 + 2 + 1.7} \times \frac{2}{2 + 5 + 6.45} = 0.07 \text{ A.}$$

Again considering the 2 A source acting alone, the current through the 2 Ω resistor is

$$I' = 2 \times \frac{10}{10 + \frac{5 \times 3.7}{5 + 3.7}} \times \frac{5}{5 + 2 + 1.7} \text{ A} = 0.4739 \text{ A}$$

Hence the current due to the 2 A current source through $R_L$ is

$$I_2 = 0.4739 \times \frac{2}{2 + 5 + 6.45} \text{ A} = 0.07 \text{ A}$$

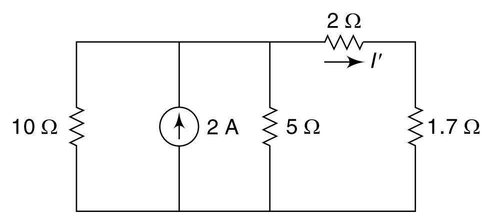



**Fig. 1.113(c)** *Determination of current through 2 Ω resistor for 2 A source only*

Applying the superposition theorem current through $R_L$ (when both the sources are acting simultaneously) is

$$I = I_1 + I_2 = 0.07 + 0.07 = 0.14 \text{ A}$$

∴ Maximum power transferred across $R_L$ is

$$I^2 R_L = (0.14)^2 \times 6.45 = 0.126 \text{ W.}$$

**1.83** Find the value of $R$ in the circuit of Fig. 1.114 such that maximum power transfer takes place. What is the amount of this power?

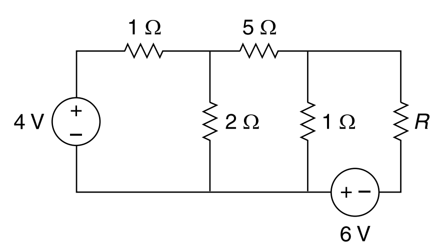



**Fig. 1.114** *Circuit of Ex. 1.83*

***Solution***

Deactivating all the sources, internal resistance $R_i$ of the network is found out as shown in Fig. 1.114(a).

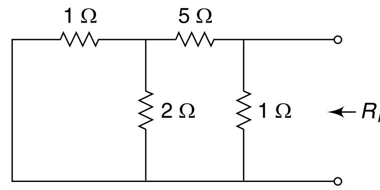



**Fig. 1.114(a)** *Determination of ($R_i$)*

$$R_i = \left(\frac{2 \times 1}{2+1} + 5\right) \parallel 1$$

$$= \frac{5.67 \times 1}{5.67 + 1} = 0.85\ \Omega$$

According to the maximum power transfer theorem the maximum power takes place across $R$ when $R = R_i = 0.85\ \Omega$.

The current through $R$ due to the 4 V source acting alone is

$$I_1 = \frac{4}{\left[\left(\frac{0.85 \times 1}{0.85+1} + 5\right) \parallel 2\right] + 1} \times \frac{2}{2 + 5 + \frac{1 \times 0.85}{1 + 0.85}} \times \frac{1}{1 + 0.85}$$

$$= \frac{4}{\frac{5.46 \times 2}{5.46 + 2} + 1} \times \frac{2}{13.8} = 0.235\ \text{A}$$

The current through ($R$) due to 6 V source acting alone is

$$I_2 = \frac{6}{\left[\left(\frac{1 \times 2}{1+2} + 5\right) \parallel 1\right] + 0.85} = \frac{6}{\frac{5.67 \times 1}{5.67 + 1} + 0.85} = 3.53\ \text{A}$$

According to superposition, the current through $R$ when both the sources are acting simultaneously is

$$I = I_1 + I_2 = 0.235 + 3.53 = 3.765\ \text{A}.$$

Thus the maximum power is $I^2R = (3.765)^2 \times 0.85 = 12$ W.

**1.84** Assuming maximum power transfer from the source to $R$ find the value of this power in the circuit of Fig. 1.115.

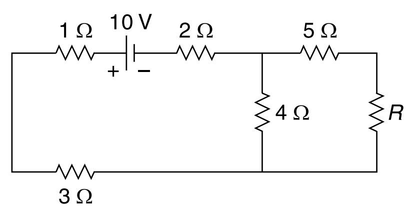



**Fig. 1.115** *Circuit of Ex. 1.84*

***Solution***

Deactivating the source the internal resistance $R_i$ of the network is found from Fig. 1.115(a);

$$R_i = 5 + \frac{4\,(1+2+3)}{4 + (1+2+3)}$$

$$= 5 + \frac{4 \times 6}{10} = 5 + 2.4 = 7.4\ \Omega$$

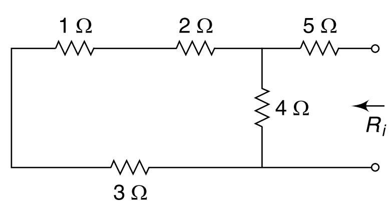



**Fig. 1.115(a)** *Determination of ($R_i$)*

According to the maximum power transfer theorem, the maximum power transfer from the source to $R$ occurs when

$$R = R_i = 7.4\ \Omega.$$

The current through $R$ due to 10 V source is

$$I = \frac{10}{1 + 2 + 3 + \frac{4 \times (5 + 7.4)}{4 + (5 + 7.4)}} \left(\frac{4}{4 + 5 + R}\right) \text{A} = 0.27\ \text{A}$$

Hence the maximum power transfer from source to $R$ is

$$I^2R = (0.27)^2 \times 7.4 = 0.539 \text{ W.}$$

**1.85** Find the value of $R_L$ for which the power transfer across $R_L$ is maximum and find the value of this maximum power [Fig. 1.116].

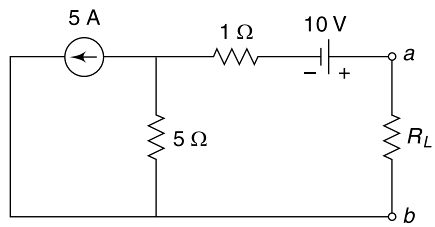



**Fig. 1.116** *Circuit of Ex. 1.85*

***Solution***

Deactivating the sources the internal resistance of the network is found out looking back from the open circuited terminals of $R_L$ [as shown Fig. 1.116(a)].

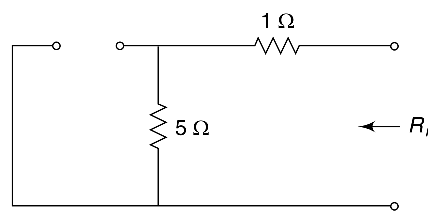



**Fig. 1.116(a)** *Determination of ($R_i$)*

$\therefore$ $$R_i = 1 + 5 = 6\ \Omega.$$

Power transfer across $R_L$ is maximum when

$$R_i = R_L = 6\ \Omega.$$

The current through ($R_L$) is

$$I = \frac{10}{6+5+1} - 5 \times \frac{5}{5+1+6} \quad \text{(from } a \text{ to } b\text{)}$$

$$= -1.25 \text{ A (from } a \text{ to } b\text{)} \quad \text{or} \quad 1.25 \text{ A from (} b \text{ to } a\text{)}$$

The value of the maixmum power is obtained as

$$I^2R_L = (1.25)^2 \times 6 = 9.375 \text{ W.}$$

## ADDITIONAL PROBLEMS

**A1.1** Determine the Thevenin's equivalent of the bridge network shown in Fig. 1.117 as seen from the galvanometer terminals $B$ and $D$ and hence determine the galvanometer current when $R_G = 50\ \Omega$.

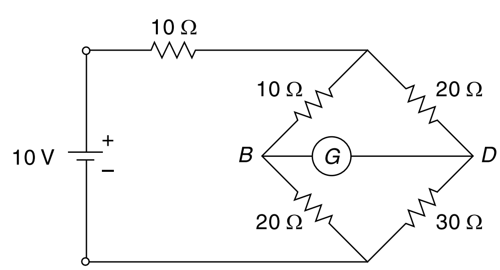



**Fig. 1.117** *Circuit of Ex. A1.1*

***Solution***

To find the Thevenin's equivalent voltage across *BD*, the galvanometer is open-circuited and the corresponding figure is shown in Fig. 1.117(a). The circuit of Fig. 1.117(a) can then be reduced to that shown in Fig. 1.117(b).

$$\text{Current through the } 30\ \Omega \text{ resistor} = \frac{10}{10 + \dfrac{30 \times 50}{30 + 50}} \times \frac{50}{50 + 30} = 0.217 \text{ A.}$$

$$\text{Current through the } 50\ \Omega \text{ resistor} = \frac{10 \times 30}{2300} \text{ A} = 0.13 \text{ A.}$$

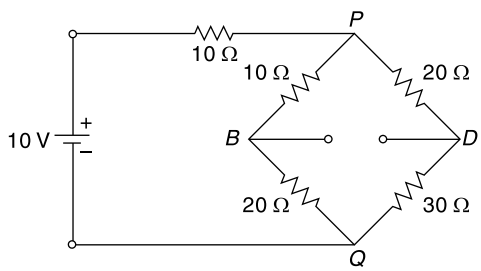



**Fig. 1.117(a)** ***Circuit configuration with galvanometer removed***

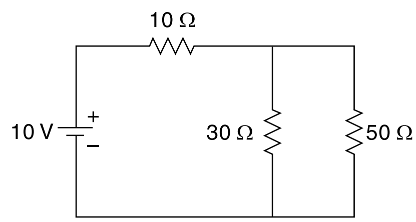



**Fig. 1.117(b)** ***Reduced network***

∴ Currents through *PB* and *PD* in Fig. 1.117(a) are 0.217 A and 0.13 A respectively.

$V_{\text{Th}} = V_{BD} = V_{PD} - V_{PB}$

$= 20 \times 0.13 - 0.217 \times 10 = 0.43$ V

To find Thevenin's equivalent resistance the voltage source is short circuited as shown in Fig. 1.117(c)

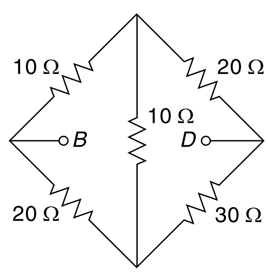



**Fig. 1.117(c)** ***Finding of $R_{Th}$***

Converting delta network into equivalent star network Fig. 1.117(d) is obtained.

$$R_1 = \frac{10 \times 10}{10 + 20 + 10} = 2.5\ \Omega$$

$$R_2 = \frac{10 \times 20}{40} = 5\ \Omega$$

$$R_3 = \frac{20 \times 10}{40} = 5\ \Omega$$

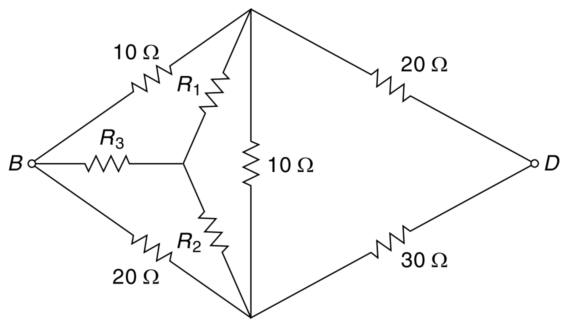



**Fig. 1.117(d)** *Network reduction for network shown in Fig. 1.117(c)*

The equivalent resistance across terminal *BD* can be found out from Fig. 1.117(e) as

$$R_{\text{Th}} = 5 + \frac{22.5 \times 35}{22.5 + 35} = 18.696\ \Omega.$$

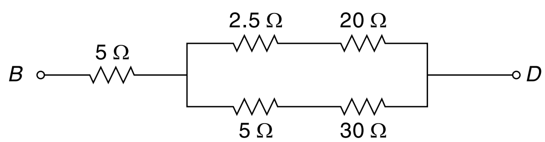



**Fig. 1.117(e)** *Finally reduced network*

Thevenin's equivalent of the bridge network is shown in Fig. 1.117(f).

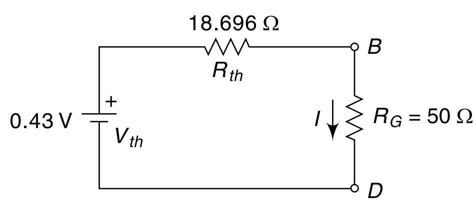



**Fig. 1.117(f)** *Thevenin's equivalent of Ex. A1.1*

The galvanometer current is given by

$$I = \frac{0.43}{18.696 + 50}\ \text{A} = 0.0063\ \text{A} = 6.3\ \text{mA}$$

**A1.2** From the circuit shown in Fig. 1.118, use loop analysis to determine the loop currents $I_1, I_2, I_3$.

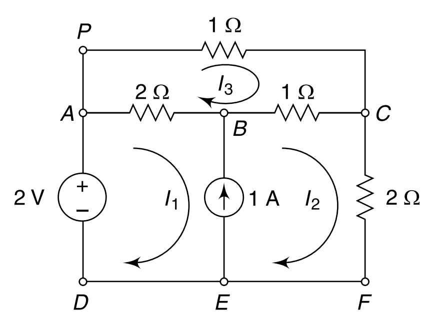



**Fig. 1.118** *Circuit of Ex. A1.2*

***Solution***

From Fig. 1.118 the current source of 1 A is equivalent to $(I_2 - I_1)$, i.e., $I_2 - I_1 = 1$ A. (i)

From loop A*BCPA*

$$2(I_3 - I_1) + 1(I_3 - I_2) + I_3 \times 1 = 0$$

or $-2I_1 - I_2 + 4I_3 = 0$ (ii)

From loop A$BCFEDA$

$$2(I_1 - I_3) + 1(I_2 - I_3) + 2I_2 - 2 = 0$$

or $\quad 2I_1 + 3I_2 - 3I_3 = 2 \qquad$ (iii)

Solving the three equations (i), (ii), (iii) we obtain

$$I_1 = -\frac{1}{11}\text{ A}, \quad I_2 = \frac{10}{11}\text{ A} \quad \text{and} \quad I_3 = \frac{2}{11}\text{ A}.$$

**A1.3** Determine the Thevenin's equivalent circuit with respect to terminals A, $B$ for the network shown in Fig. 1.119.

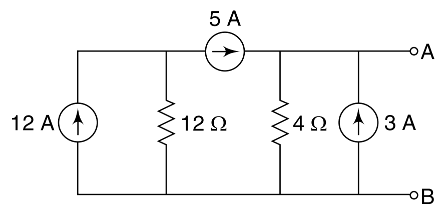



**Fig. 1.119** *Circuit of Ex. A1.3*

***Solution***

From Fig. 1.119 it is evident that open circuit voltage $V_{oc}$ between A and $B$ is the voltage across the 4 Ω resistor. The current through the 4 Ω resistor due to the 12 A source is zero due to the presence of 5 A current source. The other two current sources deliver 5 A and 3 A current in the same direction through the 4 Ω resistor. So voltage across the 4 Ω resistor $V_{oc}$ is [4 × (5 + 3) = 32 V]. Next, removing all the sources Thevenin's equivalent resistance can be obtained as shown in Fig. 1.119(a).

Here $\qquad R_{\text{Th}} = 4\ \Omega$

The Thevenin's equivalent circuit is shown in Fig. 1.119(b).

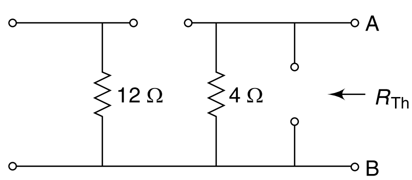



**Fig. 1.119(a)** *Determination of Thevenin's equivalent only resistance*

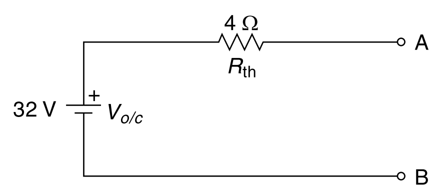



**Fig. 1.119(b)** *Thevenin's equivalent network of Ex. A1.3*

**A1.4** In the network shown in Fig. 1.120 determine all branch currents and the voltage across the 5 Ω resistor by loop current analysis.

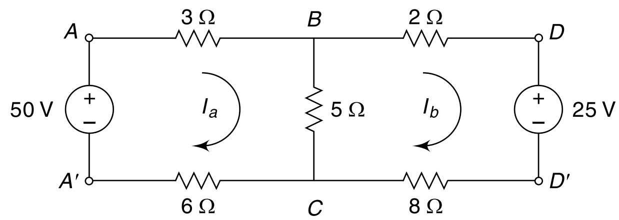



**Fig. 1.120** *Circuit of Ex. A1.4*

***Solution***

Let $I_a$ and $I_b$ be the loop currents. Applying KVL to loop *ABCA′A*

$$3I_a + 5(I_a - I_b) + 6I_a = 50$$

or $$14I_a - 5I_b = 50. \quad \text{(i)}$$

Applying KVL to loop *BDD′CB*

$$2I_b + 25 + 8I_b + 5(I_b - I_a) = 0$$

or $$-5I_a + 15I_b = -25. \quad \text{(ii)}$$

Solution of equations (i) and (ii) yields

$$I_a = 3.3784 \text{ A and } I_b = -0.541 \text{ A}$$

The current through 3 Ω and 6 Ω resistors is thus 3.3784 A from *A* to *B* and *C* to *A′* respectively. The current through 2 Ω and 8 Ω resistors is 0.541 A from *D* to *B* and *C* to *D′* respectively, while the curent through 5 Ω resistor is

$$I_a - I_b = 3.9194 \text{ A from } B \text{ to } C.$$

Voltage across 5 Ω resistor is 5 × 3.9194 = 19.597 V.

**A1.5** In the circuit shown in Fig. 1.121 determine the voltages at nodes 1 and 2 with respect to the reference point.

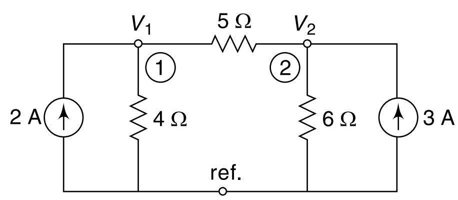



**Fig. 1.121** *Circuit of Ex. A1.5*

***Solution***

Applying nodal analysis at node (1),

$$\frac{V_1}{4} + \frac{V_1 - V_2}{5} - 2 = 0$$

or $$9V_1 - 4V_2 = 40 \quad \text{(i)}$$

Applying nodal analysis at node (2),

$$\frac{V_2 - V_1}{5} + \frac{V_2}{6} - 3 = 0$$

or $$-6V_1 + 11V_2 = 90 \quad \text{(ii)}$$

Solving equations (i) and (ii)

$$V_1 = 10.667\ V \text{ and } V_2 = 14 \text{ V.}$$

**A1.6** Find Norton's equivalent circuit at terminals *A* and *B* for the network shown in Fig. 1.122 and hence determine the power dissipated in a 5 Ω resistor to be connected between terminals *A* and *B*.

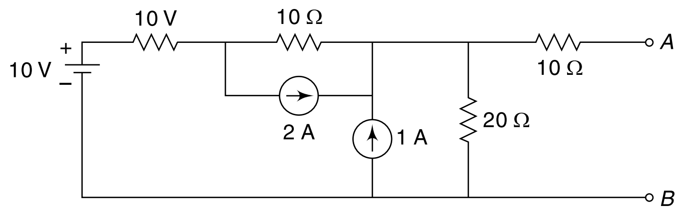



**Fig. 1.122** *Circuit of Ex. A1.6*

***Solution***

First we convert the current sources into equivalent voltage sources and short circuit terminals A*B* [Fig. 1.122(a)].

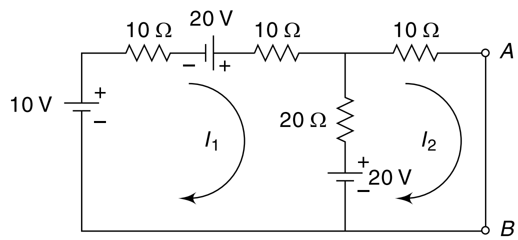



**Fig. 1.122(a)** *Conversion of sources*

If $I_1$ and $I_2$ be the loop currents then

$$-10 + (10 + 10)\, I_1 + 20(I_1 - I_2) = 0 \quad \text{(i)}$$

and
$$-20 + 20(I_2 - I_1) + 10I_2 = 0 \quad \text{(ii)}$$

Solving Eqs (i) and (ii) we get

$$I_2 = 1.25 \text{ A}$$

Now, Norton's equivalent current i.e., the current through short-circuited path A*B* is given by

$$\therefore \quad I_N = 1.25 \text{ A}$$

To find Norton's equivalent resistance, A*B* is open circuited and the sources are removed as shown in Fig. 1.122(b).

$$R_N = 10 + \frac{20 \times 20}{20 + 20} = 20\ \Omega$$

Norton's equivalent circuit is shown in Fig. 1.132(c).

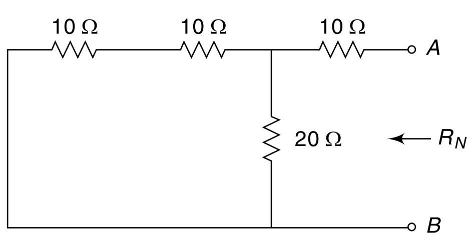



**Fig. 1.122(b)** *Finding of $R_N$*

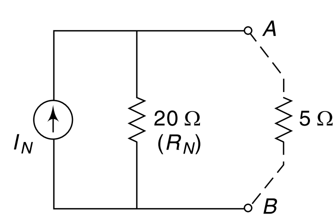



**Fig. 1.122(c)** *Norton's equivalent network of Ex. A1.6*

So, the current $I$ through 5 Ω resistor connected between terminals A and $B$ is

$$I = 1.25 \times \frac{20}{20+5} = 1 \text{ A}$$

Hence power dissipated through 5 Ω resistor = $1^2 \times 5 = 5$ W.

**A1.7** In the circuit shown in Fig. 1.123 find voltage at node A.

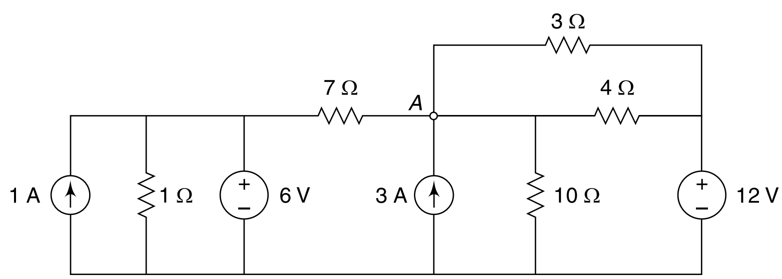



**Fig. 1.123** *Circuit of Ex. A1.7*

***Solution***

Since the 3 A current source is in parallel with the 10 Ω resistor, hence converting the current source into the equivalent voltage source and replacing the parallel combination of 3 Ω and 4 Ω by a single resistance [Fig. 1.123(a)] we can write nodal equation at node A as

$$\frac{V_A - 30}{10} + \frac{V_A - 6}{7} + \frac{V_A - 12}{\frac{12}{7}} = 0$$

or $$V_A\left(\frac{1}{10} + \frac{1}{7} + \frac{7}{12}\right) - 3 - \frac{6}{7} - 7 = 0$$

or $$V_A = 13.14 \text{ V}$$

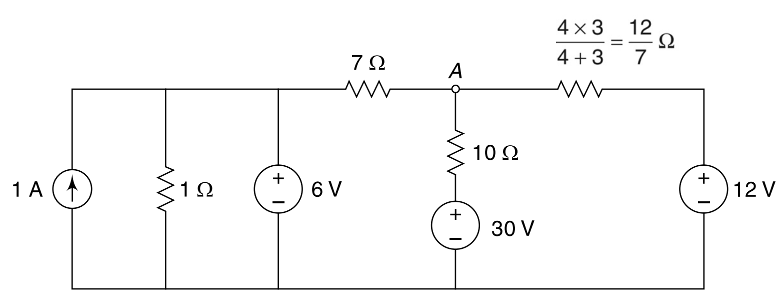



**Fig. 1.123(a)** *Network reduction of the circuit shown in Fig. 1.123*

**A1.8** In the circuit shown in Fig. 1.124 find current $I_a$.

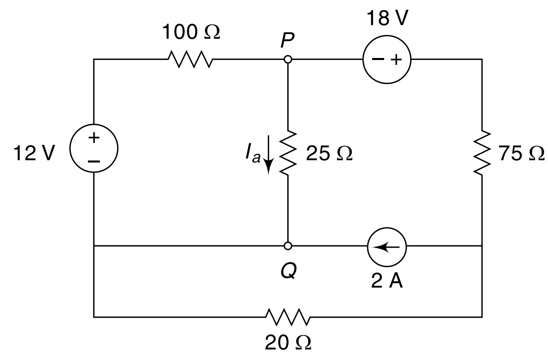



**Fig. 1.124** *Circuit of Ex. A1.8*

***Solution***

The circuit is redrawn as shown in Fig. 1.124(a).

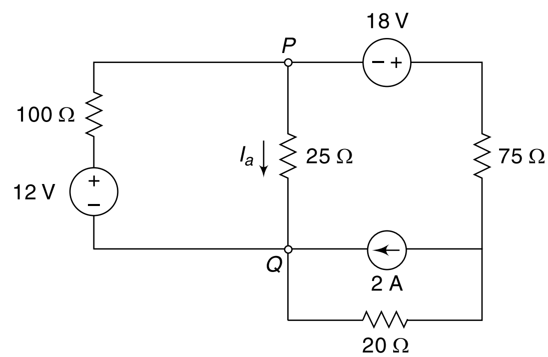



**Fig. 1.124(a)** *Circuit of Ex. A1.8 redrawn*

The 2 A current source can be replaced by an equivalent voltage source of $20 \times 2 = 40$ V in series with a 20 Ω resistance and the modified circuit is shown in Fig. 1.124(b).

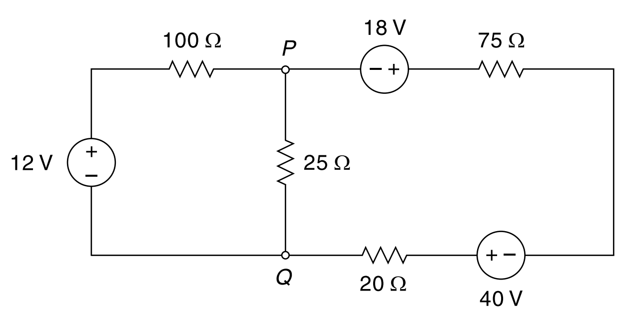



**Fig. 1.124(b)** *Modified circuit*

The two-voltage sources in series can be combined into a single source as shown in Fig. 1.124(c).

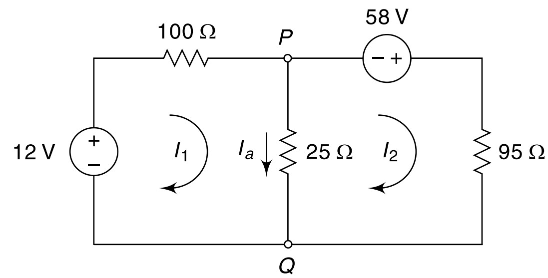



**Fig. 1.124(c)** ***Finally reduced circuit of Ex. A1.8***

Let $I_1$ and $I_2$ be the loop currents applying KVL in these two loops

$$100\,I_1 + 25(I_1 - I_2) = 12 \qquad \text{(i)}$$

and $$95I_2 + 25(I_2 - I_1) = 58 \qquad \text{(ii)}$$

Solving the two equations (i) and (ii).

$$I_1 = 0.201 \text{ A and } I_2 = 0.525 \text{ A}$$

∴ $$I_a\ (= I_1 - I_2) = -0.3242 \text{ A (from } P \text{ to } Q)$$

or, $$I_a = 0.3242 \text{ A (from } Q \text{ to } P).$$

**A1.9** Find the Thevenin's equivalent circuit at terminals A*B* for the network shown in Fig. 1.125 and hence determine the power dissipated in a 5 Ω resistor connected between A and *B*.

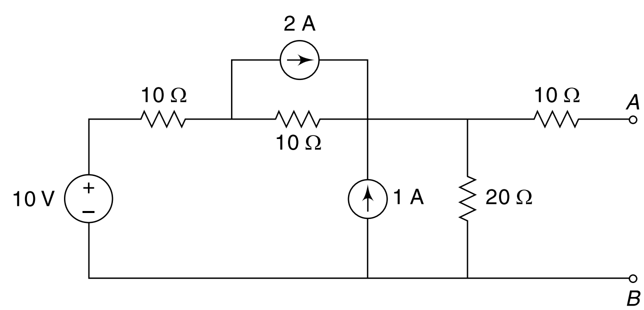



**Fig. 1.125** ***Circuit of Ex. A1.9***

***Solution***

Converting the current sources into equivalent voltage sources [Fig. 1.125(a)], current *I* through 20 Ω resistor is given as

$$I = \frac{20 + 10 - 20}{10 + 10 + 20} = 0.25 \text{ A}$$

∴ $$V_{\text{Th}} = \text{Voltage across CD}$$

$$= 20 + 20 \times 0.25 = 25 \text{ V}$$

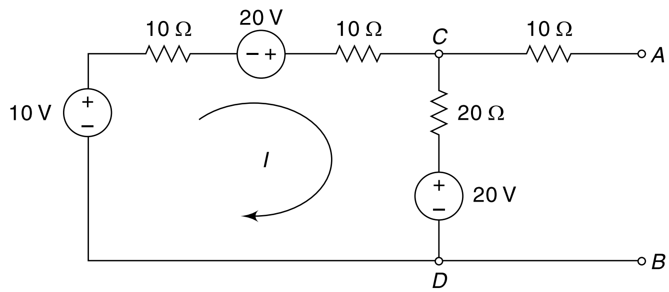



**Fig. 1.125(a)** *Modified circuit*

To find $R_{Th}$, deactivating all sources [Fig. 1.125(b)] we get

$$R_{Th} = 10 + \frac{20 \times (10 + 10)}{20 + (10 + 10)} = 20\ \Omega$$

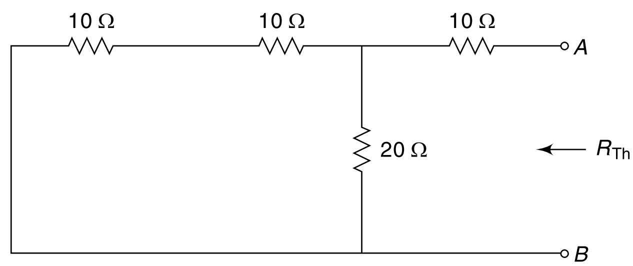



**Fig. 1.125(b)** *Determination of $R_{TH}$*

Thevenin's equivalent circuit is shown in Fig. 1.125(c).

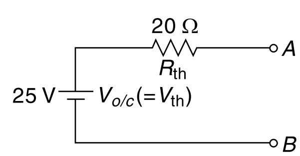



**Fig. 1.125(c)** *Thevenin's equivalent circuit of Ex. A1.9*

**A1.10** Determine the current in the conductor of 2 Siemens of the network shown in Fig. 1.126 using node voltage analysis.

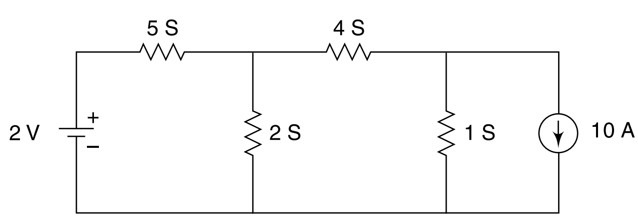



**Fig. 1.126** *Circuit of Ex. A1.10*

***Solution***

Replacing the current source by an equivalent voltage source the new transformed circuit is shown in Fig. 1.126(a). Here there are two nodes (1) and (2). Let node (2) be taken as the reference node and let $V$ be the potential at node (1).

Hence $(V - 2)5 + V \times 2 + (V + 10)(4 + 1) = 0$

or $V = -\dfrac{40}{12} = -3.333.$

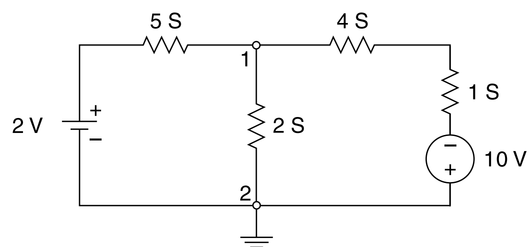



**Fig. 1.126(a)** ***Transformed circuit of Fig. 1.126***

Hence current through conductance of 2 Siemens is $-3.333 \times 2 = -6.67$ A. This current is directed from (2) to (1).

**A1.11** Using mesh analysis obtain the values of all mesh currents of the network shown in Fig. 1.127.

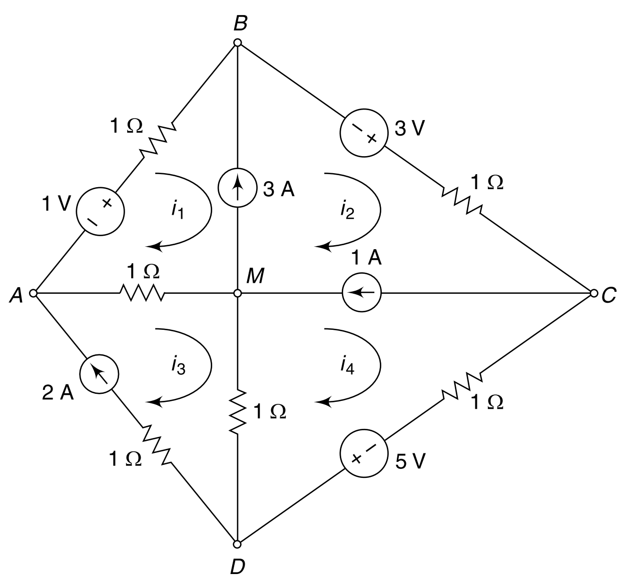



**Fig. 1.127** ***Circuit of Ex. A1.11***

***Solution***

From Fig. 1.127 it can easily be observed that

$$i_3 = 2\text{ A}$$

$$i_2 - i_1 = 3\text{ A}$$

and $\quad i_2 - i_4 = 1\text{ A}$

By applying KVL to a closed loop which does not have any current source (loop *ABCDMA*) we obtain

$$-1 + i_1 - 3 + i_2 + i_4 - 5 + (i_4 - i_3) + (i_1 - i_3) = 0$$

or $\quad 2i_1 + i_2 + 2i_4 = 13$

As $i_1 = (i_2 - 3)$ and $i_4 = (i_2 - 1)$, hence from above we can write,

$$2(i_2 - 3) + i_2 + 2(i_2 - 1) = 13$$

or $\quad i_2 = 4.2\text{ A}$

Hence, $\quad i_1 = 4.2 - 3 = 1.2\text{ A}$ and $i_4 = 4.2 - 1 = 3.2\text{ A}$.

$\therefore \quad i_1 = 1.2\text{ A}; i_2 = 4.2\text{ A}; i_3 = 2\text{ A}; i_4 = 3.2\text{ A}.$

**A1.12** Use node voltage analysis determine the power in the 2 Ω and 4 Ω resistor in the network of Fig. 1.128.

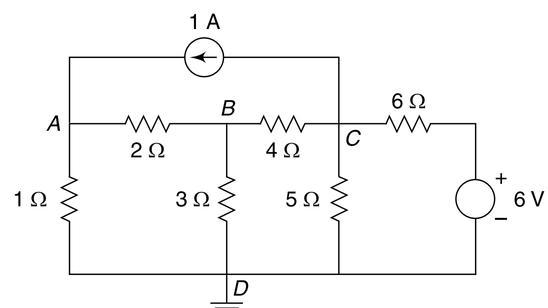



**Fig. 1.128** ***Circuit of Ex. A1.12***

***Solution***

There are four nodes in the network of which *D* is considered as the reference node.

At node A

$$\frac{V_A - V_B}{2} + V_A - 1 = 0$$

or $\quad 3V_A - V_B = 2 \qquad$ (i)

At node *B*

$$-\frac{V_A - V_B}{2} + \frac{V_B - V_C}{4} + \frac{V_B}{3} = 0$$

or $\quad 6V_B - 6V_A + 3V_B - 3V_C + 4V_B = 0$

or $\quad -6V_A + 13V_B - 3V_C = 0 \qquad$ (ii)

At node *C*

$$\frac{V_C - V_B}{4} + \frac{V_C}{5} + \frac{V_C - 6}{6} + 1 = 0$$

or $15V_C - 15V_B + 12V_C + 10V_C - 60 + 60 = 0$

or $-15V_B + 37V_C = 0$ (iii)

Solving equations (i), (ii) and (iii)

$$V_A = 0.8031 \text{ V}, V_B = 0.4088 \text{ V and } V_C = 0.1658 \text{ V}$$

$$\therefore \text{ Power in the 2 } \Omega \text{ resistor} = \frac{(V_A - V_B)^2}{2} = \frac{(0.8031 - 0.4088)^2}{2}$$

$$= 0.078 \text{ W}$$

$$\text{Power in 4 } \Omega \text{ resistor} = \frac{(V_B - V_C)^2}{4} \left( = \frac{(0.4088 - 0.1658)^2}{4} \right) = 0.01476 \text{ W.}$$

**A1.13** Determine the current through the 1 Ω resistor connected across *A, B* of the network shown in Fig. 1.129 using Norton's theorem.

***Solution***

Removing the 1 Ω resistor and short-circuiting the terminals *AB* the circuit is redrawn as shown in Fig. 1.129(a). The 1 A current source has been transformed into voltage source.

Applying KVL to the three loops we get the following three equations:

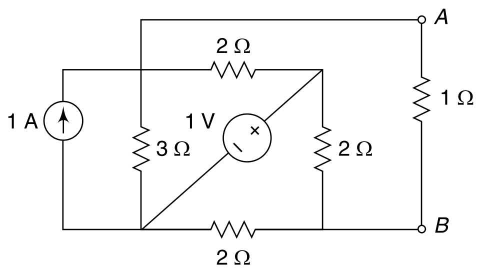



**Fig. 1.129** ***Circuit of Ex. A1.13***

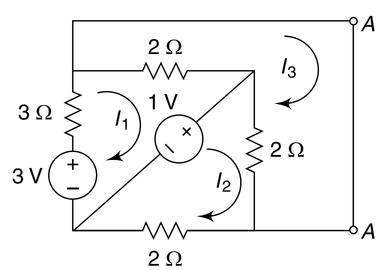



**Fig. 1.129(a)** ***Determination of ($I_N$)***

$$3I_1 + 2(I_1 - I_3) + 1 - 3 = 0$$

or $5I_1 - 2I_3 = 2$ (i)

$$2I_2 - 1 + 2(I_2 - I_3) = 0$$

or $4I_2 - 2I_3 = 1$ (ii)

and $2(I_3 - I_2) + 2(I_3 - I_1) = 0$

or $-2I_1 - 2I_2 + 4I_3 = 0$ (iii)

Solving the three equations (i), (ii) and (iii) we get $I_3 = 0.59$ A. Hence the current through the short circuited path *AB* is $I_3 = 0.59$ A, i.e. $I_N = 0.59$ A.

To find $R_N$, all the sources are deactivated and open circuiting terminals *AB* [Fig. 1.132(b)], we get

$$R_N = \frac{3 \times 2}{3 + 2} + \frac{2 \times 2}{2 + 2} = 2.2 \ \Omega$$

From Fig. 1.129(c) the current through the 1 Ω resistor is

$$0.59 \times \frac{2.2}{2.2 + 1} = 0.4056 \text{ A.}$$

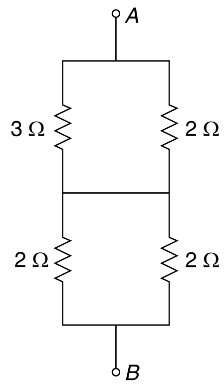



**Fig. 1.129(b)** ***Determination of ($R_N$)***

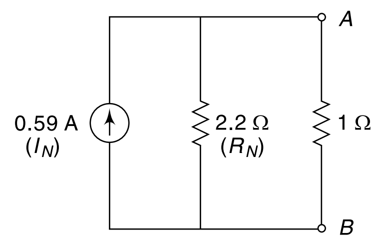



**Fig. 1.129(c)** ***Norton's equivalent circuit of Ex. A1.13***

**A1.14** Solve the above problem (Example A1.13) using the superposition theorem.

***Solution***

Considering a 1 A current source acting alone, the circuit shown in Fig. 1.129, transforms into the circuit shown in Fig. 1.130.

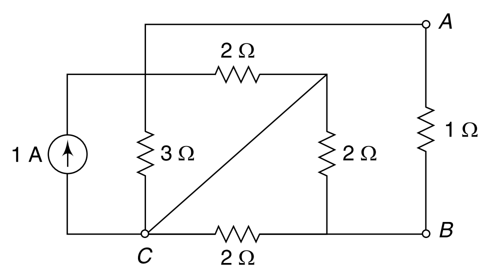



**Fig. 1.130** ***1 A source is acting alone in circuit of Fig. 1.129***

The circuit further reduces as shown in Fig. 1.130(a).

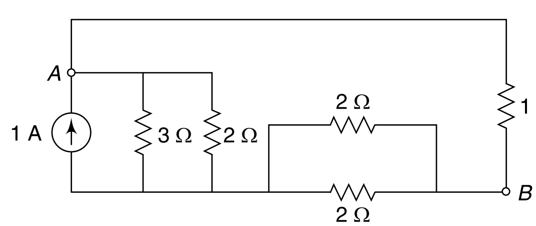



**Fig. 1.130(a)** ***Reduced circuit***

Next, Fig. 1.130(a) is simplified into Fig. 1.130(b) and then into Fig. 1.130(c).

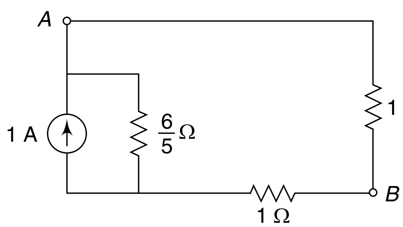



**Fig. 1.130(b)** *Network reduction*

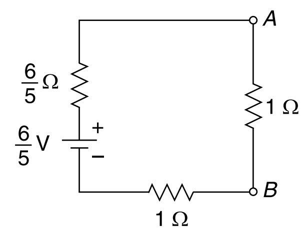



**Fig. 1.130(c)** *Finally reduced circuit with 1 A source acting only*

The current through the 1 Ω resistor when the current source acts alone is given by

$$\frac{\frac{6}{5}}{\frac{6}{5}+1+1}\text{ A} = 0.375 \text{ A (from } A \text{ to } B)$$

Next, considering the voltage source acting alone, the network in Fig. 1.129 transforms into Fig. 1.130(d).

Applying KVL in the three loops the following three equations are obtained:

$$2(I_1 - I_3) + 1 + 3I_1 = 0 \quad \text{(i)}$$

$$2(I_2 - I_3) + 2I_2 - 1 = 0 \quad \text{(ii)}$$

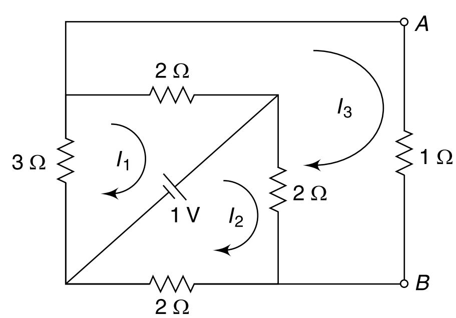



**Fig. 1.130(d)** *Voltage source acting alone in circuit of Fig. 1.132*

and $$I_3 + 2(I_3 - I_2) + 2(I_3 - I_1) = 0 \quad \text{(iii)}$$

Solving these three equations,

$$I_3 = 0.03125 \text{ A (from A to } B).$$

Applying superposition theorem current through the 1 Ω resistor (when both the sources are acting simultaneously) is 0.375 + 0.03125 = 0.40625 A (from $A$ to $B$). ▪▪▪▪▪▪▪

**A1.15** Determine $R_L$ in Fig. 1.131 for maximum power transfer to the load.

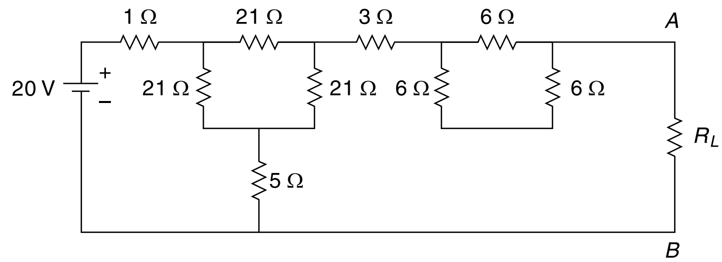



**Fig. 1.131** *Circuit of Ex. A1.15*

***Solution***

The two-delta networks, one formed by 3 numbers of 6 Ω resistors and another by 3 numbers of 21 Ω resistors, are first converted into equivalent star network.

Here
$$R_1 = \frac{21 \times 21}{21 + 21 + 21}\ \Omega = 7\ \Omega$$

$$R_2 = \frac{6 \times 6}{6 + 6 + 6} = 2\ \Omega$$

The corresponding network is shown in Fig. 1.131(a).

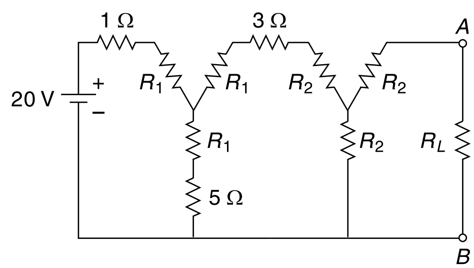



**Fig. 1.131(a)** *Circuit reduction*

The network shown in Fig. 1.131(a) can further be reduced to Fig. 1.131(b).

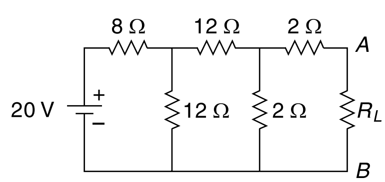



**Fig. 1.131(b)** *Finally reduced circuit*

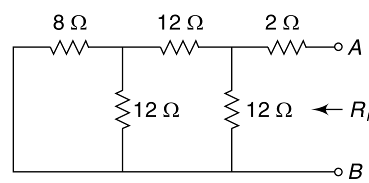



**Fig. 1.131(c)** *Finding of ($R_i$)*

For maximum power transfer to the load $R_L$ the value $R_L$ should be equal to $R_i$ which is equal to the internal resistance of the network. $R_i$ can be found from Fig. 1.131(c) removing the source and open circuiting terminals A*B* (Fig. 1.131(c)).

$$R_L = R_i = \left\{ \left( \frac{8 \times 12}{8 + 12} + 12 \right) \parallel 12 \right\} + 2$$

$$= \frac{16.8 \times 12}{16.8 + 12} + 2 = 9\ \Omega$$

**A1.16** Find the current through the 2 Ω resistor as shown in Fig. 1.132 using the Superposition theorem.

***Solution***

Considering the 2 A source acting alone, the corresponding circuit is shown in Fig. 1.132(a).

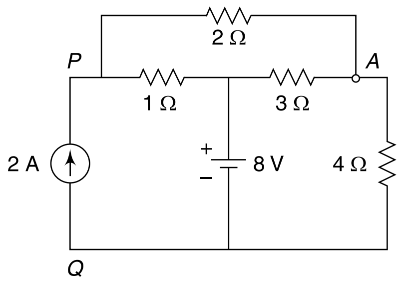



**Fig. 1.132** *Circuit of Ex. A1.16*

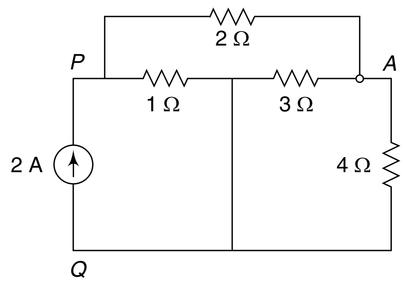



**Fig. 1.132(a)** *2 A source acting alone*

Fig. 1.132(a) is redrawn in Fig. 1.132(b).

Now the current through the 2 Ω resistor is

$$I_1 = 2 \times \frac{1}{1 + 2 + \dfrac{4 \times 3}{4 + 3}} = \frac{2}{4.71} = 0.424 \text{ A (from } P \text{ to } A)$$

Considering the 8 V source acting alone, the corresponding circuit is shown in Fig. 1.132(c).

Current through the 2 Ω resistor is

$$I_2 = \frac{8}{4 + \dfrac{3}{2}} \times \frac{1}{2} = \frac{4}{5.5} = 0.727 \text{ A (from } P \text{ to } A)$$

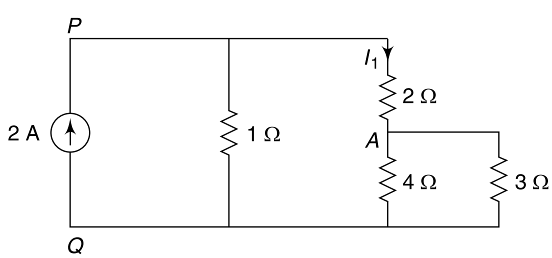



**Fig. 1.132(b)** *Simplified circuit*

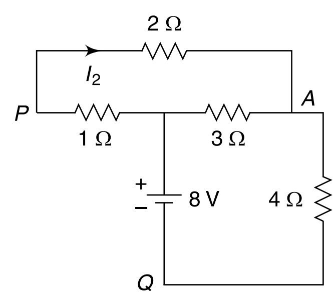



**Fig. 1.132(c)** *8 V Source acting alone*

Using superposition theorem, net current through 2 Ω resistor is $I_1 + I_2 = 0.424 + 0.727 = 1.151$ A.

## HARDER PROBLEMS

**H1.1** Using loop equations obtain the current in the 12 Ω resistor of the network shown in Fig. 1.133.

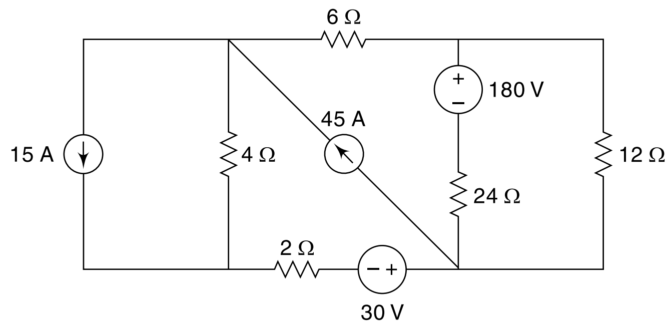



**Fig. 1.133** *Circuit of Ex. H1.1*

***Solution***

Let us first replace 15 A current source by an equivalent voltage source; the corresponding figure is shown in Fig. 1.133(a).

Here $I_2 - I_1 = 45$ (i)

Applying KVL in loop A*BCDA*

$$6I_2 + 180 + 24(I_2 - I_3) + 30 + 6I_1 + 60 = 0$$

or $6I_1 + 30I_2 - 24I_3 = -270$

or $I_1 + 5I_2 - 4I_3 = -45$ (ii)

Applying KVL in loop *BEFCB*

$$12I_3 + 24(I_3 - I_2) - 180 = 0$$

or $-24I_2 + 36I_3 = 180$

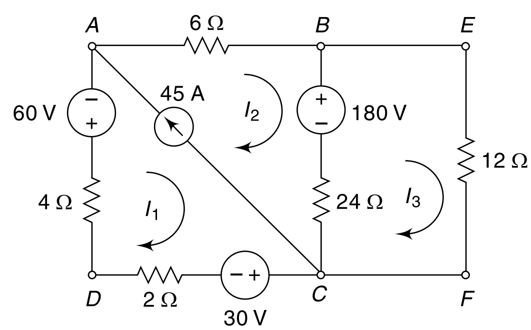



**Fig. 1.133(a)** *Transformed circuit of Fig. 1.133*

or $-2I_2 + 3I_3 = 15$ (iii)

Solving equations (i), (ii) and (iii)

$$I_2 = 6 \text{ A}, I_3 = 9 \text{ A and } I_1 = -39 \text{ A}$$

∴ Current in the 12 Ω resistor is $I_3 = 9$ A.

**H1.2** Obtain Thevenin's equivalent circuit with respect to terminals A and *B* of the network shown in Fig. 1.134.

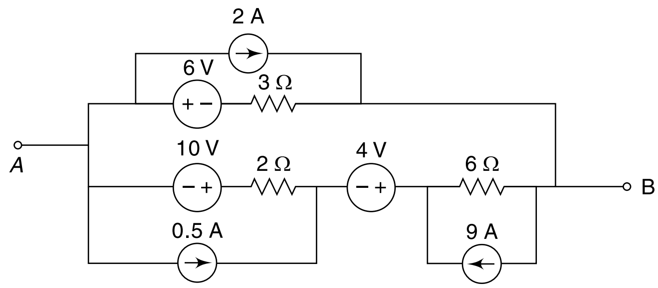



**Fig. 1.134** *Circuit of Ex. H1.2*

### *Solution*

Let us first convert 6 V and 10 V voltage sources into corresponding current sources and 9 A current source into voltage source [Fig. 1.134(a)]. Next, Fig. 1.134(a) is reduced to Fig. 1.134(b).

Next we convert 5.5 A current source into equivalent voltage source as shown in Fig. 1.134(c). Figure 1.134(d) shows further network reduction.

The current *I* through the loop in Fig. 1.128(d) is

$$I = \frac{39}{8+3} = 3.55 \text{ A}$$

$\therefore \quad V_{Th}$ = Voltage across the 3 Ω resistor

$= 3 \times 3.55 \text{ V} = 10.65 \text{ V}.$

Thevenin's equivalent resistance

$$R_{Th} = \frac{8 \times 3}{3+8} = 1.18 \ \Omega.$$

Thevenin's equivalent circuit is shown in Fig. 1.134(e).

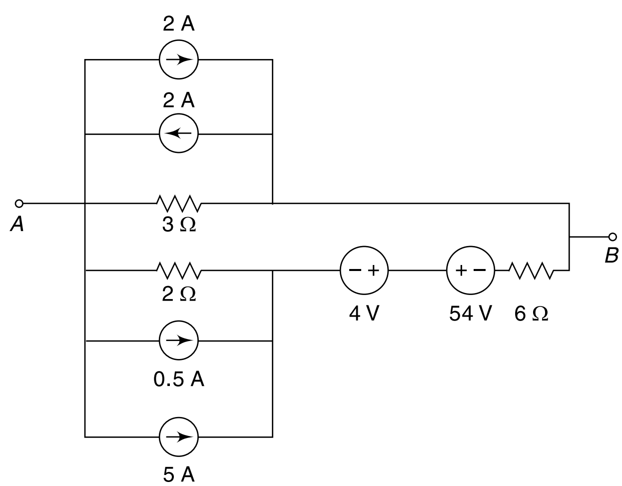



**Fig. 1.134(a)** *Conversion of sources*

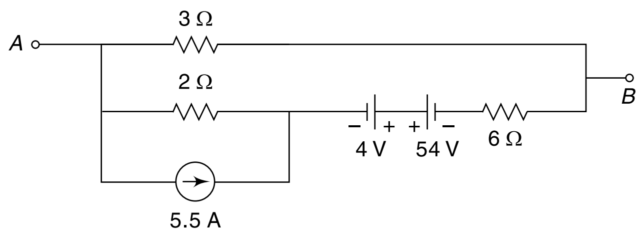



**Fig. 1.134(b)** *Network reduction*

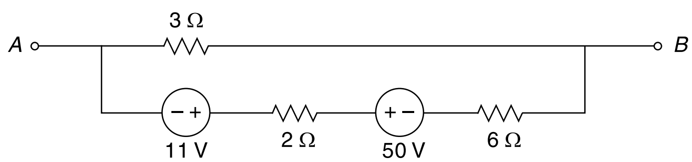



**Fig. 1.134(c)** *Reduced network*

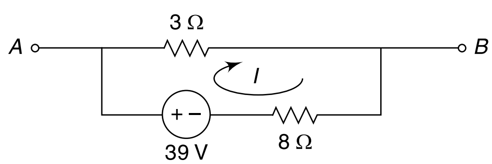



**Fig. 1.134(d)** *Finally reduced network*

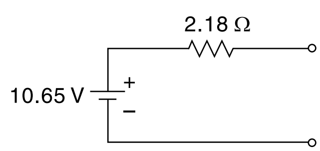



**Fig. 1.134(e)** *Thevenin's equivalent network of Ex. H1.2*

**H1.3** In Fig. 1.135 the galvanometer *G* has a conductance of 10 S. Determine the current through the galvanomneter using Thevenin's theorem.

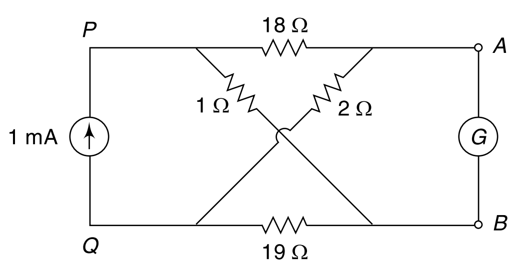



**Fig. 1.135** *Circuit of Ex. H1.3*

***Solution***

Let us first see open-circuiting terminals *AB* [Fig. 1.135(a)]

Figure 1.135(a) is redrawn as shown in Fig. 1.135(b).

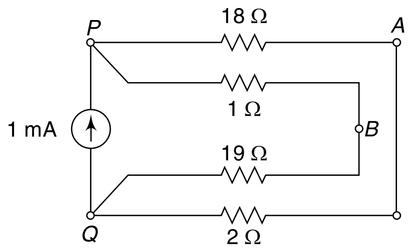



**Fig. 1.135(a)** ***Circuit with galvanometer removed***

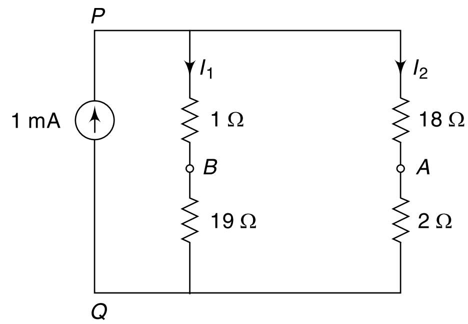



**Fig. 1.135(b)** ***Modified circuit of Fig. 1.135(a)***

From Fig. 1.135(b) current through the 1 Ω resistor is

$$I_1 = 1 \times \frac{20}{20+20} = 0.5 \text{ mA}$$

and current through 18 Ω resistor is also

$$I_2 = 1 \times \frac{20}{20+20} = 0.5 \text{ mA.}$$

Now $\quad V_{AB} = V_{PB} - V_{PA}$

$$= 1 \times 0.5 - 18 \times 0.5 = -8.5 \text{ mV}$$

$\therefore \quad V_{\text{Th}} = V_{BA} = 8.5 \text{ mV}$

[terminal *B* is at higher potential].

To find Thevenin's equivalent resistance current source is open-circuited and the network of Fig. 1.135(c) is obtained.

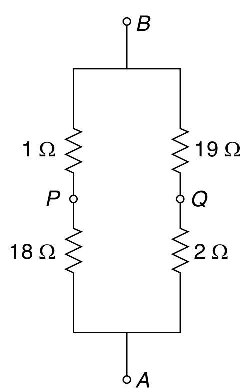



**Fig. 1.135(c)** ***Determination of $R_{TH}$***

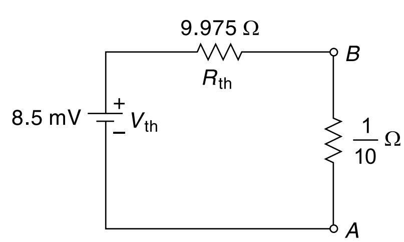



**Fig. 1.135(d)** ***Thevenin's equivalent of Ex. H1.3***

Hence $$R_{Th} = \frac{(18+1)\,(19+2)}{(18+1)+(19+2)} = 9.975\ \Omega$$

From Fig. 1.135(d) current through the galvanometer of 10 S, i.e. 1/10 Ω resistance is

$$\frac{8.5\times10^{-3}}{9.975+\frac{1}{10}}\text{ A} = 0.844\times10^{-3}\text{ A}$$

$$= 0.844\text{ mA}$$

**H1.4** Find the current through the 2 Ω resistor of Fig. 1.132 using Norton's theorem.

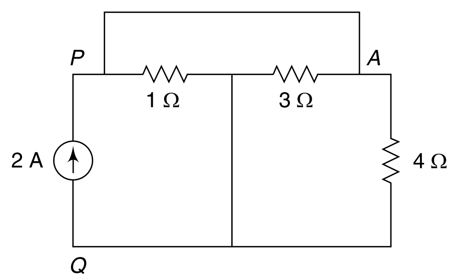



**Fig. 1.136** ***Circuit of Ex. H1.4***

### *Solution*

Let us short-circuit the terminals *PA* after removing the 2 Ω resistor. Now we consider the 2 A source acting alone the (corresponding circuit being shown in Fig. 1.136).

Figure 1.136 can be further reduced to the circuit shown in Fig. 1.136(a).

The short circuit current due to the 2 A source acting alone is

$$I_{sc_1} = 2\times\frac{1}{1+\frac{4\times3}{4+3}} = \frac{2\times7}{19} = \frac{14}{19}\text{ A} \qquad \text{(from } P \text{ to } A\text{).}$$

Considering the 8 V source acting alone, the current through the short circuited path can be found from Fig. 1.136(b). Current through short circuited path due to the 8 V source acting alone is

$$I_{sc_2} = \frac{8}{4+\frac{3\times1}{3+1}}\times\frac{3}{3+1} = \frac{24}{19} \quad \text{(from } P \text{ to } A\text{).}$$

Applying the superposition theorem the current through the short circuited path when both the sources are acting simultaneously is

$$I_{sc} = \left(\frac{14}{19}+\frac{24}{19}\right) = 2\text{ A}$$

Hence, Norton's equivalent current $I_N = 2$ A.

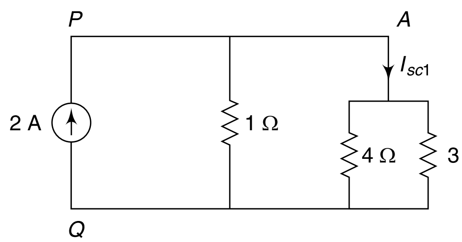



**Fig. 1.136(a)** *Reduced network with 2 A source acting alone*

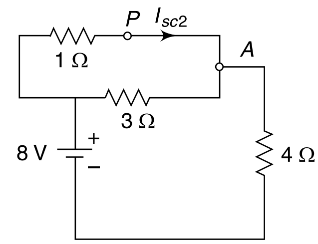



**Fig. 1.136(b)** *8 V source acting alone*

Now to find Norton's equivalent resistance $R_N$, all the sources are deactivated and open circuiting terminals *PA* the circuit configuration shown in Fig. 1.136(c) is obtained.

$$R_N = 1 + \frac{3 \times 4}{3 + 4} = 1 + \frac{12}{7} = \frac{19}{7}\ \Omega.$$

Norton's equivalent circuit is shown in Fig. 1.136(d).

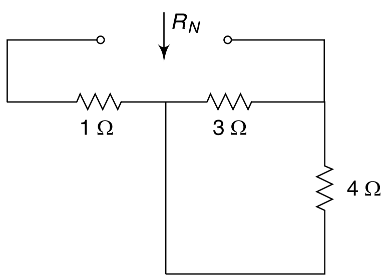



**Fig. 1.136(c)** *Determination of ($R_N$)*

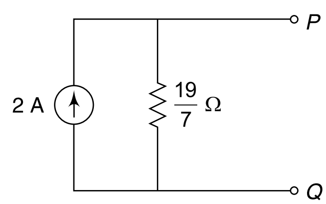



**Fig. 1.136(d)** *Norton's equivalent circuit of Ex. H1.4*

The current through the 2 Ω resistor connected between terminals *P&A* using Norton's theorem is $2 \times \dfrac{\frac{19}{7}}{2 + \frac{19}{7}} = \dfrac{2 \times 19}{33}$ A = 1.151 A.

**H1.5** Using the superposition theorem find the voltage across the 20 Ω resistor of the circuit shown in Fig. 1.137.

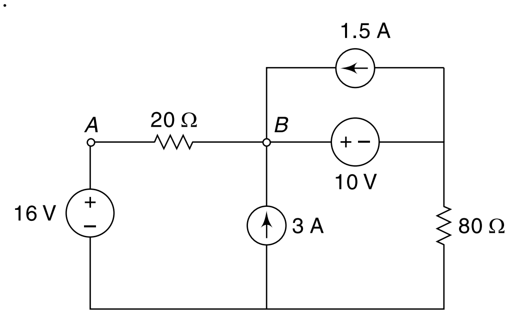



**Fig. 1.137** *Circuit of Ex. H1.5*

***Solution***

Let us consider that the 16 V source acts alone; removing the other sources the circuit configuration is shown in Fig. 1.137(a).

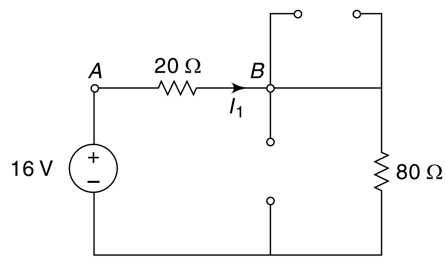



**Fig. 1.137(a)** ***16 V source acting alone***

The current through the 20 Ω resistor is

$$I_1 = \frac{16}{20 + 80} = 0.16 \text{ A} \quad \text{from A to } B$$

Considering 10 V source acting alone the circuit is redrawn as shown in Fig. 1.137(b).

Current through the 20 Ω is $I_2 = \dfrac{10}{20 + 80}$ A = 0.1 from $B$ to $A$.

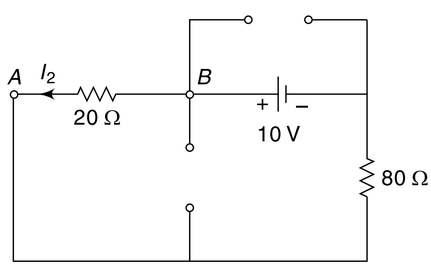



**Fig. 1.137(b)** ***10 V source acting alone***

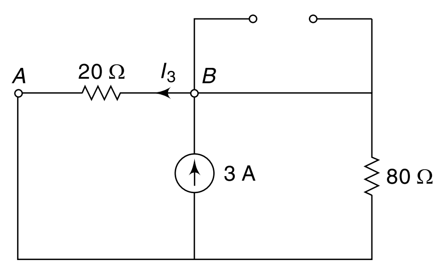



**Fig. 1.137(c)** ***3 A source is acting alone***

Next, considering 3 A source acting alone the corresponding circuit is shown in Fig. 1.137(c).

Current in the 20 Ω resistor is $I_3 = 3 \times \dfrac{80}{20 + 80}$ A = 2.4 A from $B$ to $A$.

Considering the 1.5 A source acting alone the correspoding circuit is shown in Fig. 1.137(d).

As there is a short circuit path in parallel with 1.5 A current source, hence no current flows through 20 Ω resistor due to this source.

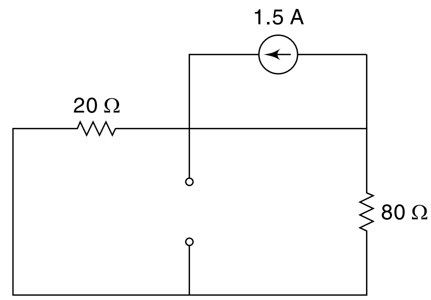



**Fig. 1.137(d)** *1.5 A source acting alone*

Applying superposition theorem, when all the sources are acting simultaneously the current through the 20 Ω resistor is $(I_2 + I_3 - I_1) = (0.1 + 2.4 - 0.16) = 2.34$ A from *B* to *A*.
or voltage across the 20 Ω resistor is $2.34 \times 20 = 46.8$ V.

**H1.6** Using the superposition theorem, find the current through $R_L$ in the circuit shown in Fig. 1.138.

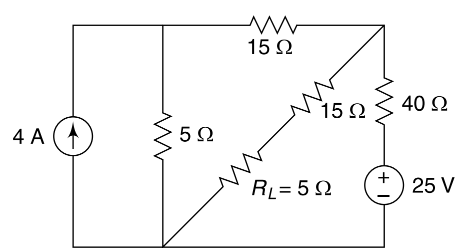



**Fig. 1.138** *Circuit of Ex. H1.6*

***Solution***

Converting the current source into equivalent voltage source the transformed network is shown in Fig. 1.138(a)

Considering the 20 V source acting alone, the circuit is shown in Fig. 1.138(b).

The current through $R_L$ is

$$I_1 = \frac{20}{20 + \frac{20 \times 40}{20 + 40}} \times \frac{40}{40 + 20}$$

$$= \frac{20 \times 40}{20 \times 60 + 20 \times 40}$$

$$= \frac{800}{1200 + 800} = \frac{8}{20}$$

$$= 0.4 \text{ A} \quad (\text{from } A \text{ to } B)$$

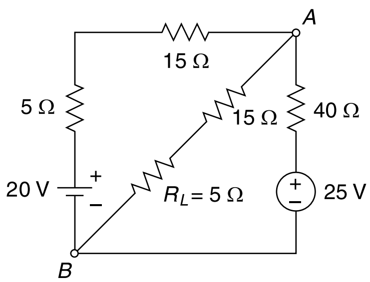



**Fig. 1.138(a)** *Conversion of source*

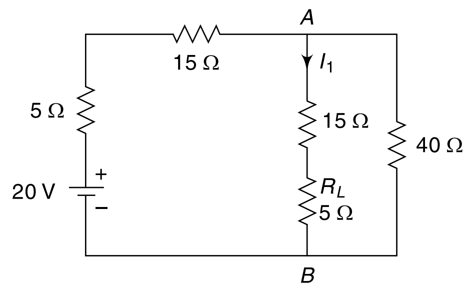



**Fig. 1.138(b)** *20 V source acting alone*

Considering the 25 V source acting alone from the circuit, is shown in Fig. 1.138(c), the current through ($R_L$) is

$$I_2 = \frac{25}{40+\frac{20}{2}} \times \frac{1}{2} = \frac{25}{2 \times 50} = 0.25 \text{ A} \qquad \text{(from } A \text{ to } B\text{).}$$

Applying the superposition theorem when both the sources are acting simultaneously the current through $R_L$ is

$$I_1 + I_2 = 0.4 + 0.25 = 0.65 \text{ A (from } A \text{ to } B\text{)}$$

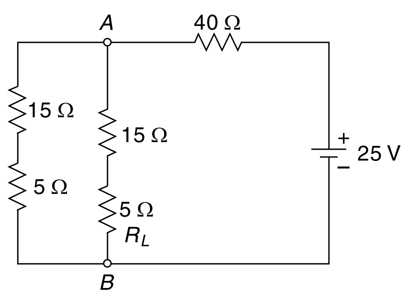



**Fig. 1.138(c)** *25 V source acting alone*

**H1.7** Find the value of $V_R$ in the circuit shown in Fig. 1.139.

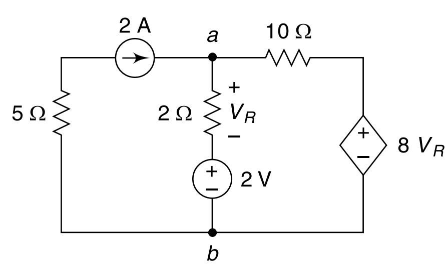



**Fig. 1.139** *Circuit of Ex. H1.7*

***Solution***

Let $V_a$ be the voltage at node $a$. Applying KCL at node $a$

$$\frac{V_a - 2}{2} + \frac{V_a - 8V_R}{10} - 2 = 0$$

or $\quad 5V_a - 10 + V_a - 8V_R = 20$

or $\quad 6V_a - 8V_R = 30$

Again $\quad V_R = V_a - 2$ or, $V_a = V_R + 2$

Hence, $\quad 6\,(V_R + 2) - 8V_R = 30$

or $\quad -2V_R = 18$

or $\quad V_R = -9$ V [this means node "a" is of negative polarity.]

**H1.8** Applying KCL find the value of current $i$ in the circuit shown in Fig. 1.140.

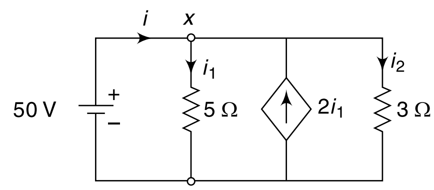



**Fig. 1.140** ***Circuit of Ex. H1.8***

***Solution***

Applying KCL at node ($x$),

$$i - i_1 + 2i_1 - i_2 = 0$$

or $\quad i + i_1 - i_2 = 0$

$\because \quad i_1 = \frac{50}{5} = 10$ A and $i_2 = \frac{50}{3}$ A,

$$i + 10 - \frac{50}{3} = 0$$

or, $\quad i = \frac{50}{3} - 10 = \frac{20}{3} = 6.67$ A

**H1.9** Find the power dissipated in the 100 Ω resistor and find the voltage rating of the dependent source in Fig. 1.141.

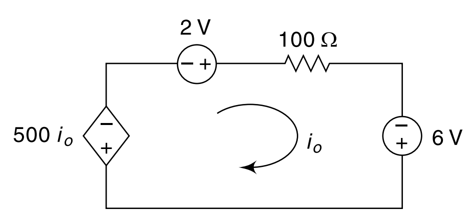



**Fig. 1.141** ***Circuit of Ex. H1.9***

***Solution***

Applying KVL in the given figure,

$$6 - 500i_o + 2 - 100i_o = 0$$

or $$i_o = \frac{8}{600} = 13.33 \text{ mA}.$$

Power dissipated in the 100 Ω resistor = (100) × $(0.0133)^2$ = 17.7 m Watts. Hence voltage rating of the dependent source is 500 × $i_o$ = 500 × 0.0133 = 6.65 V.

**H1.10** Using node analysis find the value of $\alpha$ for the circuit shown in Fig. 1.142 when the power loss in the 1 Ω resistor is 9 W.

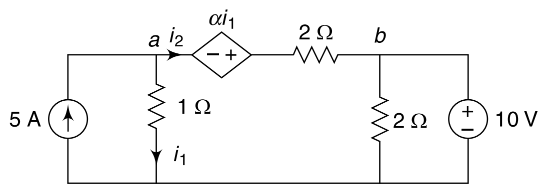



**Fig. 1.142** ***Circuit of Ex. H1.10***

***Solution***

Power loss in the 1 Ω resistor is

$$i_1^2 \times 1 = 9$$

or $$i_1 = 3 \text{ A}$$

and $$v_a = 3 \times 1 = 3 \text{ V}.$$

Applying KCL at node $a$

$$i_1 + i_2 = 5$$

or $$i_2 = 5 - i_1 = 5 - 3 = 2 \text{ A}$$

Also $$v_a + \alpha i_1 - 2i_2 = v_b$$

or $$3 + 3\alpha - 4 = v_b$$

Since $$v_b = 10 \text{ V}$$

Hence $$3\alpha = 10 + 1 = 11$$

or $$\alpha = 3.67.$$

**H1.11** Find $i_1$ and $i_2$ in the circuit shown in Fig. 1.143 using superposition theorem.

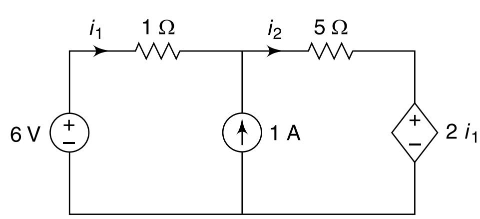



**Fig. 1.143** ***Circuit of Ex. H1.11***

***Solution***

Considering 6 V source acting alone and removing the current source (as shown in Fig. 1.143(a)), we get

$$6 - 2i_1 = (1+5)i_1 \quad \text{or} \quad i_1 = \frac{6}{8} = \frac{3}{4}\,\text{A}$$

Also $\quad i_2 = \frac{3}{4}\,\text{A}$

Now, considering the 1 A current source acting alone and removing the others, from the corresponding circuit (shown in Fig. 1.143(b)), we have

$$1 + i_1 - \frac{v_a - 2i_1}{5} = 0$$

or $\quad 5 + 5i_1 - v_a + 2i_1 = 0$

or $\quad 7i_1 - v_a + 5 = 0$

$\because \quad \frac{v_a}{1} = -i_1,$

hence $\quad 7i_1 + i_1 + 5 = 0$

or $\quad i_1 = -\frac{5}{8}\,\text{A}$

and $\quad i_2 = \frac{v_a - 2i_1}{5} = \frac{-i_1 - 2i_1}{5} = \frac{3}{5} \times \frac{5}{8} = \frac{3}{8}\,\text{A}$

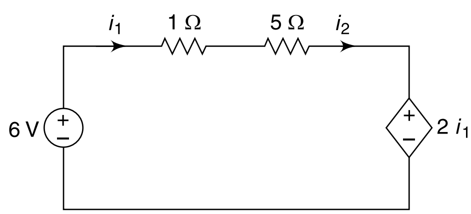



**Fig. 1.143(a)** ***6 V source acting alone***

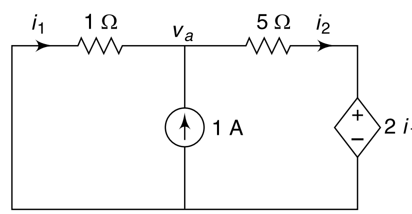



**Fig. 1.143(b)** ***1 A source acting alone***

Applying superposition theorem when both the sources are acting simultaneously

$$i_1 = \frac{3}{4} - \frac{5}{8} = \frac{6-5}{8} = \frac{1}{8} = 0.125\,\text{A}$$

and $\quad i_2 = \frac{3}{4} + \frac{3}{8} = \frac{6+3}{8} = \frac{9}{8} = 1.125\,\text{A}$

**H1.12** Find power loss in the 2 Ω resistor shown in Fig. 1.144 using superposition theorem.

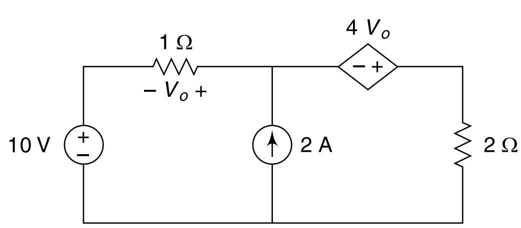



**Fig. 1.144** ***Circuit of Ex. H1.12***

***Solution***

Considering the 10 V source acting alone in the circuit [Fig. 1.144(a)] the loop equation

$$10 + V_o + 4V_o - 2.i = 0$$

or $$5V_o - 2.i + 10 = 0$$

Now $$1 \times i = -V_o$$

Hence $$5(-i) - 2i + 10 = 0$$

or $$-7i + 10 = 0, \text{ i.e. } \quad i = \frac{10}{7} \text{ A} = 1.43 \text{ A}$$

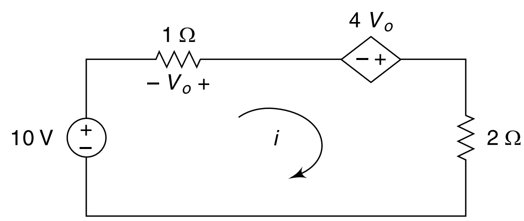



**Fig. 1.144(a)** ***10 V source acting alone***

Considering 2 A source acting alone and applying KCL at node *a* we have Fig. 1.144(b)

$$2 - \frac{v_a}{1} - \frac{v_a + 4V_o}{2} = 0$$

or $$4 - 2v_a - v_a - 4V_o = 0$$

or $$4 - 3v_a - 4v_o = 0$$

Now, from the given figure, $v_a = V_o$

Hence from (i) $4 - 7\,V_o = 0$ i.e. $V_o = \frac{4}{7} = 0.57$ V

Current through 2 Ω resistor is

$$\frac{v_a + 4V_o}{2} = \frac{V_o + 4V_o}{2} = \frac{5}{2} \times 0.57 = 1.425 \text{ A}$$

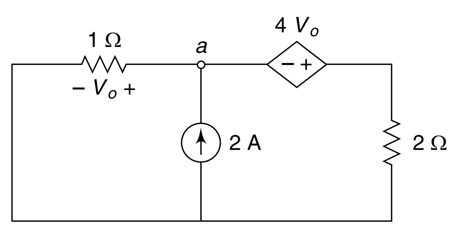



**Fig. 1.144(b)** ***2 A source acting alone***

Applying superposition theorem the current through 2 Ω resistor is thus (1.43 + 1.425) A i.e 2.855 A.

Hence power loss in 2 Ω resistor is $(2.855)^2 \times 2 = 16.31$ W.

**H1.13** Find $v$ in the circuit shown in Fig. 1.145 using superposition theorem.

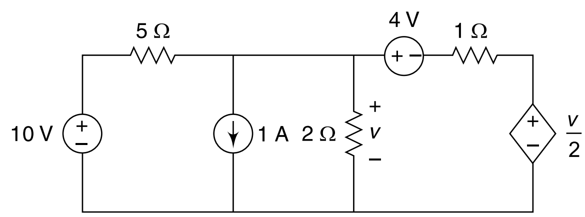



**Fig. 1.145** *Circuit of Ex. H1.13*

### *Solution*

Let us consider the 10 V source only removing the 1 A and 4 V source. The corresponding circuit is shown in Fig. 1.145(a)

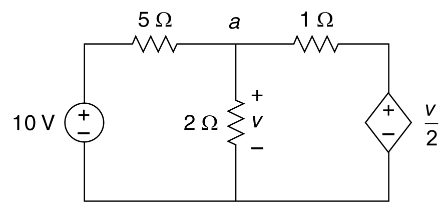



**Fig. 1.145(a)** *10 V source considered only*

At node $a$,

$$\frac{v-10}{5}+\frac{v-\frac{v}{2}}{1}+\frac{v}{2}=0$$

or $\quad 2v - 20 + 10v - 5v + 5v = 0$

or $\quad 12v = 20$ or, $v = 1.67$ V.

Now, let us consider 1 A source acting alone. The corresponding figure is shown in Fig. 1.145(b).

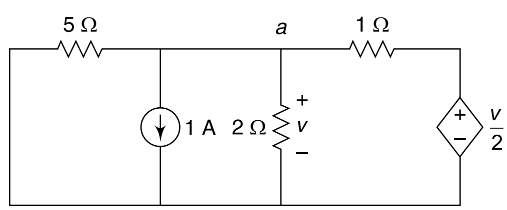



**Fig. 1.145(b)** *1 A source acting alone*

At node $a$,

$$\frac{v}{2}+\frac{v}{5}+1+\frac{v-\frac{v}{2}}{1}=0$$

or $\quad v + 5 + 5v = 0$

or $\quad v = -\frac{5}{6} = -0.833$ V

Finally let us consider 4 V source acting alone [as shown in Fig. 1.145(c)].

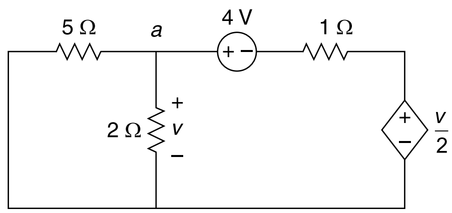



**Fig. 1.145(c)** ***4 V source acting alone***

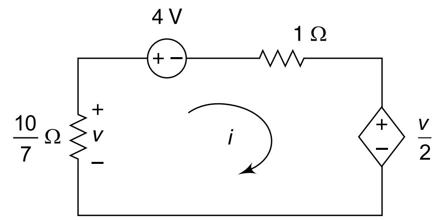



**Fig. 1.145(d)** ***Simplified network with 4 V source***

Here 5 Ω and 2 Ω are in parallel. The transformed network is shown in Fig. 1.145(d).

In the circuit of Fig. 1.145(d),

$$v - 4 - 1 \times i - \frac{v}{2} = 0$$

or $$\frac{v}{2} - i = 4$$

Again, $$v = -\frac{10}{7} \times i$$

Hence $$-\frac{10i}{2 \times 7} - i = 4$$

or $$i = -2.33$$

∴ $$v = \frac{10}{7} \times 2.33 = 3.33 \text{ V}$$

Using superposition theorem, when all the sources are acting simultaneously we have

$$v = 1.67 - 0.833 + 3.33 = 4.17 \text{ V}.$$

**H1.14** Solve Example H1.12 using Thevenin's theorem.

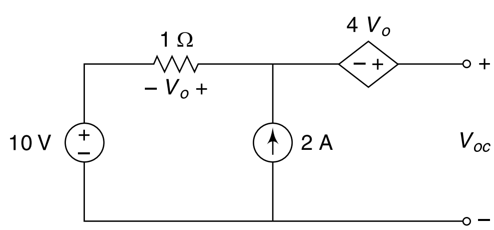



**Fig. 1.146** ***Determination of $V_{OC}$***

***Solution***

Removing 2 Ω resistor as shown in Fig. 1.146, the open circuit voltage is obtained and

$$V_{oc} = 10 + V_o + 4V_o = 10 + 5V_o$$

However, $V_o$ = voltage across the 1 Ω resistor

= 1 × current through the 1 Ω resistor

= 1 × 2 = 2 V

Hence $V_{oc} = 10 + 5 \times 2 = 20$ V.

To find out $R_{Th}$, let us first short-circuit the output terminals as shown in Fig. 1.146(a).

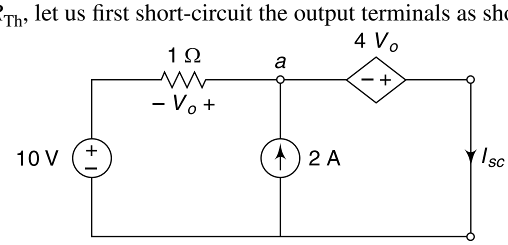



**Fig. 1.146(a)** ***Determination of $I_{SC}$ and $R_{Th}$***

Applying KVL in the circuit,

$$10 + V_o + 4V_o = 0$$

or $$V_o = -\frac{10}{5} = -2$$

Applying nodal analysis at node $a$,

$$2 - \frac{V_o}{1} - I_{sc} = 0$$

or $$I_{sc} = 2 - \frac{-2}{1} = 4 \text{ A}$$

Hence $$R_{Th} = \frac{V_{oc}}{I_{sc}} = 5\ \Omega.$$

The Thevenin's equivalent circuit is shown in Fig. 1.146(b)

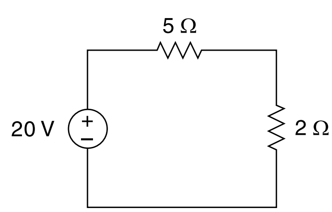



**Fig. 1.146(b)** ***Thevenin's equivalent circuit***

The current through 2 Ω resistor $= \frac{20}{7}$ A = 2.857 A.

Hence the power loss in the 2 Ω resistor is $(2.857)^2 \times 2 = 16.31$ W

**H1.15** Find $i_1$, $i_2$ and $i_3$ in Fig. 1.147.

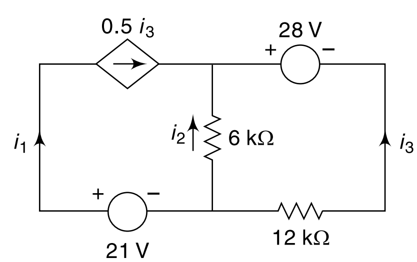



**Fig. 1.147** ***Circuit of Ex. 1.111***

***Solution***

Let us consider mesh currents $i_x$ and $i_y$ in the two meshes as shown in Fig. 1.147(a).

Applying loop equations in the two meshes

$$6 \times 10^3(i_x - i_y) - 21 = 0$$

and $$6 \times 10^3(i_y - i_x) + 12 \times 10^3 i_y + 28 = 0$$

or $$i_x = i_y + \frac{21}{6 \times 10^3} \quad \text{(i)}$$

and $$18 \times 10^3 i_y - 6 \times 10^3 i_x + 28 = 0 \quad \text{(ii)}$$

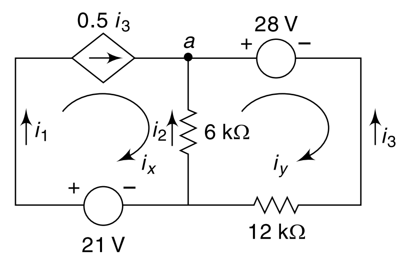



**Fig. 1.147(a)** ***Network of Fig. 1.147 with mesh currents***

Solving these two equations

$$i_y = -0.583 \text{ mA} \quad \text{and} \quad i_x = 2.917 \text{ mA}$$

From Fig. 1.147(a) it is evident that

$$0.5 i_3 = 2.917 \text{ mA}$$

or $$i_3 = 5.834 \text{ mA}.$$

Applying KCL at node $a$

$$0.5 i_3 + i_2 + i_3 = 0$$

or $$i_2 + 1.5 i_3 = 0$$

or $$i_2 = -1.5 \times 5.834$$
$$= -8.751 \text{ mA}$$

Also, $i_1 = 0.5 i_3$
$$= 0.5 \times 5.834 = 2.917 \text{ mA}.$$

**H1.16** Determine the resistance connected across terminals $a - b$ which will transfer maximum power across it in the circuit shown in Fig. 1.148.

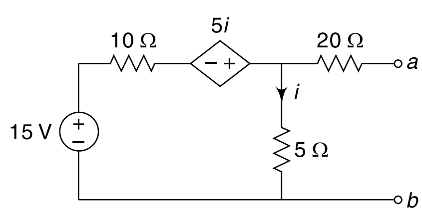



**Fig. 1.148** ***Circuit of Ex. H1.16***

***Solution***

Applying KVL in the closed loop (Fig. 1.148), we have

$$15 - 10i + 5i - 5i = 0$$

or $$i = \frac{15}{10} = 1.5 \text{ A}$$

Hence $V_{ab} = 1.5 \times 5 = 7.5$ V

For finding out the internal resistance $R_i$ of the circuit let us short circuit the path $ab$ as shown in Fig. 1.148(a).

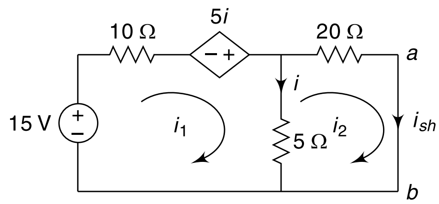



**Fig. 1.148(a)** ***Determination of $R_i$***

The mesh equations in the two loops are

$$-15 + 10i_1 - 5i + 5(i_1 - i_2) = 0$$

i.e., $\quad -15i_1 + 5i_2 + 5i + 15 = 0$

and $\quad 20i_2 + 5(i_2 - i_1) = 0$

i.e. $\quad 5i_1 - 25i_2 = 0$

$\therefore \quad i_1 = 5i_2$

Also $\quad i = i_1 - i_2$

Hence $\quad -15(5i_2) + 5i_2 + 5(i_1 - i_2) + 15 = 0$

With $\quad i_1 = 5i_2$ we get

$$i_2 = 0.3 \text{ A } (= i_{sh})$$

$$\therefore \quad R_i = \frac{v_{ab}}{i_2} = 25\ \Omega$$

According to the maximum power transfer theorem maximum power will be transferred across *ab* when the resistance connected across *ab* is equal to $R_i$ i.e., 25 Ω.

**H1.17** In the circuit shown in Fig. 1.149, find *I*.

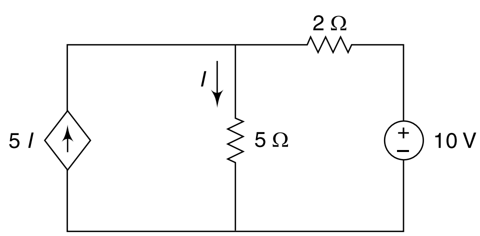



**Fig. 1.149** ***Circuit of Ex. H1.17***

### *Solution*

We redraw the circuit with arbitrary current distribution and node number [Fig. 1.149(a)]

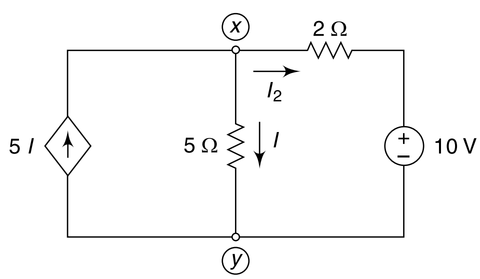



**Fig. 1.149(a)** ***Circuit of Ex. H1.17 redrawn with currents and nodes designated***

At node ⓧ,

$$5I = I + I_2 = \frac{V}{5} + \frac{V - 10}{2}$$

[Assuming voltage at node $x$ to be ($V$)]

$$\therefore \quad 5I = \frac{7V - 50}{10}$$

$$\text{or} \quad 50I = 7V - 50 \qquad (1)$$

$$\text{But} \quad I = \frac{V}{5}$$

$\therefore$ From (1), 10 V = 7 V – 50

$$\text{or} \quad V = -\frac{50}{3} = -16.67 \text{ V}$$

$$\text{Thus} \quad I = \frac{V}{5} = -\frac{16.67}{5} = -3.34 \text{ A}$$

The actual current $I$ is directed upwards (i.e., towards $x$ from $y$) in Fig. 1.149(a).

**H1.18** In the network shown in Fig. 1.150, find the value of the dependent source using (i) nodal method and (ii) superposition theorem.

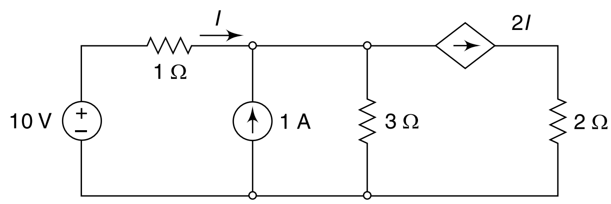



**Fig. 1.150** *Circuit of Ex. H1.18*

***Solution***

***Using nodal method:*** We first redraw the figure and assign a node ⓧ for application of nodal method (Fig. 1.150(a))

At the node ⓧ, we have, $\frac{10 - V}{1} + 1 = \frac{V}{3} + 2I$ [($V$) being the node voltage at ⓧ]

Since $I = \frac{10 - V}{1}$, we can write, $\frac{10 - V}{1} + 1 = \frac{V}{3} + 2\left(\frac{10 - V}{1}\right)$

Simplification yields $V = 13.5$ V.

Thus, $I = \frac{10 - 13.5}{1} = -3.5$ and the dependent source would have value $2I$, i.e., $2 \times (-3.5)$ i.e., –7 A.

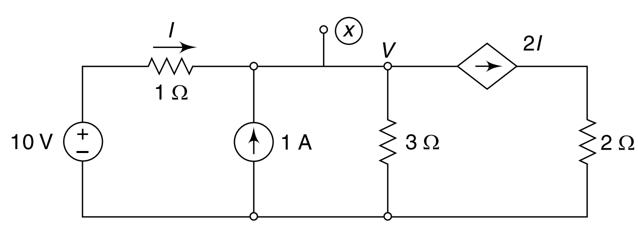



**Fig. 1.150(a)** *Solution by nodal method*

It may be noted here that the actual direction of the currents $I$ and $2I$ would be just the reverse than given in the question.

***Using Superposition Theorem***

Let us first assume the 10 V source only (Fig. 1.150(b)).

At node ⓐ, we find

$$\frac{10 - V}{1} = \frac{V}{3} + 2\left(\frac{10 - V}{1}\right)$$

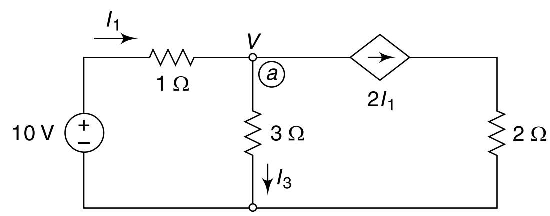



**Fig. 1.150(b)** ***Solution by superposition method (10 V source acting only)***

Simplifying,

$$V = 15 \text{ Volts (at node } a)$$

∴ $$I_1 = \frac{10 - V}{1} = \frac{10 - 15}{1} = -5 \text{ A}$$

while $$2I_1 = -10 \text{ A}$$

Next we consider the constant current source 1 A only (Fig. 1.150(c)).

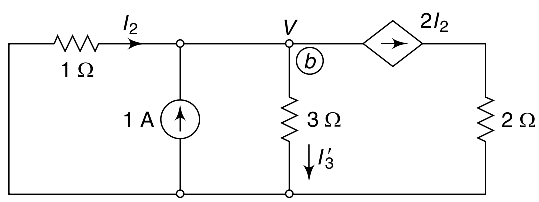



**Fig. 1.150(c)** ***1 A source acting only***

We select node ⓑ where we find

$$1 + I_2 = \frac{V}{3} + 2I_2$$

or $$\frac{V}{3} + I_2 = 1 \qquad \text{(i)}$$

But $$I_2 = \frac{-V}{1} \qquad \text{(ii)}$$

Using (ii) in (i), we get,

$$\frac{V}{3} - V = 1, \quad \text{i.e.,} \quad V = -1.5 \text{ V.}$$

∴ $$I_2 = 1.5 \text{ A}; \; 2I_2 = 3 \text{ A.}$$

Finally, using the principle of superposition, we get

$$2I = 2I_1 + 2I_2 = -10 + 3 = -7 \text{ A}$$

(the same result that we obtained earlier).

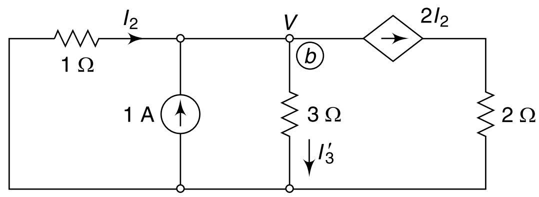



**Fig. 1.150(c)** *1 A source acting only*

**H1.19** Applying Kirchhoff's voltage law find the values of current $i$ and the voltages $v_1$ and $v_2$ in the circuit shown in Fig. 1.151.

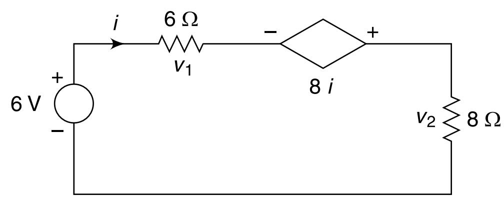



**Fig. 1.151** *Circuit of Ex. H1.19*

***Solution***

Applying Kirchhoff's voltage law in Fig. 1.151

$$6 - v_1 + 8i - v_2 = 0$$

or $v_1 + v_2 = 6 + 8i$

Now $v_1 = 6i$ and $v_2 = 8i$

Hence $6i + 8i = 6 + 8i$

or $i = 1$

Therefore $v_1 = 6 \times 1 = 6$ Volts and $v_2 = 8 \times 1 = 8$ V.

**H1.20** Find the current through $R_L$ in the circuit shown in Fig. 1.152 using Norton's theorem.

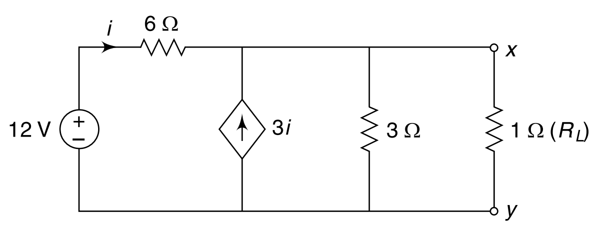



**Fig. 1.152** *Circuit of Ex. H1.20*

***Solution***

Let us short-circuit the terminals $xy$ to find out the Norton's equivalent current (Fig. 1.152(a)).

$$i_N = i + 3i = 4i$$

Now, $i = \frac{12}{6}\text{ A} = 2\text{ A}$

Hence $i_N = 4 \times 2 = 8$ A

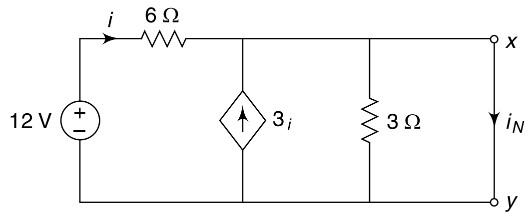



**Fig. 1.152(a)** ***Determination of $i_N$***

To find Norton's equivalent resistance $R_N$ let us open circuit terminals *xy*. The corresponding circuit is shown in Fig. 1.152(b).

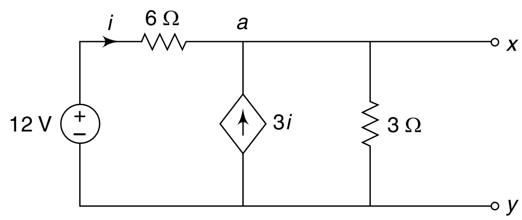



**Fig. 1.152(b)** ***Determination of $V_{xy}$***

At node *a*,

$$i + 3i - \frac{V_{xy}}{3} = 0$$

$$V_{xy} = 12i$$

But, $$i = \frac{12 - V_{xy}}{6}$$

Hence $$V_{xy} = 12\,\frac{12 - V_{xy}}{6} = 24 - 2V_{xy}$$

$$3V_{xy} = 24 \text{ i.e. } V_{xy} = 8 \text{ V.}$$

Therefore $$R_N = \frac{8}{8} = 1\ \Omega.$$

Norton's equivalent circuit is shown in Fig. 1.152(c).

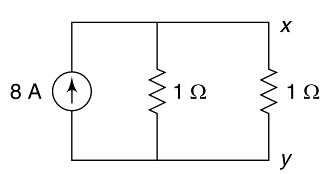



**Fig. 1.152(c)** ***Norton's equivalent circuit of Ex. H1.20***

Hence current through 1 Ω resistor $= 8 \times \frac{1}{1+1} = 4$ A.

**H1.21** Obtain Thevenin's equivalent circuit across terminals $a - b$ in the Fig. 1.153.

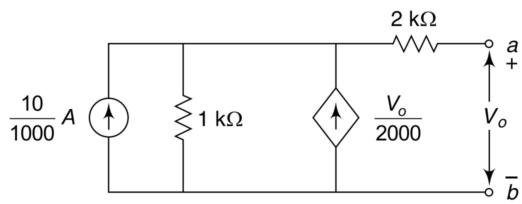



**Fig. 1.153** ***Circuit of Ex. H1.21***

***Solution***

The current through 1 kΩ resistor is $\left(\frac{10}{1000} + \frac{V_o}{2000}\right)$ A

Open circuit voltage across $a - b$ is the voltage across the 1 kΩ resistor

Hence $$V_o = \left(\frac{10}{1000} + \frac{V_o}{2000}\right) \times 1000 = 10 + 0.5\, V_o$$

i.e. $$V_o = 20 \text{ V}$$

To find out Thevenin's equivalent resistance ($R_{Th}$) let us short circuit terminals *ab* as shown in Fig. 1.153(a).

As *ab* is short-circuited $V_o$ is zero. The network reduces to that shown in Fig. 1.153(b).

Hence $$i_{sh} = \frac{10}{1000} \times \frac{1}{1+2} = \frac{10}{3000} \text{ A}$$

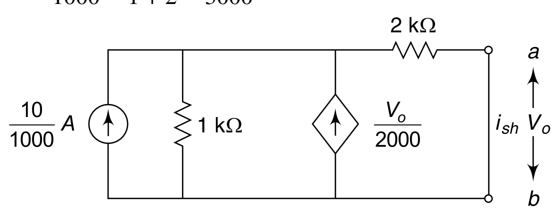



**Fig. 1.153(a)** ***Determination of $R_{Th}$***

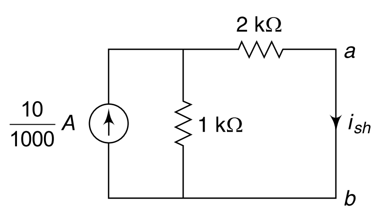



**Fig. 1.153(b)** ***Reduced network of the circuit shown in Fig. 1.153(a)***

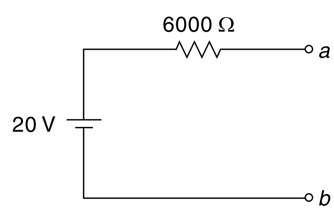



**Fig. 1.153(c)** ***Thevenin's equivalent circuit of Ex. H1.21***

Therefore, $$R_{Th} = \frac{V_o}{i_{sh}} = \frac{20}{10} \times 3000 = 6000\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 1.153(c).

**H1.22** Using maximum power transfer theorem find the value of the load resistance $R_L$ so that the maximum power is transfered across $R_L$ in the circuit shown in Fig. 1.154.

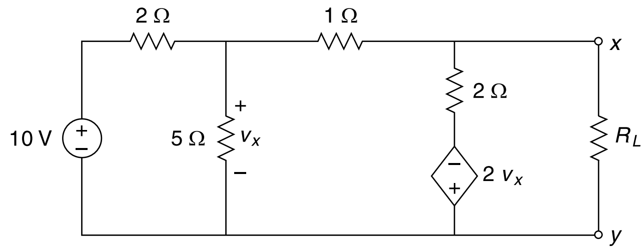



**Fig. 1.154** ***Circuit of Ex. H1.22***

### *Solution*

Let us remove $R_L$ and open circuit terminals $xy$ to find out the internal resistance $R_i$ of the circuit. According to maximum power transfer theorem the maximum power will be transfered through $R_L$ when

$$R_L = R_i$$

From Fig. 1.154(a) applying KVL,

$$10 - 2i_1 - 5(i_1 - i_2) = 0$$

i.e. $\quad 7i_1 - 5i_2 = 10 \qquad$ (i)

and $\quad 3v_x - 5(i_2 - i_1) - i_2 \times 1 - 2i_2 = 0$

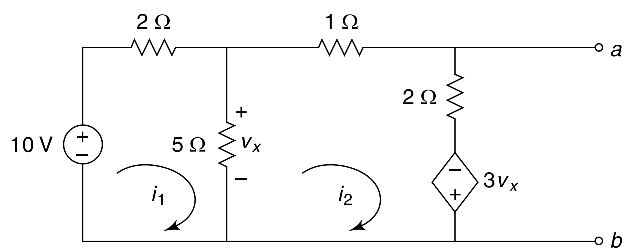



**Fig. 1.154(a)** ***Determination of $V_{ab}$***

i.e. $\quad 3 \times 5(i_1 - i_2) - 5(i_2 - i_1) - 3i_2 = 0 \qquad [\because v_x = 5(i_1 - i_2)]$

or $\quad 20i_1 - 23i_2 = 0$ i.e., $i_1 = \dfrac{23}{20} i_2 \qquad$ (ii)

Using equation (ii) in equation (i) we get

Hence $\quad 7 \times \dfrac{23}{20} i_2 - 5i_2 = 10$

or $\quad i_2 = 3.28$ A

and $\quad i_1 = 3.772$ A

$\therefore \quad v_x = 5(3.772 - 3.28) = 2.46$ V.

Now, $\quad v_{ab} = -3v_x + 2i_2$

$$= -3 \times 2.46 + 2 \times 3.28$$

$$= -0.82 \text{ V.}$$

Let us now short circuit the terminals *xy* as shown in Fig. 1.154(b).

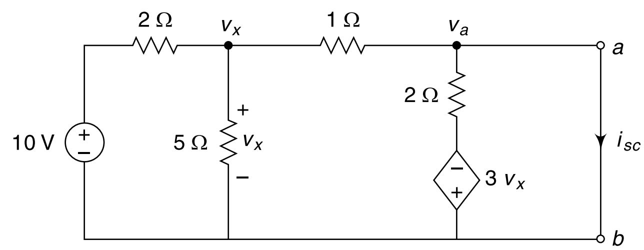



**Fig. 1.154(b)** ***Determination of*** $i_{SC}$

At node *a*,

$$\frac{v_a - v_x}{1} + \frac{v_a + 3v_x}{2} + i_{sc} = 0$$

As $a - b$ are shorted $v_a = 0$

Hence $- v_x + 1.5v_x + i_{sc} = 0$

i.e. $i_{sc} = - 0.5v_x$

Now, $v_x$ = Voltage across 5 Ω resistor. Current through 5 Ω resistor

$$I_{5\Omega} = \frac{10}{2 + \frac{5 \times 1}{5 + 1}} \times \frac{1}{5 + 1}$$

$$= \frac{60}{12 + 5} \times \frac{1}{6} = 0.588 \text{ A}$$

Hence $v_x = 5 \times 0.588 = 2.94$ V

and $i_{sc} = -0.5 \times 2.94 = -1.47$ A

Therefore, $R_L = R_i = \frac{-0.82}{-1.47} = 0.558\ \Omega.$

## EXERCISES

1. State and explain Thevenin's theorem. What are the limitations of this theorem?
2. State and prove maximum power transfer theorem.
3. State and explain Kirchhoff's voltage law and current law.
4. Distinguish between dependent and independent sources. How do you transform a voltage source into a current source?
5. Distinguish between
   (a) Linear and non-linear elements
   (b) Active and passive elements
   (c) Unilateral and bilateral elements.
6. State superposition theorem and explain it.
7. State Norton's theorem and explain it.
8. Prove that under maximum power transfer condition the power transfer efficiency of the circuit is only 50%.

9. Find the equivalent resistance for the circuit shown in Fig. 1.155. [*Ans:* 10 Ω]

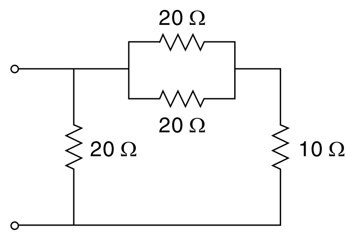



**Fig. 1.155**

10. Find $R_1$ and $R_2$ for the potential divider in Fig. 1.156 so that current $I$ is limited to 1 A when $V_o = 20$ V.

[*Ans:* $R_2 = 20\ \Omega$, $R_1 = 80\ \Omega$]

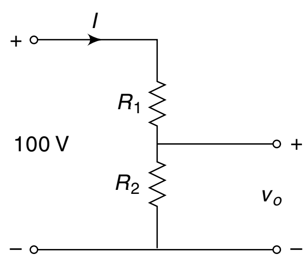



**Fig. 1.156**

11. Use mesh currents in the network shown in Fig. 1.157 to find the current supplied by a 60 V source. [*Ans:* 6 A]

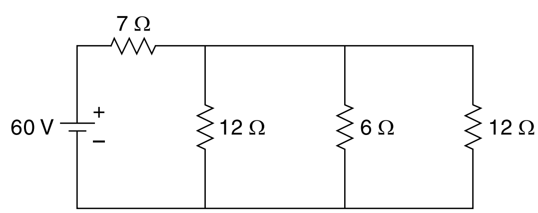



**Fig. 1.157**

12. Solve problem no. 11 by nodal current method.
13. Two ammeters $x$ and $y$ are connected in series and a current of 20 A flows through them. The potential difference across the ammeters are 0.2 V and 0.3 V respectively. Find how the same current will divide between $x$ and $y$ when they are connected in parallel. [*Ans:* 12 A and 8 A]
14. Obtain the source current $I$ and the power delivered to the circuit in Fig. 1.158. [*Ans:* 6 A, 228 W]

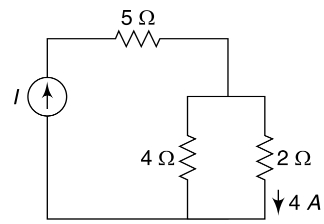



Fig. 1.158

[Hint: $I_{2\Omega} = 4$ A; $V_{\text{drop}(2\Omega)} = 4 \times 2 = 8$ V;

Hence $I_{4\Omega} = \frac{8}{4} = 2$ A.

$\therefore\ I = I_{2\Omega} + I_{4\Omega} = 6$ A;

$P = 6^2 \times 5 + 2^2 \times 4 + 4^2 \times 2 = 228$ W]

15. For the circuit shown in Fig. 1.159 find the potential difference between $x$ and $y$. [*Ans:* –2.85 V]

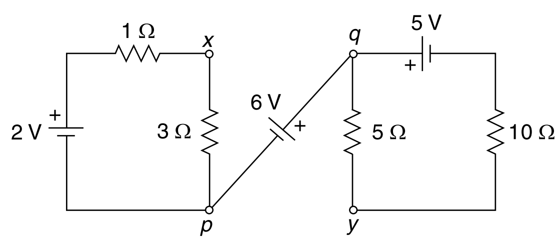



Fig. 1.159

[Hint: In left loop, $I = \frac{2}{1+3} = 0.5$ A; in right loop, $I = \frac{5}{15} = 0.33$ A.

$V_{x\text{-}y} = V_{xp} + (-6) + V_{qy} = 0.5 \times 3 - 6 + 0.33 \times 5 = -2.85$ V]

16. Reduce the circuit in Fig. 1.160 to a voltage source in series with a resistance between terminals $A$ and $B$.

$\left[Ans:\ V = \frac{90}{23}\ \text{V and } R = \frac{15}{23}\ \Omega\right]$

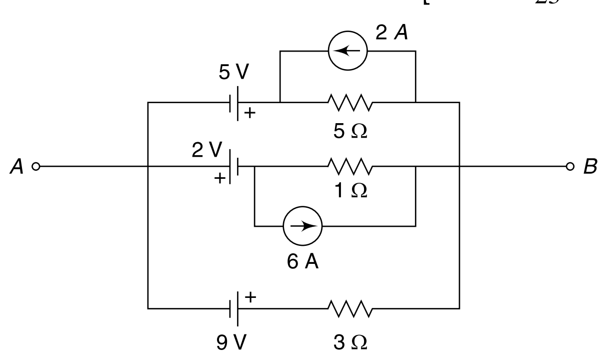



Fig. 1.160

17. For the network shown in Fig. 1.161 find $V$ which makes $I = 7.5$ mA.

[*Ans:* 1.02 V]

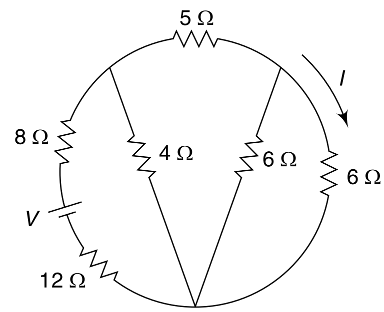



**Fig. 1.161**

$\left[ I = 7.5 \text{ mA}; I_{6\Omega} = \dfrac{7.5 \text{ mA} \times 6\ \Omega}{6\ \Omega} = 7.5 \text{ mA} \right.$

$\therefore I_{5\Omega} = 15$ mA. Drop in 5 Ω = 15 mA × 5 Ω = 75 mV.

Then drop across 4 Ω is 75 mV + 7.5 mA × 6 Ω = 120 mV.

$\therefore I_{4\Omega} = \dfrac{120}{4} = 30$ mA. Current from battery is then (30 + 15) i.e., 45 mA. Hence voltage drop in 8 Ω is 45 mA × 8 = 360 mV. Drop in 12 Ω is 45 mA × 12 = 540 mV.

$\therefore V = 360 + 75 + 45 + 540 = 1020$ mV. i.e., 1.02 V]

18. In the network shown in Fig. 1.162 find the resistance between (i) $A$ and $B$ (ii) $C$ and $A$.

$\left[ \textit{Ans:} \text{ (i) } \dfrac{7}{8}\ \Omega \text{ (ii) } \dfrac{7}{8}\ \Omega \right]$

[Hint: Convert delta (*abc*) to star first and proceed]

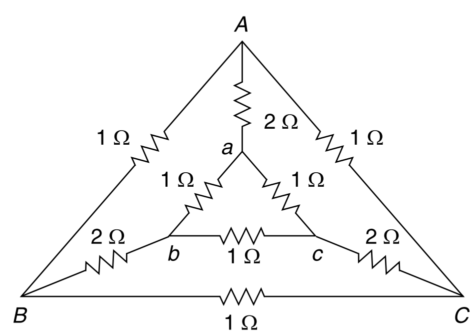



**Fig. 1.162**

19. For the ladder network shown in Fig. 1.163, find the applied voltage $V$.

[*Ans:* 800 V]

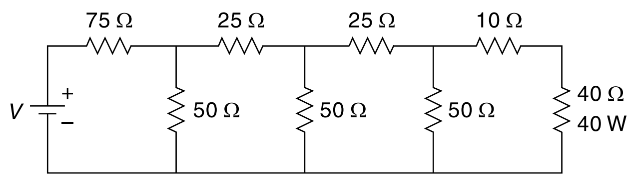



**Fig. 1.163**

[Hint: Find current through the 40 Ω resistor and then proceed as shown in Problem no. 17]

20. Find the open circuit voltage across in the 10 Ω resistance in the network shown in Fig. 1.164. [*Ans:* 139.36 V]

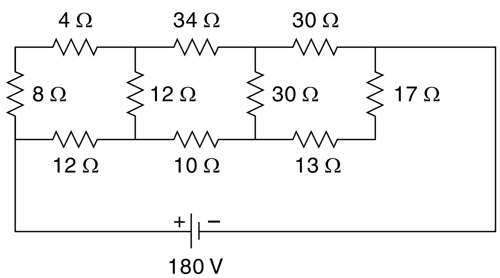



**Fig. 1.164**

21. In the network shown in Fig. 1.165, find Thevenin's equivalent network across $x - y$ terminals. [*Ans:* $V_{\text{o/c}} = V_{x-y} = 25$ V; $R_{\text{Th}} = 17\ \Omega$]

[Hint: 10 Ω resistor is removed. $V_{o/c} = 5\text{ A} \times 5\ \Omega = 25$ V. Next $x - y$ is shorted. At node $z$, we can write

$5 = \frac{V}{5} + 2I_{s/c} + I_{s/c}$; But $I_{s/c} = \frac{V}{2}$ ($V$ being the voltage at node $z$).

$\therefore I_{s/c} = 1.47$ A and $R_{\text{Th}} = \frac{V_{o/c}}{I_{s/c}} = 17\ \Omega$]

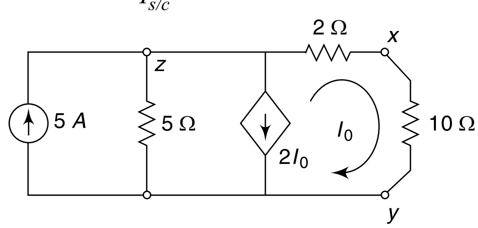



**Fig. 1.165**

22. Find the Thevenin's equivalent circuit at terminals $A$, $B$ for the network shown in Fig. 1.166. [*Ans:* $V_{\text{Th}} = 25$ V, $R_{\text{Th}} = 20\ \Omega$]

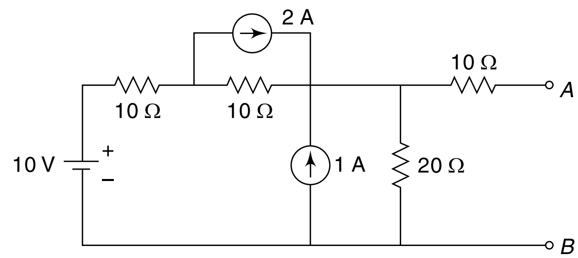



**Fig. 1.166**

23. Find $V_1$ and $V_2$ in Fig. 1.167 using nodal voltages analysis method.
[*Ans:* $V_1$ = 2.468 V and $V_2$ = 1.156 V]

[Hint: At node $A$, $20 = \frac{V_1}{0.3} + \frac{V_1 - V_2}{0.2}$; At node $B$, $5 = \frac{V_2}{0.1} + \frac{V_2 - V_1}{0.2}$].

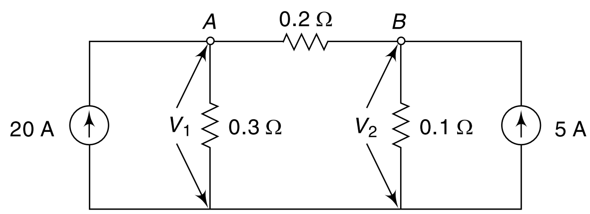



**Fig. 1.167**

24. Find $i_0$, $i_2$ and the value of the dependent source for the network shown in Fig. 1.168.
[*Ans:* 2 A, –4 A ; 4 A]

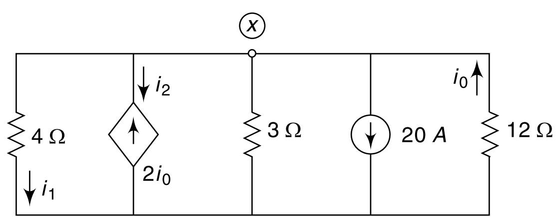



**Fig. 1.168**

[Hint: At node $x$, assuming node voltage to be $v$, we have, $\frac{v}{4} + \frac{v}{3} + \frac{v}{12} + 20 = 2i_0$

However, $-i_0 = \frac{v}{12}$ A.

$\therefore$ $v = -24$ V and $i_1 = \frac{v}{4} = -6$ A; $i_0 = -\frac{v}{12} = 2$ A.

Value of dependent source is 4 A.

$i_2 = -2i_0 = -4$ A.]

25. Find the current in the 6 Ω resistor of Fig. 1.169 using Thevenin's theorem.

[*Ans:* 0.67 A]

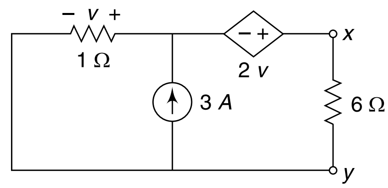



**Fig. 1.169**

26. What is the power supplied by the dependent source in the circuit of Fig. 1.170. [*Ans:* –84 W]

[Hint: In the right loop, $-5 - V_0 + 2i + 2V_0 = 0 \quad \therefore V_0 + 2i = 5$.

But $V_0 = -(i + 1) \times 1 = -i - 1$

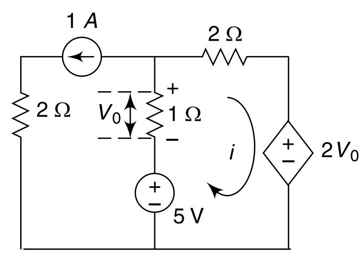



**Fig. 1.170**

Solving, $i = 6$ A and $V_0 = -7$ V

$\therefore$ Power supplied by the dependent source is $2\,V_0 \times i = -84$ W]

27. Find Norton's equivalent circuit of the network shown in Fig. 1.171.

[*Ans:* 1.17 A, 6 Ω]

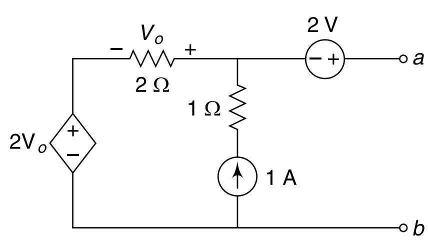



**Fig. 1.171**

28. Using Norton's theorem find the current in the 5 Ω resistor in the network shown in Fig. 1.172. [*Ans:* 1.65 A]

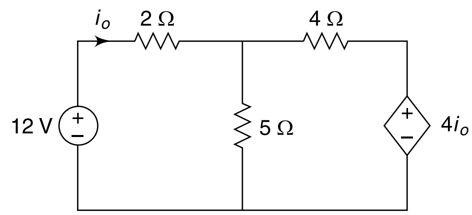



**Fig. 1.172**

29. In the circuit of Fig. 1.173, if $r = 5\ \Omega$, $R_L = 10\ \Omega$, $v_o = 10$ V, $i_o = 2$ A, find the current through $R_L$ using Thevenin's theorem. [*Ans:* 1.33 A]

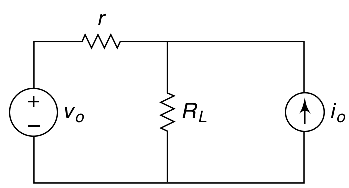



**Fig. 1.173**

Hint: $R_L$ is removed.

$$V_{o/c} = i_o \times r + v_o = 20 \text{ V}$$

$$R_{\text{Th}} = 5\ \Omega\ (= r)$$

$$\therefore \quad I_{R_L} = \frac{V_o/c}{R_{\text{Th}} + R_L} = 1.33 \text{ A}$$

30. Find $v$ by superposition theorem (Fig. 1.174).

[*Ans:* $v = 23.37$ V]

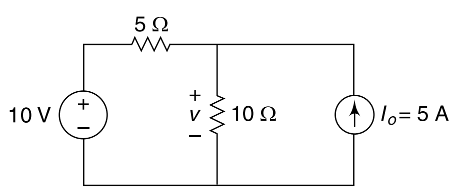



**Fig. 1.174**

[Hint: With 10 V source only,

$$v_1 = 10 \times i$$

$$= 10 \times \frac{10}{5 + 10}$$

$$= 6.67 \text{ V}$$

With 5 A source only,

$$i_{10} = 5 \times \frac{5}{5 + 10} = 1.67 \text{ A}$$

$$\therefore \quad v_2 = 1.67 \times 10 = 16.7 \text{ V}$$

Thus, $v = v_1 + v_2 = 23.37$ V]

31. The galvanometer in Fig. 1.175 has a resistance of 5 Ω. Find the current through the galvanometer using Thevenin's theorem. [*Ans:* 15.9 *mA*]

[Hint: Open circuiting *BD*, current through 10 Ω resistor

$$I_1 = \frac{10}{10 + 15} \text{ A} = 0.4 \text{ A}.$$

Current through the 12 Ω resistor

$$I_2 = \frac{10}{12 + 16} \text{ A} = 0.357 \text{ A}.$$

$$V_{\text{Th}} = V_{BD} = V_{AD} - V_{AB} = 12 \times 0.357 - 10 \times 0.4 = 0.284 \text{ V}$$

$$R_{\text{Th}} = \frac{10 \times 15}{10 + 15} + \frac{12 \times 16}{12 + 16} = 12.857\ \Omega.$$

Currrent through galvanometer $= \dfrac{0.284}{12.857 + 5}$ A = 0.0159 A [*B* to *D*]]

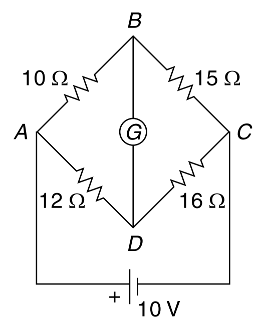



**Fig. 1.175**

32. For the electrical network shown in Fig. 1.176 find the value of load resistance $R_L$ for which source will supply maximum power to the load. Find also the maximum power. [*Ans:* 8 W]

[Hint: $R_L = \dfrac{6 \times 3}{6 + 3}\ \Omega = \dfrac{18}{9}\ \Omega = 2\ \Omega$

$= (R_{\text{int}})$

$$\therefore \quad I_L = \frac{12}{3 + \dfrac{6 \times 2}{6 + 2}} \times \frac{6}{6 + 2} = 2 \text{ A}$$

$$P_{\max} = (2)^2 \times 2 \text{ W} = 8 \text{ W}]$$

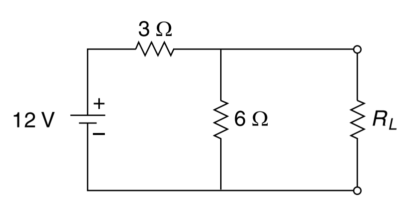



**Fig. 1.176**

33. Determine the current passing through the 20 Ω (*BD*) resistor of the network as shown in Fig. 1.177 with the help of Thevenin's theorem.

[*Ans:* $I(B \text{ to } D) = -7.79$ A]

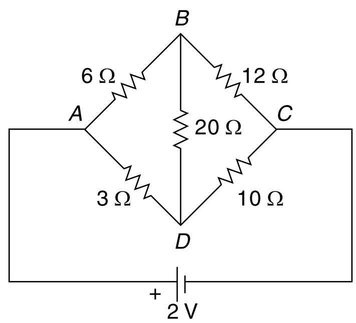



**Fig. 1.177**

[Hint: Removing the 20 Ω resistor the open circuit voltage

$$V_{BD} = V_{\text{Th}} = V_{AD} - V_{AB}$$

$$= \frac{2}{3+10} \times 3 - \frac{2}{6+12} \times 6 = -\,0.205 \text{ V}$$

$$R_{\text{Th}} = \frac{6 \times 12}{6+12} + \frac{3 \times 10}{3+10} = 6.307\ \Omega.$$

Current through the 20 Ω resistor $= \frac{-\,0.205}{20 + 6.307}$ A from *B* to *D* or 7.79 mA from *D* to *B*]

34. Find the current in each branch of the network shown in Fig. 1.178 using Kirchhoff's law.

[*Ans:* $I_{1\Omega} = 1.978$ A; $I_{2\Omega} = 1.12$ A (*AD*)

$I_{4\Omega} = 0.066$ A (*BD*);

$I_{2\Omega} = 1.912$ A (*BC*)

$I_{3\Omega} = 1.186$ A (*DC*);

Current through battery = 3.098 A]

[Hint: Taking 3 mesh currents $I_1$, $I_2$ and $I_3$ in loops *ABDA*, *BCDB* and *ADC* (12 V) A,

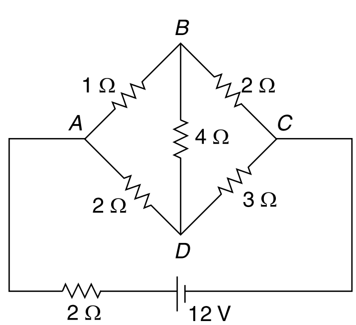



**Fig. 1.178**

$I_1 + (I_1 - I_2)4 + (I_1 - I_3)2 = 0$

$2I_2 + 3(I_2 - I_3) + 4(I_2 - I_1) = 0$

$2I_3 + 2(I_3 - I_1) + 3(I_3 - I_2) = 12$

Solving $I_1$ = 1.978 A; $I_2$ = 1.912 A; $I_3$ = 3.098 A currents in all branches can be found out from $I_1$, $I_2$ and $I_3$]

## MULTIPLE CHOICE QUESTIONS

1. Identify the passive elements among the following.
   (a) Voltage source (b) Current source
   (c) Inductor (d) Transistor
2. Determine the total inductance of a parallel combination of 100 mH, 50 mH and 10 mH.
   (a) 7.69 mH (b) 160 mH
   (c) 60 mH (d) 110 mH
3. If the voltage across a given capacitor is increased, the amount of stored charge
   (a) increases (b) decreases
   (c) remains same (d) is exactly doubled
4. How much energy is stored by a 100 mH inductance with a current of 1 A?
   (a) 100 J (b) 1 J
   (c) 0.05 J (d) 0.01 J
5. The following voltage drops are measured across each of three resistors in series 5.2 V, 8.5 V and 12.3 V. What is the value of the source voltage to which these resistors are connected?
   (a) 8.2 V (b) 12.3 V
   (c) 5.2 V (d) 26 V
6. A certain series circuit has 100 Ω, 270 Ω and 330 Ω resistors in series. If the 270 Ω resistor is removed, the current will
   (a) increase (b) become zero
   (c) decrease (d) remain constant
7. A series circuit consists of a 4.7 kΩ, 5.6 kΩ, 9 kΩ and 10 kΩ resistors. Which resistor has the highest voltage across it?
   (a) 4.7 kΩ (b) 5.6 kΩ
   (c) 9 kΩ (d) 10 kΩ
8. The total power in a series circuit is 10 W. There are five equal value resistors in the circuit. How much power does each resistor dissipate?
   (a) 10 W (b) 5 W
   (c) 2 W (d) 1 W

9. When a 1.2 kΩ resistor, 100 Ω resistor, 1 kΩ resistor and 50 Ω resistor are in parallel, the total resistance is less than
   (a) 100 Ω (b) 50 Ω
   (c) 1.2 kΩ (d) 1 kΩ
10. If one of the resistors in a parallel circuit is removed, what happens to the total resistance?
   (a) Decreases (b) Increases
   (c) Exactly doubles (d) Remains constant
11. Six light bulbs are connected in parallel across 110 V. Each bulb is rated at 75 W. How much current flows through each bulb?
   (a) 0.682 A (b) 0.7 A
   (c) 75 A (d) 110 A
12. Superposition theorem is valid only for
   (a) linear circuits (b) non-linear circuits
   (c) both (a) and (b) (d) neither (a) nor (b)
13. When superposition theorem is applied to any circuit, the dependent voltage source is always
   (a) opened (b) shorted
   (c) active (d) none of the above
14. Maximum power is transferred when the load resistance is
   (a) equal to source in resistance
   (b) equal to half of the source resistance
   (c) equal to zero
   (d) none of the above
15. The superposition theorem is not valid for
   (a) voltage responses (b) current responses
   (c) power responses (d) all the above
16. Determine the current *I* in the circuit (Fig. 1.179)

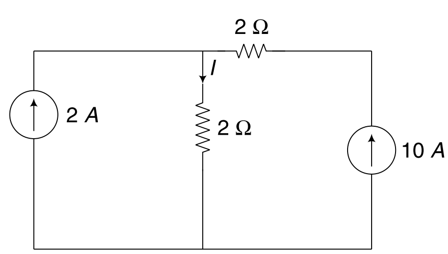



**Fig. 1.179**

   (a) 2.5 A (b) 1 A
   (c) 12 A (d) 4.5 A

17. The reciprocity theorem is applicable to
    (a) linear networks only (b) bilateral networks only
    (c) both (a) and (b) (d) neither (a) nor (b)
18. Thevenin voltage in the circuit shown in Fig. 1.180 is
    (a) 3V (b) 2.5 V
    (c) 2 V (d) 0.1 V

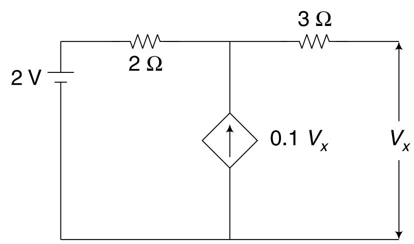



**Fig. 1.180**

19. Three equal resistances of 3 Ω are connected in star what is the resistance in one of the arms in an equivalent delta circuit?
    (a) 10 Ω (b) 3 Ω
    (c) 9 Ω (d) 27 Ω
20. Three equal resistances of 5 Ω are connected in delta. What is the resistance in one of the arms of the equivalent star circuit?
    (a) 5 Ω (b) 1.67 Ω
    (c) 10 Ω (d) 15 Ω
21. Norton's current in the circuit (Fig. 1.181) is given by

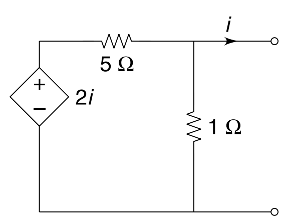



**Fig. 1.181**

    (a) (2i/5) (b) zero
    (c) infinite (d) none
22. The nodal method of circuit analysis is based on
    (a) KVL and Ohm's law (b) KCL and Ohm's law
    (c) KVL and KCL (d) both (a) and (b)
23. A practical voltage source consists of an ideal voltage source in
    (a) series with an internal resistance
    (b) parallel with an internal resistance
    (c) both (a) and (b)
    (d) neither (a) nor (b)

24. Find the voltage between *A* and *B* in a voltage divider network (Fig. 1.182)

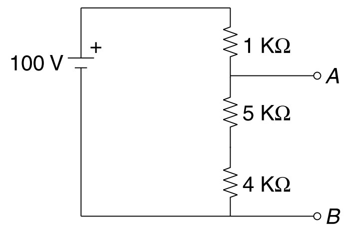



**Fig. 1.182**

(a) 90 V (b) 9 V
(c) 100 V (d) 0 V

25. The algebraic sum of all the currents meeting a junction is equal to
(a) 1 (b) –1
(c) zero (d) can't say

26. Norton's equivalent current of the circuit in Fig. 1.183 is

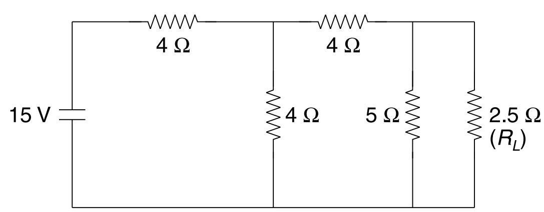



**Fig. 1.183**

(a) 1.67 A (b) 2.5 A
(c) 2 A (d) 1.25 A

27. Norton's equivalent current of the circuit in Fig. 1.184 is

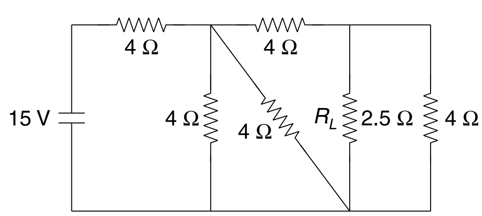



**Fig. 1.184**

(a) 1.875 A (b) 0.9375 A
(c) 2 A (d) 1 A

28. Which one of the following can be applied to analyse communication networks?
    (a) Thevenin's Theorem (b) Norton's Theorem
    (c) Superposition Theorem (d) Maximum-power Transfer Theorem
29. Thevenin's Theorem cannot he applied to a network whist contains
    (a) resistors (b) linear impedance
    (c) non-linear impedance (d) none of these
30. Superposition Theorem is valid for
    (a) non-linear bilateral network
    (b) linear bilateral network
    (c) non-linear unilateral network
    (d) linear unilateral network
31. The equivalent voltage source of the current source as shown in Fig. 1.185 is

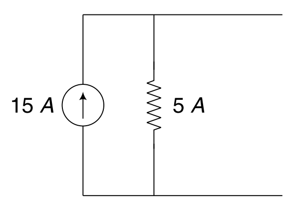



**Fig. 1.185**

    (a) 3 V (b) 75 V
    (c) $\frac{3}{5}$ V (d) 5 Volt
32. A resistor is a/an
    (a) non-linear element (b) active element
    (c) Unilateral element (d) none of these
33. The current flowing through the resistors of 10 Ω, 20 Ω, and 30 Ω connected in series is 2 A. The circuit is connected across a dc supply of
    (a) 240 V (b) 60 V
    (c) 120 V (d) 30 V
34. The value of the current *I* in the single-loop circuit of Fig. 1.186 is

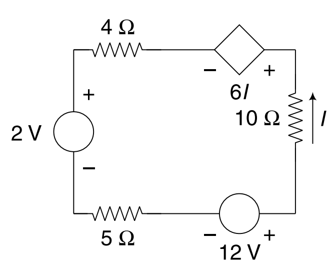



**Fig. 1.186**

(a) $\frac{2}{5}$ A (b) $\frac{7}{25}$ A

(c) 3 A (d) $\frac{9}{16}$ A

35. Efficiency of maximum power transfer is
    (a) 100% (b) 50%
    (c) 25% (d) 10%
36. The power dissipated across the 3 Ω resistor of Fig. 1.187 is

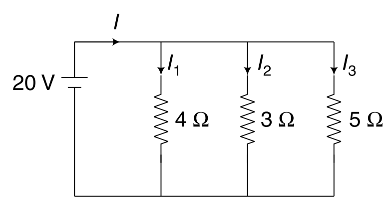



**Fig. 1.187**

    (a) 133.33 W (b) 93.29 W
    (c) 127.6 W (d) 146.91 W
37. The value of $V_1$ of the circuit in Fig. 1.188 is
    (a) 10.6 V
    (b) 18.2 V
    (c) 16.7 V
    (d) 21.13 V

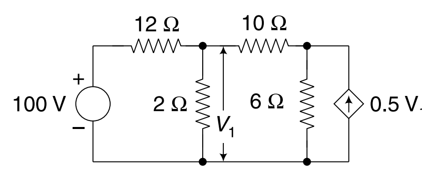



**Fig. 1.188**

38. The unit of energy is the
    (a) ampere (b) volt
    (c) watt (d) joule
39. According to KVL, the algebraic sum of all *IR* drops and emfs in any closed loop of a network is always.
    (a) zero (b) positive
    (c) negative (d) determined by battery emf
40. The load resistance needed to extract maximum power from the circuit of Fig. 1.189 is

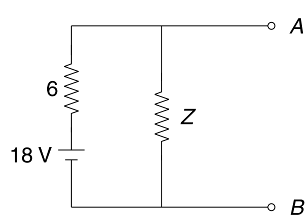



**Fig. 1.189**

(a) 2 Ω (b) 9 Ω
(c) 6 Ω (d) 18 Ω

41. A 12 Volt source with an internal resistance of 1.2 Ω is connected across a wire resistor. Maximum power will be dissipated in the resistor when its resistance is equal to
 (a) zero (b) 1.2 Ω
 (c) 12 Ω (d) infinity
42. Which of the following elements is unilateral?
 (a) Diode (b) Resistor
 (c) Capacitor (d) Inductor
43. Two 6 V, 2 Ω batteries are connected in series. This combination can be replaced by a single equivalent current generator with a parallel resistance of
 (a) 3 A, 4 Ω (b) 3 A, 2 Ω
 (c) 3 A, 1 Ω (d) 6 A, 2 Ω
44. If two identical 3 A, 4 Ω Norton equivalent circuits are connected in parallel with like polarity to like, the combined Norton equivalent circuit is
 (a) 6 A, 4 Ω (b) 6 A, 2 Ω
 (c) 3 A, 2 Ω (d) 6 A, 8 Ω
45. Two capacitances having 20 F and 5 F capacitances are connected in series. Their equivalent capacitance is
 (a) 5 F (b) 20 F
 (c) 25 F (d) 4 F
46. Kirchhoff's voltage law is concerned with
 (a) *IR* drops (b) battery emfs
 (c) junction voltages (d) both (a) and (b)
47. A good electric conductor is one that
 (a) has low conductance
 (b) is always made of copper wire
 (c) produces a minimum voltage drop
 (d) has few free electrons
48. Which of the following material has nearly zero temperature coefficient of resistance?
 (a) Carbon (b) Porcelain
 (c) Copper (d) Manganin
49. The positive terminal of a 6 V battery is connected to the negative terminal of a 12 V battery whose positive terminal is grounded. The potential at that negative terminal of the 6 V battery is
 (a) +6 V (b) –6 V
 (c) –18 V (d) +18 V

50. In the above question, the potential at the positive terminal of the 6 V battery is ______ volt

(a) +6 (b) –6

(c) –12 (d) +12

51. What is the equivalent resistance in Ω between points. *A* and *B* of Fig. 1.190?

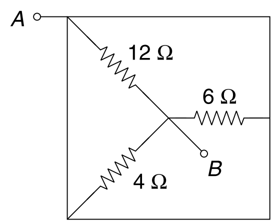



**Fig. 1.190**

(a) 12 (b) 14.4

(c) 22 (d) 2

52. In the circuit shown in Fig. 1.191, Thevenin's equivalents voltage is

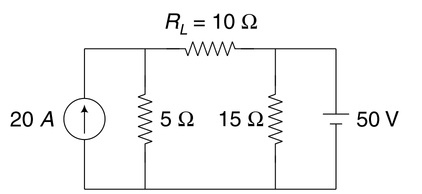



**Fig. 1.191**

(a) 50 V (b) 100 V

(c) 10 V (d) 150 V

53. Thevenin's equivalent resistance for circuit shown in Fig. 1.192 is

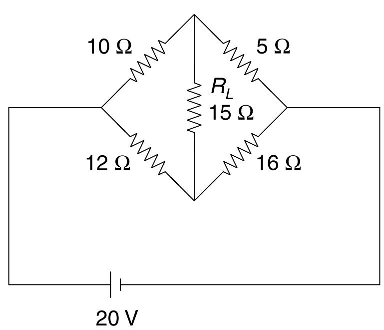



**Fig. 1.192**

(a) 0 V (b) 10.2 Ω

(c) 20 Ω (d) 300 Ω

54. How many electrons pass a given point in a conductor in 10s if the current strength is 18 A?
    (a) $1.6 \times 10^{-19}$ (b) $18 \times 10^{19}$
    (c) $112.5 \times 10^{19}$ (d) $1800 \times 10^{11}$
55. Delta-star conversion of each equal resistances in each branch leads to
    (a) decrease of resistance (b) increase of resistance
    (c) same resistances (d) none of these
56. Tesla is the unit of
    (a) magnetic flux (b) magnetic flux density
    (c) reluctance (d) flux intensity
57. One radian is equal to
    (a) $\pi^\circ$ (b) $180^\circ$
    (c) $\frac{\pi^\circ}{180^\circ}$ (d) $\frac{180}{\pi^\circ}$
58. The unit of conductance is
    (a) Coulomb (b) Siemens
    (c) Farad (d) Henry
59. In an electrical circuit, the base voltage is 4 kV while the base current is 100 amp. The base power would be
    (a) 25 KVA (b) 400 KVA
    (c) 40 KVA (d) 250 VA
60. When the current comes out from the +ve polarity of the device, the current is called
    (a) +ve Current (b) –ve Current
    (c) zero sequence Current (d) none of these
61. A conductor has a cross-sectional area of 3 $mm^2$ while the length is 100 m. If the conductor offers 8 $\Omega$ resistance, the conductivity of the material is
    (a) $2.5 \times 10^6$ Siemens/m (b) $9.2 \times 10^6$ Siemens/m
    (c) $4.17 \times 10^6$ Siemens/m (d) $6.19 \times 10^6$ Siemens/m
62. A charge of 100 C passes through a conductor in 20 seconds. What is the corresponding current in amperes?
    (a) 5 A (b) 2.5 A
    (c) 10 A (d) 7.5 A
63. The temperature coefficient of resistance is given by
    (a) $\frac{(R_1 - t_1)/(R_2 - t_2)}{R_1}$ (b) $\frac{(R_2 - t_2)/(R_1 - t_1)}{R_1}$
    (c) $\frac{(t_2 - t_1)/(R_2 - R_1)}{t_1}$ (d) $\frac{(R_2 - R_1)/(t_2 - t_1)}{R_1}$

64. Per unit impedance (*Zpu*) of an electric power network in terms of the base voltage ($V_B$) and base voltampenes $(VA)_B$

(a) $\dfrac{Z(VA)_B}{V_B^2}$ (b) $Z\dfrac{V_B^2}{(VA)_B}$

(c) $\dfrac{(VA)_B}{ZV_B^2}$ (d) $V_B^2/Z(VA)_B$

65. Parallel connection of voltage sources is not permitted because it violates

(a) Ohm's law (b) KCL

(c) KVL (d) Superposition principle

## ANSWERS

| | | | | | |
|---|---|---|---|---|---|
| 1. (c) | 2. (a) | 3. (a) | 4. (c) | 5. (d) | 6. (a) |
| 7. (d) | 8. (c) | 9. (b) | 10. (b) | 11. (a) | 12. (a) |
| 13. (c) | 14. (a) | 15. (c) | 16. (c) | 17. (c) | 18. (b) |
| 19. (c) | 20. (b) | 21. (a) | 22. (b) | 23. (a) | 24. (a) |
| 25. (c) | 26. (d) | 27. (b) | 28. (d) | 29. (c) | 30. (b) |
| 31. (b) | 32. (d) | 33. (c) | 34. (a) | 35. (b) | 36. (a) |
| 37. (b) | 38. (d) | 39. (a) | 40. (a) | 41. (b) | 42. (a) |
| 43. (a) | 44. (b) | 45. (d) | 46. (d) | 47. (c) | 48. (d) |
| 49. (c) | 50. (c) | 51. (d) | 52. (a) | 53. (b) | 54. (c) |
| 55. (a) | 56. (b) | 57. (c) | 58. (b) | 59. (b) | 60. (a) |
| 61. (c) | 62. (a) | 63. (d) | 64. (a) | 65. (c) | |

## UNIVERSITY QUESTIONS WITH ANSWERS

1. Thevenin resistance of the circuit given in figure below across its terminals *A* and *B* is 10 ohm. [2004]

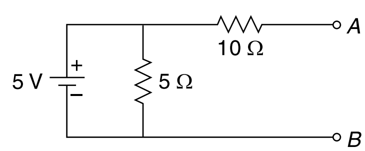



**Fig. 1**

*Answer:* True

Removing the voltage source and short circuiting we find that 5 Ω resistance is shorted out. Hence, Thevenin's resistance of the circuit is 10 Ω.

2. (a) State and explain superposition theorem. Mention its limitations. [2004]

*Answer*: Article 1.11

(b) For the electrical network as shown in fig. 2, find the value of load resistance ($R_L$) for which source will supply maximum power to the load. Find also the maximum power. [2004]

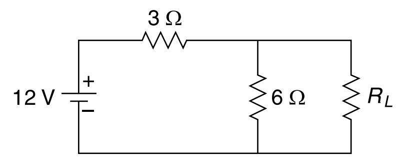



**Fig. 2**

*Answer*:

According to maximum power transfer theorem the power across load is maximum when resistance ($R_L$) is equal to the internal resistance ($R_i$) of the network. Open circuiting the load resistance ($R_L$) and removing the source as shown in Fig. 2(a).

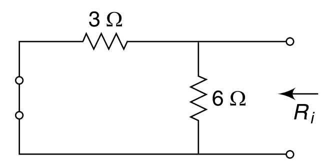



**Fig. 2(a)**

$$R_i = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\ \Omega$$

Hence, $R_L = 2\ \Omega$ for which source will supply maximum power to the load.

Now, when $R_L = 2\ \Omega$, total current supplied by the source

$$(I) = \frac{12}{3 + \frac{6 \times 2}{6+2}} = \frac{12}{3 + \frac{3}{2}} = \frac{24}{9}\ \text{A}$$

Hence, current through $R_L$ is $I_L = \frac{24}{9} = \frac{6}{6+2}$

$$= \frac{24 \times 6}{9 \times 8} = 2\ \text{A}$$

∴ Maximum power across $R_L$ is $I_L^2 R_L = (2)^2 \times 2 = 8$ watts.

3. (a) Determine the current passing through 20 Ω (*BD*) resistor of the network as shown in Fig. 3 with the help of Thevenin's theorem. [2005]

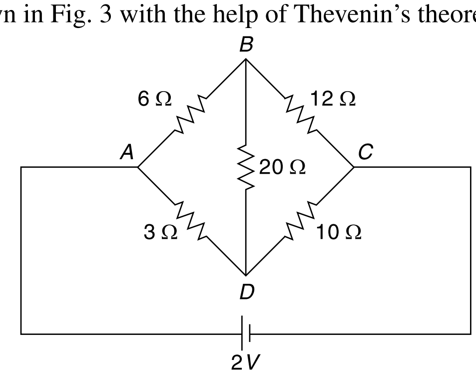



**Fig. 3**

*Answer*: Open circuiting the terminals of 20 Ω resistor the network is redrawn as shown in Fig. 3(a)

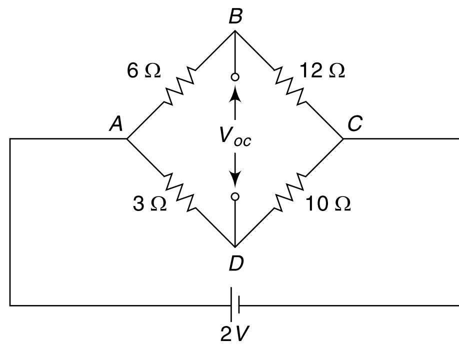



**Fig. 3(a)**

Current through *AB* is $(I_1) = \dfrac{2}{6+12} = \dfrac{1}{9}$ A

Current through *AD* is $(I_2) = \dfrac{2}{3+10} = \dfrac{2}{13}$ A

$\therefore$ Voltage across *AB* is $(V_{AB}) = 6 \times \dfrac{1}{9} = \dfrac{2}{3}$ V

Voltage across *AD* is $(V_{AD}) = 3 \times \dfrac{2}{13} = \dfrac{6}{13}$ V

$\therefore$ Open circuit voltage

$$(V_{oc}) = V_{BD} = V_{AD} - V_{\text{AB}} = \frac{6}{13} - \frac{2}{3}$$

$$= \frac{18-26}{39}$$

$$= -\frac{8}{39}\text{ V}$$

$$\therefore \quad V_{DB} = \frac{8}{39}\text{ V}$$

$\therefore$ Thevenin's equivalent voltage $V_{\text{Th}} = \dfrac{8}{39}$ V

For finding Thevenin's equivalent resistance $R_{\text{Th}}$ the voltage source is removed as shown in Fig. 3(b).

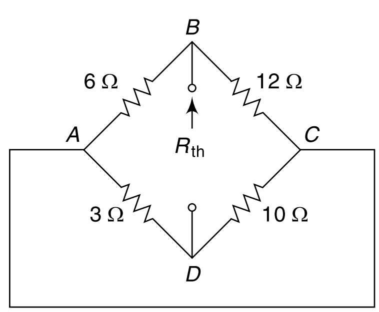



**Fig. 3(b)**

$$R_{Th} = \frac{6 \times 12}{6 + 12} + \frac{3 \times 10}{3 + 10} = \frac{72}{18} + \frac{30}{13}$$

$$= 6.3\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 3(c).

∴ Current through 20 Ω resistor is

$$(I) = \frac{\frac{8}{39}}{6.3 + 20} = 0.007799\text{ A}$$

$$= 7.799\text{ mA}$$

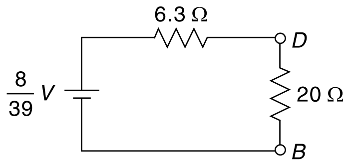



**Fig. 3(c)**

4. Three resistors of 4 Ω, 6 Ω and 8 Ω are in parallel. In which resistor power dissipated will be maximum? [2006, 2013, 2014]
   (i) 4 Ω (ii) 6 Ω
   (iii) 8 Ω (iv) equal in all resistors.
   *Answer*: (i) 4 W.
5. AC voltmeters and ammeters are normally calibrated in [2006]
   (i) average value (ii) instantaneous value
   (iii) peak value (iv) rms value
   *Answer*: (iv) rms value.
6. Two lamps (Bulbs) each rated at 100 W, 230 volts are connected in series across 230 volt, a.c. main. Each of the lamps will glow at [2006]
   (i) 50 watts (ii) 100 watts
   (iii) 200 watts (iv) 25 watts
   *Answer*: (iv) 25 watts.
7. Thevenin's resistance across the terminals *A* and *B* of the circuit shown in Fig. 4 is [2006]

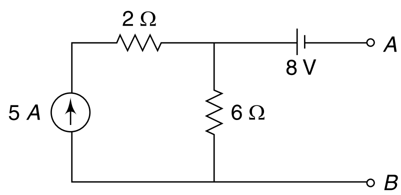



**Fig. 4**

   (i) 1.5 Ω (ii) 6 Ω
   (iii) 2 Ω (iv) 8 Ω
   *Answer*: (ii) 6 Ω.
8. (a) State and explain Superposition theorem [2006]
   *Answer*: Article 1.11

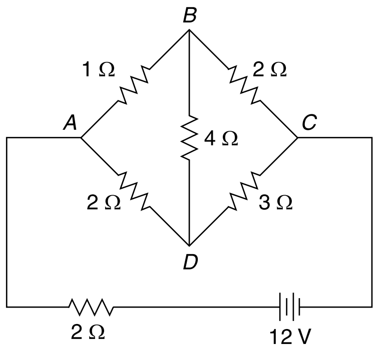



**Fig. 5**

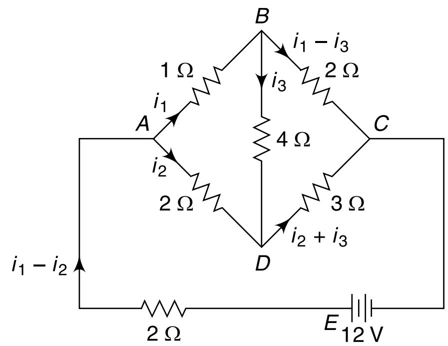



**Fig. 5(a)**

(b) Find the current in each branch of the network shown in Fig. 5 using Kirchhoff's law. [2006, 2012]

*Answer*: Consider the current in the branches *AB*, *AD* and *BD* be $i_1$, $i_2$ and $i_3$ respectively. The circuit is redrawn in Fig. 5(a) showing the branch currents.

Applying Kirchhoff's voltage law in loop *ABDA*

$$1i_1 + 4i_3 - 2i_2 = 0 \quad (1)$$

Applying Kirchhoff's voltage law in loop *BCDB*

$$2\,(i_1 - i_3) - 3\,(i_2 + i_3) - 4i_3 = 0$$

or, $$2i_1 - 3i_2 - 9i_3 = 0 \quad (2)$$

Applying Kirchhoff's voltage law in loop *ADCEA*

$$2i_2 + 3\,(i_2 + i_3) - 12 + 2\,(i_1 + i_2) = 0$$

or, $$2i_1, + 7i_2 + 3i_3 = 12 \quad (3)$$

Solving equations (1), (2) and (3) we get

$i_1 = 1.976$ A, $i_2 = 1.12$ A and $i_3 = 0.065$ A

The current through the branches are given below.

| Branch | Current |
|---|---|
| *AB* | 1.976 A |
| *AD* | 1.12 A |
| *BD* | 0.065 A |
| *BC* | (1.976 – 0.065) = 1.911 A |
| *DC* | (1.12 + 0.065) = 1.185 A |
| *CEA* | (1.976 + 1.12) = 3.096 A |

9. Calculate the current through 3 Ω resistor by superposition Theorem (Fig. 6) [2007]

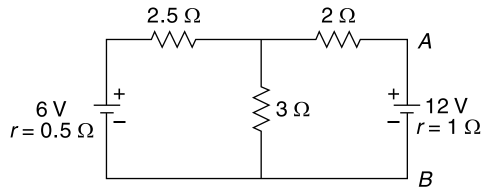



**Fig. 6**

*Answer*: Considering the 6 V voltage source acting alone as shown in Fig. 6(a) the total current supplied by the source $I_1 = \dfrac{6}{0.5 + 2.5 + \dfrac{3 \times 3}{3 + 3}} = 1.33$ A

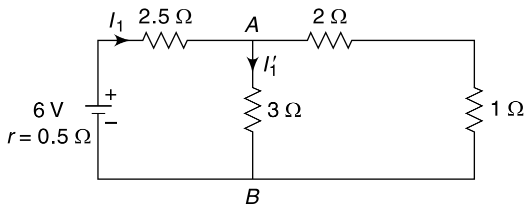



**Fig. 6(a)**

∴ Current through 3 Ω resistor when 6 V source is acting alone $I_1' = I_1 \times \dfrac{3}{3+3}$ $= 1.33 \times \dfrac{1}{2} = 0.67$ A from $A$ to $B$

Considering 12 V source acting alone as shown in Fig. 6(b), the total current supplied by the source.

$$I_2 = \frac{12}{1 + 2 + \dfrac{3 \times 3}{3 + 3}} = 2.67 \text{ A}$$

∴ Current through 3 Ω resistor when 12 V source is acting alone $I_2' = I_2 \times \dfrac{3}{3+3}$ $= 2.67 \times \dfrac{1}{2} = 1.335$ A from $A$ to $B$

Hence according to superposition theorem current through 3 Ω resistor when both the sources are acting simultaneously is $I_1' + I_2' = 0.67 + 1.335 = 2$ A

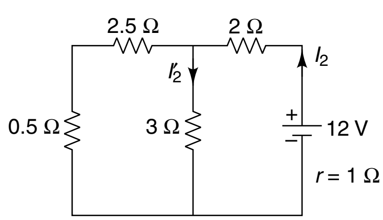



**Fig. 6(b)**

10. State and prove Maximum Power Transfer Theorem for dc networks. Determine current $I_1$ through the 15 ohm resistor in the network (Fig. 7) by Norton's Theorem. [2007]

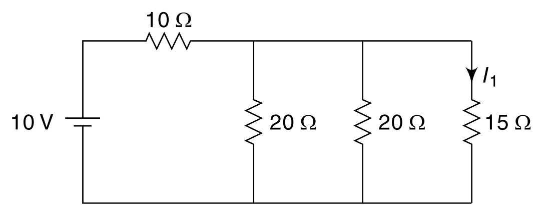



**Fig. 7**

*Answer*: Refer to the statement and proof of Maximum Power Transfer Theorem (Article 1.15)

15 Ω resistor is removed and the terminals are short circuited as shown in Fig. 7(a).

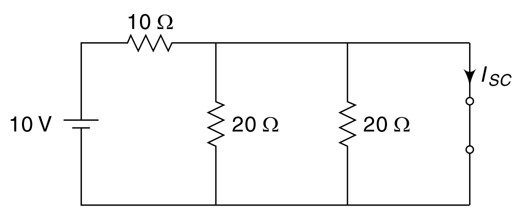



**Fig. 7(a)**

The current through the short circuited path

$$I_{sc} = \frac{10}{10} = 1 \text{ A}$$

$\therefore$ Norton's equivalent current ($I_N$) = 1 A

To find the Norton's equivalent resistance ($R_N$) the source is removed and the load terminals are open circuited as shown in Fig. 7(b).

$$\therefore \qquad R_N = \frac{1}{\frac{1}{10} + \frac{1}{20} + \frac{1}{20}}$$

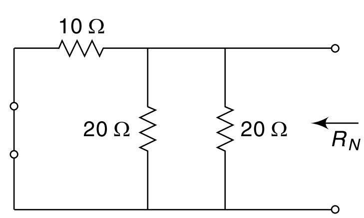



**Fig. 7(b)**

$$= \frac{20}{2 + 1 + 1} = 5 \ \Omega$$

The Norton's equivalent circuit is shown in Fig. 7(c)

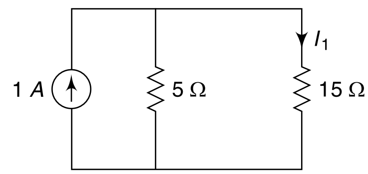



**Fig. 7(c)**

According to Norton's theorem the current through the 15 ohm resistor is $I_1 = 1 \times \frac{5}{5+15} = 0.25$ A.

11. Kirchhoff's voltage law is used for [June 2008]
 (a) loop analysis
 (b) node analysis
 (c) finding out equivalent resistance
 (d) None of these

 *Answer*: (a) loop analysis.

12. For Norton's equivalent circuit [June 2008]
 (a) $I_{sc} = V_{oc}, R_{eq}$ (b) $I_{sc} = \frac{V_{oc}}{R_{eq}}$
 (c) $V_{oc}, I_{sc}/R_q$ (d) None of these.

 *Answer:* This question seems to be incorrect.

13. A network of resistance is formed as given in the figure. Compute the resistance measured between *A* and *B*. [June 2008]

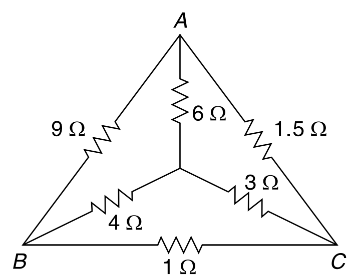



**Fig. 8**

*Answer:* Converting the delta connected network consisting of 6 Ω, 1.5 Ω and 3 Ω resistors into equivalent star network the modified network is shown in Fig. 8(a).

$$R_1 = \frac{6 \times 1.5}{6 + 1.5 + 3} = 0.857\ \Omega$$

$$R_2 = \frac{3 \times 1.5}{6 + 1.5 + 3} = 0.4286\ \Omega$$

$$R_3 = \frac{6 \times 3}{6 + 1.5 + 3} = 1.714\ \Omega$$

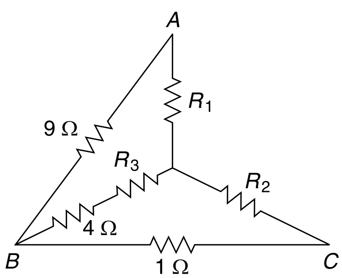



**Fig. 8(a)**

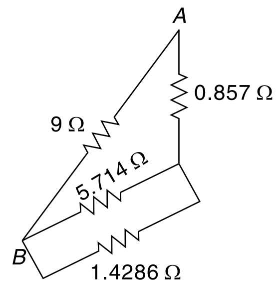



**Fig. 8(b)**

Now reduce the network shown in Fig. 8(a) to that shown in Fig. 8(b)

Now 1.4286 Ω and 5.714 Ω are in parallel and the combination is in series with 0.857 Ω.

∴ Equivalent resistance $R_{eq} = \frac{5.714 \times 1.4286}{5.714 + 1.4286} + 0.857 = 2\ \Omega$

Hence, resistance between $A$ and $B$ is $\frac{9 \times 2}{9 + 2} = 1.636\ \Omega$

14. The relation between flux density $D$ and electric field intensity $E$ is [December 2008]

(a) $D = \in_r \in_o E$ (b) $D = \in_r \in_o / E$

(c) $D = E \in_o / \in_r$ (d) $D = E \in_r / \in_o$

*Answer:* (a) $D = \in_r \delta \in_o E$

15. (a) State and prove maximum power transfer theorem for a dc network. [WBUT 2008, 2011]

*Answer:* Refer Article 1.15.

(b) Determine the current through the 15-Ω resistor in the network given by Norton's theorem.

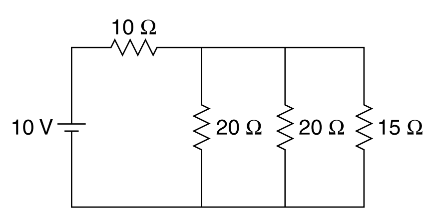



**Fig. 9**

*Answer:* The corresponding circuit after short-circuiting the 15-Ω resistor is shown in Fig. 9(a).

$$\therefore \qquad I_N = \frac{10}{10} = 1\ \text{A}$$

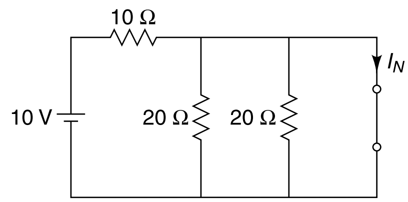



**Fig. 9(a)**

The corresponding circuit after removing the source and open-circuiting the 15-Ω resistor is shown in Fig. 9(b).

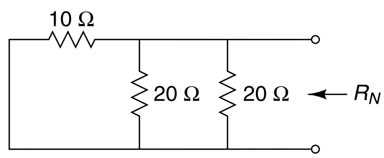



**Fig. 9(b)**

$$R_N = \frac{1}{\frac{1}{10}+\frac{1}{20}+\frac{1}{20}} = \frac{20}{2+1+1} = 5\ \Omega$$

Norton's equivalent circuit is shown in Fig. 9(c).

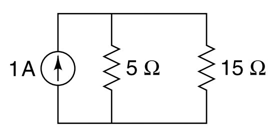



**Fig. 9(c)**

∴ current through the 15-Ω resistor

$$I = 1 \times \frac{5}{5+15} = \frac{5}{20} = 0.25$$

(c) State and explain superposition theorem.
*Answer:* Refer Article 1.11.

16. (a) State and explain Superposition theorem. [June 2009]
*Answer:* Refer Article 1.11.

(b) Find the current through 5 Ω resistor using Thevenin's theorem in figure below:

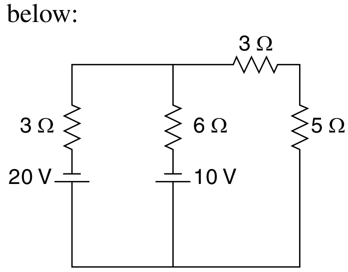



**Fig. 10**

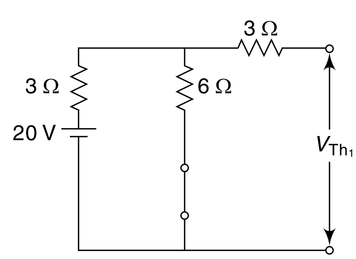



**Fig. 10 (a)**

*Answer:*

(b) Open circuiting the 5 Ω resistor and considering 20 V source acting alone the corresponding figure is shown below

$V_{Th1} = \frac{20}{3+6} \times 6 = \frac{40}{3}$ V

Considering 10 V source acting alone the correspond figure is shown below.

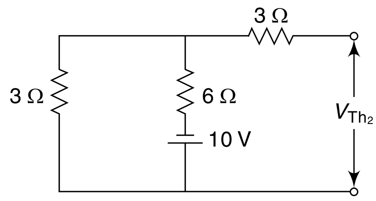



**Fig. 10(b)**

$$V_{Th2} = \frac{-10}{6+3} \times 3 = \frac{-10}{3} = 10 \text{ V}$$

Considering both the sources Thevenin's equivalent voltage

$$V_{Th} = \frac{40}{3} - \frac{40}{3} = \frac{30}{3} = 10 \text{ V}$$

Removing both the source Thevenin's equivalent resistance is found from figure below.

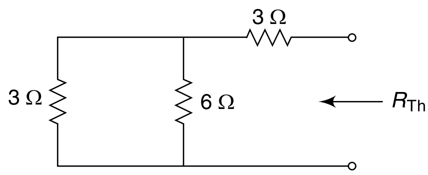



**Fig. 10(c)**

$$R_{Th} = 3 + \frac{6 \times 3}{3+6} = 5 \text{ Ω}$$

Thevenin's equivalent circuit is shown below.

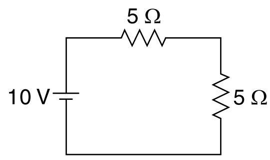



**Fig. 10(d)**

Hence current through 5 Ω resistor $\frac{10}{5+5} = 1$A

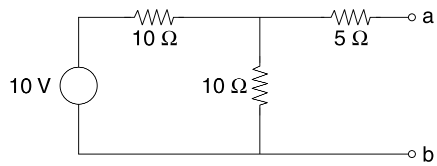
17. Three resistors of 4 Ω, 6 Ω and 8 Ω are connected in parallel. The maximum power dissipation will occur in [WBUT 2011]

(a) 4 Ω (b) 6 Ω

(c) 8 Ω (d) equal in all resistors

*Answer:* (a)

18. For the circuit shown, the Thevenin's voltage and resistance as seen at ab are [WBUT 2011, 2014]




(a) 5 V, 10 Ω (b) 10 V, 10 Ω
(c) 5 V, 5Ω (d) 15 V, 15 Ω.

*Answer:* (a)

19. (a) For the circuit shown below, determine the current $I_1$, $I_2$, $I_3$ using nodal analysis: [WBUT 2011]

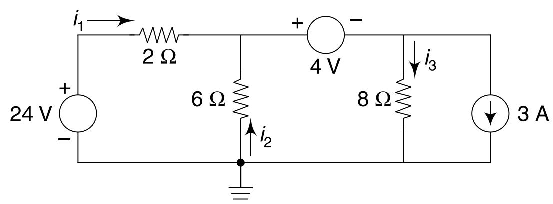



**Fig. 11 (a)**

(b) For the circuit shown below, find the potential difference between *a* and *d*:

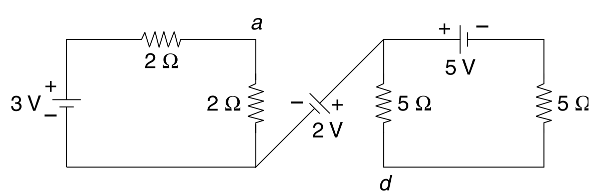



**Fig. 11 (b)**

*Answer:*

(a) Consider the node voltage $V_1$ at the node *A*.

Applying nodal analysis at *A*,

$$\frac{V_1 - 24}{2} + \frac{V_1}{6} + i_3 + 3 = 0$$ ($i_3$ + 3 current is flowing through 4 V source)

$$\therefore \quad 4V_1 + 6\, i_3 = 54$$

Now $$V_1 - 4 = 8\, i_3 \quad \Rightarrow V_1 = 4 + 8\, i_3$$

$$\therefore \quad 4\,(4 + 8\, i_3) + 6\, i_3 = 54$$

$$16 + 32\, i_3 + 6\, i_3 = 54$$

$$38\, i_3 = 38 \Rightarrow i_3 = 1 \text{ A}$$

$$\therefore \qquad i_1 = \frac{24 - V_1}{2} = \frac{24 - 4 - 8 \times 1}{2} = 6 \text{ A}$$

$$i_2 = -\frac{V_1}{6} = -\frac{4 + 8 \times 1}{6} = -2 \text{ A}$$

(b) Current through *ac* is $\frac{3}{2+2}$ A,i.e., 0.75 A from *a* to *c*.

Current through *bd* is $\frac{5}{5+5}$ A,i.e., 0.5 A from *b* to *d*.

Applying KVL in the loop *adbca*,

$$V_{ad} = 5 \times 0.5 - 2 + 2 \times 0.75 = 2 \text{ V}$$

20. Kirchhoffs voltage law is used for [WBUT 2012]
    - (a) loop analysis
    - (b) node analysis
    - (c) finding out equivalent resistance
    - (d) none of theses

    *Answer:* (a) loop analysis

21. Establish the equivalence between Thevenin's and Norton's theorems. [WBUT 2012]

    *Answer:* Refer Article 1.14.

22. Find $V_{AB}$ from the circuit if all the resistances are of same value of 1 ohm. [WBUT 2012]

    *Answer:*

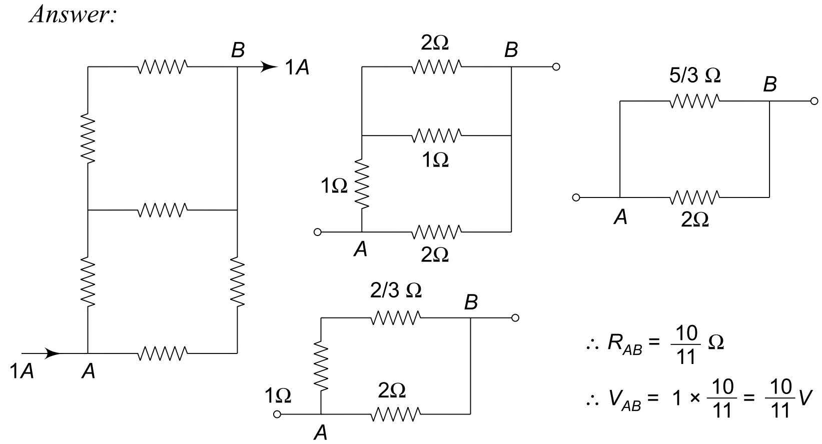



**Fig. 12**

23. Find the value of load resistance (RL) for which the power source will supply maximum power. Also find the value of the maximum power for the network as shown: [WBUT 2012]

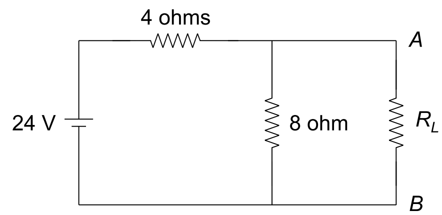



**Fig. 13**

*Answer:* $R_L = R_i = \frac{4 \times 8}{4 + 8} = \frac{8}{3}\,\Omega$

$$I_L = \frac{24}{4 + \frac{8 \times \frac{2}{3}}{8 + \frac{8}{3}}} \times \frac{8}{8 + \frac{8}{3}}$$

$$= \frac{24}{4 + \frac{64}{32}} \times \frac{24}{32} = \frac{24}{192} \times 24 = 3\text{A}$$

$\therefore$ maximum power $= 3^2 \times \frac{8}{3} = 24$ W

24. The galvanometer shown in the circuit has a resistance of 5 ohms. Find the current through the galvanometer using Thevenin's theorem. [WBUT 2012]
*Answer:* Refer in Exercise Question 31.

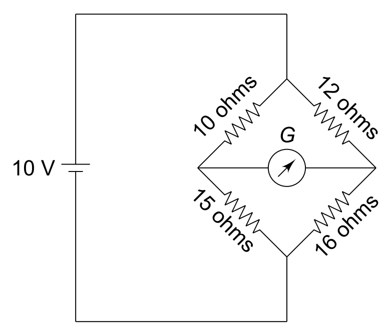



**Fig. 14**

25. Explain (a) star-delta conversion, and (b) delta-star conversion with the help of a purely resistive circuit. [WBUT 2012]
*Answer:* Refer Article 1.9.

26. State and prove maximum power Transfer Theorem. Show that under maximum power transfer condition efficiency is 50%. [WBUT 2013]

    *Answer:* Refer Article 1.15.

27. A Wheatstone bridge consists of $AB = 4\ \Omega$, $BC = 3\ \Omega$, $CD = 6\ \Omega$ and $DA = 5\ \Omega$. A 2.4 V battery is connected between points *B* and *D*. A galvano meter of 8 Ω resistance is connected between *A* and *C*. Using Thevenin's Theorem find the current through the galvano meter. [WBUT 2013]

    *Answer:* Refer Example 1.59.

28. Example Δ-*Y* conversion and *Y*-Δ conversion for a purely resistive circuit. [WBUT 2014]

    *Answer:* Refer Article 1.9.

29. Using Mesh analysis, determine the currents $I_X$ and $I_Y$ in the network as shown in the Fig. 15. [WBUT 2014]

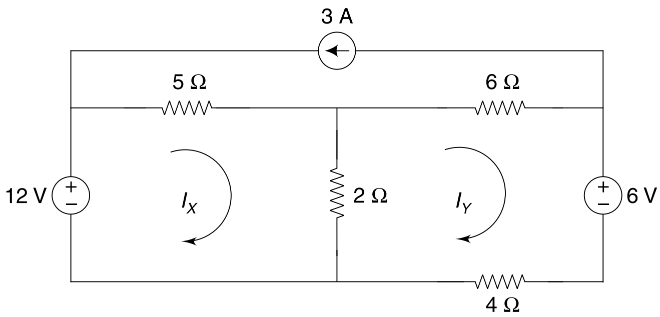



**Fig. 15**

    *Answer:* Refer Example 1.32.

30. A 10 Ω resistor is stretched to increase its length double. Its resistance will now be.

    (a) 40 Ω (b) 20 Ω
    (c) 10 Ω (d) 5 Ω

    *Answer:* (a) 40 Ω

31. A network of resistance is formed as given in the Fig. 16. Compute the resistance measured between *L* and *M*. [WBUT 2014]

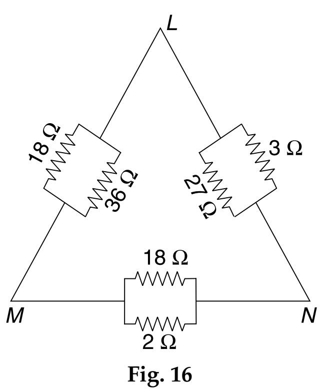



**Fig. 16**

*Answer:* The circuit can be redrawn as Fig. 16a

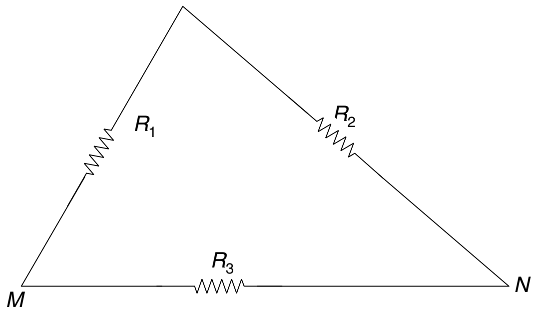



**Fig. 16a**

$$R_1 = \frac{18 \times 36}{18 + 36}\,\Omega = 12\,\Omega$$

$$R_2 = \frac{27 \times 3}{27 + 3}\,\Omega = 2.7\,\Omega$$

$$R_3 = \frac{18 \times 2}{18 + 2}\,\Omega = 1.8\,\Omega$$

Resistance between $L$ and $M = R_1 // (R_2 + R_3)$

$$= 12 // (2.7 + 1.8) = \frac{12 \times 4.5}{12 + 4.5}\,\Omega = 3.27\,\Omega$$

32. Obtain the maximum power transferred to $R_L$ in the circuit and also the value of $R_L$. [WBUT 2014]

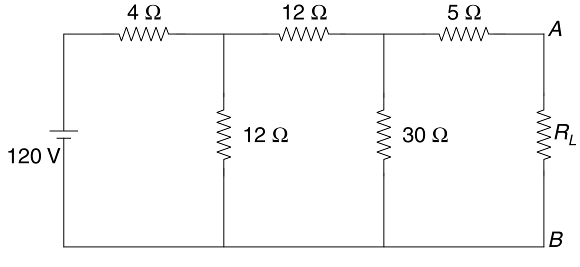



**Fig. 17**

*Answer:* Opencircuiting terminals $AB$ and removing the voltage source, the circuit is shown in the Fig. 17a.

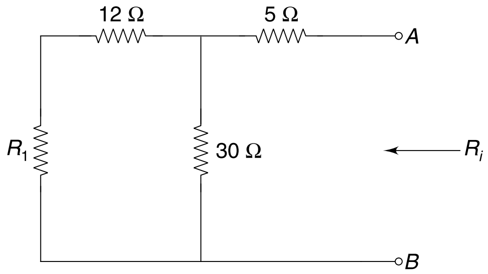



**Fig. 17a**

$$R_1 = \frac{12 \times 4}{12 + 4}\ \Omega = \frac{48}{16}\ \Omega = 3\ \Omega$$

∴ Internal resistance of the network

$$R_i = \left\{5 + \frac{30 \times (12 + 3)}{30 + 12 + 3}\right\}\ \Omega$$

$$= \left\{5 + \frac{30 \times 15}{45}\right\}\ \Omega = 15\ \Omega$$

For maximum power transfer $R_L = R_i = 15$

For obtaining $V_{Th}$ the circuit is shown in Fig. 17b.

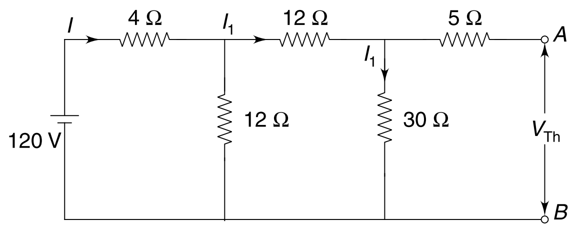



**Fig. 17b**

$$V_{Th} = \text{Voltage across } 30\ \Omega \text{ resistance}$$

$$I = \frac{120}{4 + \dfrac{12 \times (12 + 30)}{12 + 12 + 30}}\ \text{A} = \frac{120}{4 + 9.33}\ \text{A} = 9\ \text{A}$$

∴ $$I_1 = 9 \times \frac{12}{12 + 12 + 30}\ \text{A} = 2\ \text{A}$$

∴ $$V_{Th} = 30 \times 2 = 60\ \text{V}$$

Theremin's equivalent circuit is shown below

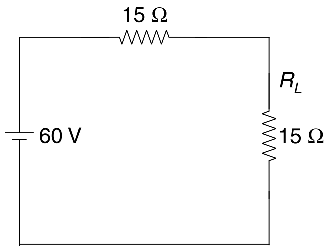



**Fig. 17c**

Hence current through $R_L$ is

$$I_L = \frac{60}{15 + 15} \text{ A} = 2 \text{ A}$$

∴ Maximum power through $R_L$ is

$$P_{max} = 2^2 \times 15 \text{ W} = 60 \text{ W}$$

33. Reduce the network given below to obtain the equivalent resistance as seen between nodes *a* and *b*. [WBUT 2014]

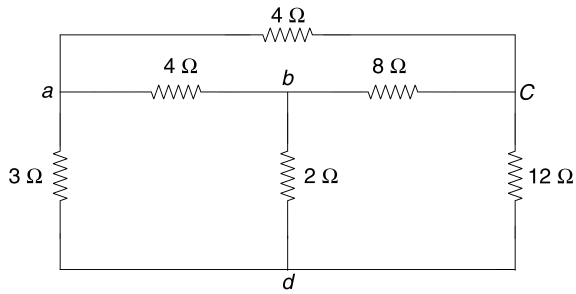



**Fig. 18**

*Answer:*

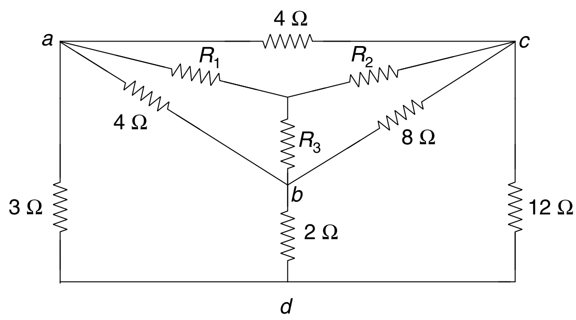



**Fig. 18a**

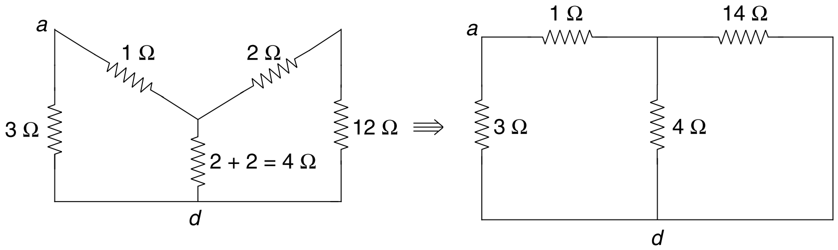



**Fig. 18b**

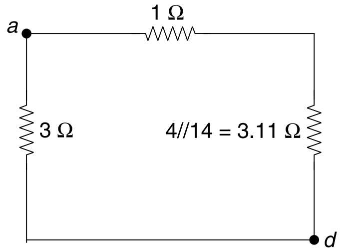



**Fig. 18c**

$$R_1 = \frac{4 \times 4}{4+4+8}\ \Omega = \frac{16}{16}\ \Omega = 1\ \Omega$$

$$R_2 = \frac{4 \times 8}{4+4+8}\ \Omega = \frac{32}{16}\ \Omega = 2\ \Omega$$

$$R_3 = \frac{4 \times 8}{4+4+8}\ \Omega = \frac{32}{16}\ \Omega = 2\ \Omega$$

$\therefore$ Rad $= \{3//(1 + 3.11)\}\ \Omega$

$$= \frac{3 \times 4.11}{3 + 4.11}\ \Omega = 1.73\ \Omega$$

34. Determine the voltage across 3 Ω resistor by applying Thevenin's theorem in the following network [WBUT 2014]

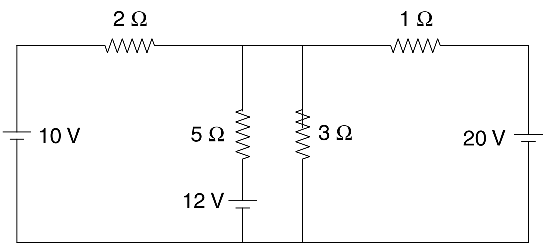



**Fig. 19**

*Answer:* Removing 3 Ω resistor the circuit is as follows:

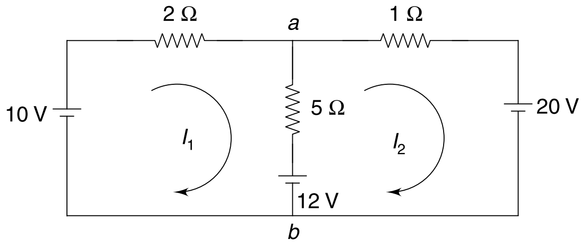



**Fig. 19a**

Using KVL in the two loops

$$10 - 2I_1 - 5(I_1 - I_2) - 12 = 0$$

$$-20 + 12 - 5(I_2 - I_1) - I_2 = 0$$

Solving $I_1 = -3.06\ A$ and $I_2 = -3.88$ A

$$V_{Th} = \text{V}_{ab} = 12 + 5(I_1 - I_2) = 16.1 \text{ V}$$

$R_{Th}$ can be found by removing the sources

$$R_{Th} = 1 // \frac{10}{7} = \frac{1 \times \frac{10}{7}}{1 + \frac{10}{7}}\ \Omega = 0.590$$

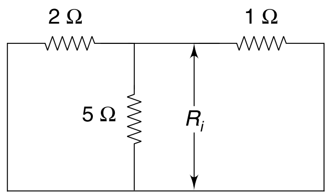



**Fig. 19b**

Thevenin's equivalent circuit is as follows

$$\therefore \qquad I = \frac{16.1}{3 + 0.59} \text{ A} = 4.48 \text{ A}$$

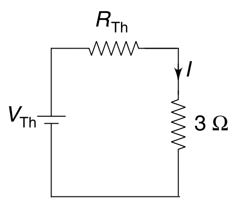



**Fig. 19c**


# ELECTRO-MAGNETISM AND MAGNETIC CIRCUITS

## 2.1 INTRODUCTION

The fascinating properties of magnets have been known since ancient times. The word 'magnet' comes from the ancient Greek city of Magnesia (The modern town Manisa in Western Turkey), where the natural magnets called lodestones were found.

The fundamental nature of magnetism is the interaction of moving electric charges. Unlike electric forces which act on electric charges whether they are moving or not, magnetic forces act only on moving charges and current-carrying conductors.

The power of a magnet by which it attracts certain substances is called *magnetism* and the materials which are attracted by a magnet are called *magnetic materials*.

Few important characteristics of magnets are:

(a) Magnets can exist only in dipole and the pole strength of its two poles is the same.

(b) Magnets always attract iron and its alloys.

(c) A magnetic field is established by a permanent magnet, by an electric current or by other moving charges.

(d) Between magnets, like poles repel and unlike poles attract (Coulomb's first law of magnetism).

(e) A magnetic substance becomes a magnet, when it is placed near a magnet. This phenomenon is known as *magnetic induction*.

(f) Magnetic field, in turn, exerts forces on other moving charges and current carrying conductors.

(g) The two poles of a magnet cannot be isolated (i.e., separated out) and hence magnetic monopole does not exist.

(h) It can be magnetically saturated.

(i) It can be demagnetized by beating, mechanical jerks, heating and with lapse of time.

(j) It produces magnetism in other materials by induction.

(k) On bending a magnet its pole strength remains unchanged but its magnetic moment changes.

(l) The magnetism of materials is mainly due to the spin motion of its electrons.

*Electromagnetism* was discovered by H.C. Oersted in 1820. He found that electric current in a conductor can produce a magnetic field around it. This discovery by Oersted provided the interaction between electricity and magnetism. The strength of this magnetic field can be increased by increasing the current in the conductor. The strength can also be increased by forming the wire into a coil of many turns as only by providing an iron core. The magnetic force has been conventionally assumed to act along a curved line from the N-pole to the S-pole.

### *Properties of Lines of Force*

(a) They are always in the closed curves existing from a N-pole and terminating on a S-pole.

(b) They never cross one another

(c) Parallel lines of forces acting in the same direction repel one another.

(d) They always take the path of least opposition.

(e) They never have an origin nor an end.

### *Coulomb's Law*

The mechanical force produced between two magnetic poles is produced to the product of their pole strengths, and inversely proportional to the square of the distance between them.

$$F \propto \frac{m_1 m_2}{d^2}$$

In SI system, the law is given by

$$F = \frac{m_1 m_2}{\mu_o \cdot \mu_r \cdot d^2} \qquad (2.1)$$

where $F$ is the force between the poles (in Newtons), $m_1$ and $m_2$ are pole strengths, $d$ is the distance between the poles in meters, $\mu_r$ is the relative permeability of the medium in which the poles are situated, and $\mu_o$ is the permeability of free space (in air). $\mu$ = Absolute permeability of air (or vacuum) × relative permeability = $\mu_o \cdot \mu_r$ = $4\pi \times 10^{-7} \times \mu_r$ wb/AT.

### *Gauss's Law of Magnetism*

If there were such a thing as a single magnetic charge (magnetic monopole), the total magnetic flux through a closed surface would be proportional to the total magnetic charge enclosed. But as no magnetic monopole has ever been observed, we can conclude that the total magnetic flux through a closed surface is zero.

$$\int \vec{B} \cdot \vec{ds} = 0 \qquad (2.2)$$

i.e. the total magnetic flux entering a closed surface equals the total magnetic flux leaving. It may be noted here that Gauss's Law of magnetism is applicable to the magnetic flux through a closed surface.

### *Intensity of Magnetism*

When a material is placed in a magnetic field, it acquires magnetic moment $M$. The intensity of magnetization is defined as the magnetic moment per unit volume. It's unit is Ampere/meter. The magnetic susceptibility is defined as the intensity of magnetisation per unit magnetizing field. It has no unit and is dimensionless.

### *Lorentz Force*

If a charge $q$ moves in a region with velocity $v$ where both electric field $\vec{E}$ and a magnetic field $\vec{B}$ are present the resultant force acting on it is

$$\vec{F} = q\,(\vec{E} + v \times \vec{B}) \tag{2.3}$$

This is called the *Lorentz force*.

The phenomenon of magnetizing an unmagnetized substance by the process magnetic induction is called *magnetization*.

The process of protecting any apparatus from the effect of earth's magnetic field is known as *magnetic shielding*.

The phenomenon of decreasing or spoiling magnetic strength of a material is known as *demagnetization*.

The state of a material after which the increase in its magnetic strength stops is known as *magnetic saturation*.

### *Curie Law*

The magnetic susceptibility of paramagnetic substances is inversely proportional to its absolute temperature, i.e. magnetic susceptivity $(x) \propto 1/T$

$$x = \frac{C}{T}$$

where $C$ = Curies constant and $T$ = absolute temperature.

On increasing the temperature, the magnetic susceptibility of paramagnet materials decreases and vice versa. The magnetic susceptibility of ferro-magnetic substances does not change according to Curie law.

### *Curie Temperature*

The temperature above which a ferromagnetic materials behaves like a paramagnetic material is defined as the Curie temperature.

For Ni, $T_c$ = 358°C

For Fe, $T_c$ = 770°C

For Co, $T_c$ = 1120°C

At this temperature the ferro magnetism of the substance suddenly vanishes.

### *Curie–Weiss Law*

At temperatures above the Curie temperature the magnetic susceptibility $(x)$ of ferromagnetic material is inversely proportional to $(T - T_c)$, i.e.

$$x \propto \frac{1}{T - T_C}, \quad x = \frac{C}{T - T_C} \tag{2.4}$$

where $T_c$ = curie-temperature, $C$ = variation constant (Curie constant).

$x - T$ curve is shown in Fig. 2.1.

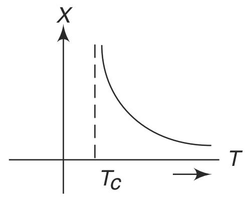
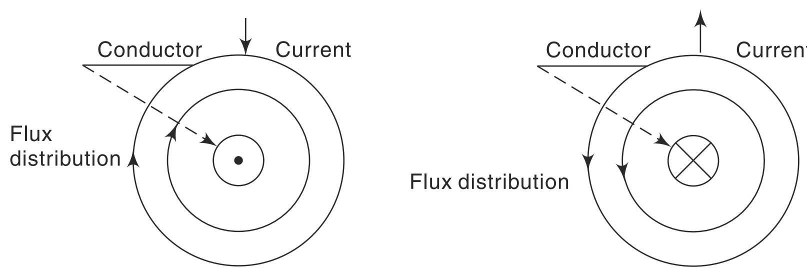



**Fig. 2.1** ***x – T characteristic***

## 2.2 MAGNETIC FIELD AROUND A CURRENT-CARRYING CONDUCTOR

If electric current passes through a conductor, a magnetic field immediately builds up due to the motion of electrons. When a magnetic field is applied to a conductor, the electrons come in motion. This is the converse phenomenon of the previous one.

When a conductor carries current downwards, i.e. away from the observer, the flux distribution is shown in Fig. 2.2(a) when it carried current upwords, i.e. towards observer, flux distribution is shown in Fig. 2.2(b).




(a) Current away from the observer (b) Current towards observer

**Fig. 2.2** ***Magnetic field distribution around a current carrying conductor***

The dot and cross symbol may be thought of a viewing an arrow in the direction of current flow and in the direction away from the current flow. Direction of the field (flux) can be determined by using Ampere's right-hand rule for a conductor, which states that when the conductor is being held by the right hand in such a way that the thumb outstretched parallel to the conductor and pointing the direction of current flow, the closed fingers then give the direction of flux around the conductor. (It can be obtained from right hand *cork screw rule*.)

### 2.2.1 Fleming's Left-hand Rule

This rule is used to determine the direction of force acting on a current carrying conductor placed in a magnetic field. According to this rule, if the middle finger, forefinger and the thumb of the left hand are at right angles to one another and if the

* Remember: Flux is a scalar quantity. It's SI unit is Weber (Wb), CGS unit is Maxwell, 1 Wb = $10^8$ Maxwell.

middle finger and forefinger represent the direction of current and magnetic field respectively, then the thumb will indicate the direction of force acting on the conductor (Fig. 2.3) This rule is used to determine the direction of motion of a conductor (rotor) in the magnetic field produced by the stator for a motor.

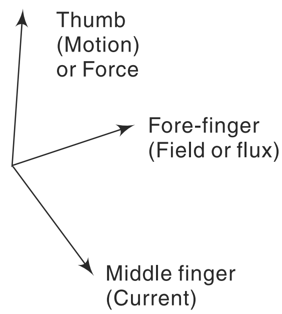



**Fig. 2.3** ***Diagramatic explanation of Fleming's left hand rule***

## 2.3 FORCE ON A CURRENT-CARRYING CONDUCTOR IN A MAGNETIC FIELD

If has been found that when a current carrying conductor is placed in a right angles to the direction of magnetic field, the conductor experiences a mechanical force, which is right angles to both the direction of magnetic field as well as flow of current. This force experienced is directly proportional to:

(a) Current ($I$) flowing through the conductor

(b) Flux density ($B$) and

(c) The length ($l$) of the conductor.

∴ Force acting on conductor,

$$F = BIl \text{ N (Newtons).} \tag{2.5a}$$

Now, say the conductor is not placed at the right angles to the field, but instead placed at an angle ($\theta$), then from natural reasoning,

$$F = BIl \sin \theta \text{ N} \tag{2.5b}$$

The direction of the mechanical force ($F$) is found by Fleming's left hand rule (described in previous section).

## 2.4 FORCE BETWEEN TWO PARALLEL CURRENT-CARRYING CONDUCTORS

Figure 2.4 shows two conductors, kept in parallel, and carrying currents $I_1$ and $I_2$, each has a length of $l$ meters and placed at distance $d$ meters from each other in air. When these two parallel conductors are carrying currents in the same direction [Fig. 2.5a], lines of force in circle each other in the same direction and as a result, resultant field tends to attract the conductor together towards each other. On the other hand, when two parallel conductors are carrying currents in the opposite direction [Fig. 2.5b] then lines of force in the same direction are crowded between the two

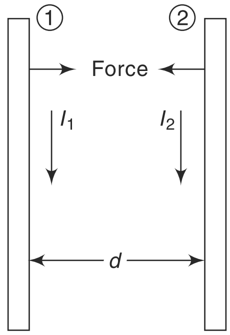



**Fig. 2.4** ***Force between two parallel current-carrying conductors***

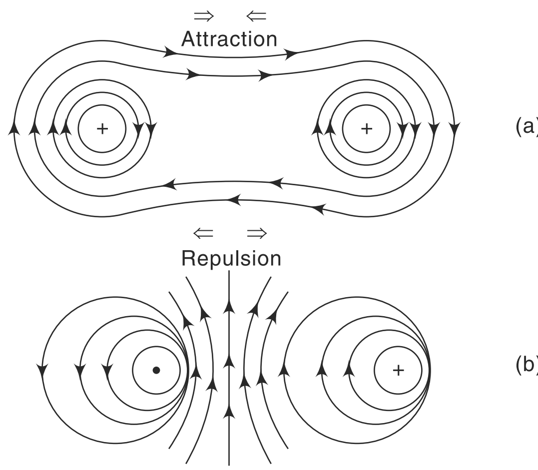



**Fig. 2.5** ***Direction of lines of force between two parallel current carrying conductors***

conductors, and thereby experience a mutual repulsive force. In fact, a mechanical force is developed between these two conductors.

In order to determine the magnitude of force between two parallel current-carrying conductors, one of the two conductors (say conductor 1) is considered placed in a magnetic field due to the other (i.e. due to conductor 2).

Magnetic field due to conductor 2 is given by

$$H = \frac{I_2}{2\pi d} \tag{2.6}$$

∴ Force acting on conductor 1 is $F = BI_1 l$, where $B$ is the *flux density* of the field due to conductor 2.

∴ $$F = \mu_o \mu_r H I_1 l \tag{2.7a}$$

where $$B = \mu_o \mu_r H$$

[$\mu_r$ is the *relative permeability* of the medium in which both the conductors are placed]

Substituting the value of ($H$) in equation (2.7a), we have

$$F = \frac{\mu_o \mu_r I_1 I_2 l}{2\pi d} \cdot \text{N}$$

$$= \frac{\mu_o I_1 I_2 l}{2\pi d} \text{ N (in air } \mu_r = 1\text{]}$$

$$= \frac{4\pi \times 10^{-7} \times I_1 I_2 l}{2\pi d} \text{ N}$$

$$= \frac{2 I_1 I_2 l \times 10^{-7}}{d} \text{ N} \tag{2.7b}$$

In differential form, we can write

$$\frac{dF}{dl} = \frac{2 \times 10^{-7} \times I_1 I_2}{d} \text{ N} \tag{2.8}$$

From the above expression it can be concluded that *the larger the currents carried by the conductor, and less is the distance between the conductors, greater is the force between them.*

**2.1** In uniform field of 1 $Wb/m^2$, a direct current of 70 A is passed through a straight wire of 1.5 m placed perpendicular to the field. Calculate:

(a) The magnitude of the mechanical force produced in the wire.

(b) The prime mover power (i.e. the mechanical power) in watts to displace the conductor against the force at a uniform velocity of 5m/sec.

(c) The emf generated in the current carrying wire.

How do you show that the electrical power produced is same as the mechanical power in creating the motion?

***Solution***

(a) Force $= BIl \sin \theta$

$= 1.0 \times 70 \times 1.5 \sin 90°$ [here $\theta = 90°$]

$= 105$ N

(b) Prime-mover, i.e. mechanical power

$= F \times v$

$= 105 \times 5 = 525$ W

(c) Emf generated $= Blv$

$= 1 \times 1.5 \times 5 = 7.5$ V.

The electrical power $= eI = 7.5 \times 70 = 525$ W. (= mechanical power)

**2.2** Calculate the force developed per meter length between two current-carrying conductors 10 cm apart and carrying 1000 A and 1500 A currents respectively.

***Solution***

Given $I_1 = 1000$ A, $I_2 = 1500$ A, $d = 10$ cm $= 0.10$ m

$$\text{Force per meter length} = \frac{2 \times 10^{-7}\, I_1 I_2}{d}$$

$$= \frac{2 \times 10^{-7} \times 1000 \times 1500}{0.10}$$

$$= 3 \text{ N}$$

**2.3** Two long straight conductors each carrying an electric current of 5.0 A, are kept parallel to each other at a separation of 2.5 cm. Calculate the magnitude of magnetic force experienced by 10 cm of a conductor.

***Solution***

The field at the side of one conductor due to the other is

$$B = \frac{\mu_o \cdot I}{2\pi d} = \frac{2 \times 10^{-7} \times 5}{2.5 \times 10^{-2}} = 4.0 \times 10^{-5}\ T$$

∴ The force experienced by 10 cm of the conductor due to the other is

$$F = IlB$$
$$= 5.0 \times 10 \times 10^{-2} \times 4.0 \times 10^{-5}$$
$$= 2 \times 10^{-5} \text{ N.}$$

**2.4** Two long straight parallel wires situated in air at a distance of 2.5 cm having a current of 100 A in each wire in the same direction. Determine the magnetic force on each wire. Justify whether the force is of the repulsion or attraction type.

***Solution***

Given, $d = 2.5 \text{ cm} = 2.5 \times 10^{-2} \text{ m}$

$I_1 = I_2 = 100 \text{ A.}$

∴ Force on each wire per meter length

$$F = \frac{4 \times 10^{-7} \times I_1 \cdot I_2}{2d} = \frac{2 \times 10^{-7} \times 100 \times 100}{2.5 \times 10^{-2}} = 0.08 \text{ N}$$

Force is attraction type since currents in both the wires are flowing in same direction.

**2.5** Two infinite parallel conductors carrying parallel currents of 50 A each. Find the magnitude and direction of the force between the conductors per meter length if the gap between them is 25 cm.

***Solution***

We know $F = \mu_0 \cdot \dfrac{I_1 I_2 d}{2\pi d}$

$$= \frac{2 \times 10^{-7} \times 50 \times 50 \times 1}{25 \times 10^{-2}} \text{ N}$$
$$= 10^2 \times 10^2 \times 10^{-7} \times 2$$
$$= 2 \times 10^{-3} \text{ N.}$$

The direction of force will depend on whether the two currents are flowing in the same direction or not. For same direction, it will be force of attraction and for opposite direction it will be force of repulsion.

**2.6** A wire of 1 m length long is bend to form a square. The plane of the square is right angled to a uniform field having a flux density of 2 m Wb/mm$^2$. Determine the work done if the wire carries a current of 20 Amps through it, is changed to a circular shape.

***Solution***

Side of the square $= \frac{1}{4} \text{ m} = 0.25 \text{ m}$

$A_1$ = area of square $= (0.25)^2 = 0.0625 \text{ m}^2$

Circumference of a circle is $1 \text{ m} = 2\pi r$

∴ $$r = \frac{1}{2\pi} = 0.15909 \text{ m}$$

∴ $$A_2 = \text{area of the circle} = \pi r^2$$
$$= \frac{22}{7} \times (0.15909)^2$$
$$= 0.0795 \text{ m}^2$$

$\therefore$ Work done = difference between the torques = $BIA_2 - BIA_1$

$= 2 \times 10^{-3} \times 20\,(0.0795 - 0.0625)$

$= 0.00068$ J

## 2.5 BIOT-SAVART'S (OR LAPLACE'S) LAW

If a current carrying conductor is placed at an angle ($\theta$) to the direction of a magnetic field, the effective length being ($l \sin \theta$), the force on the conductor (Fig. 2.6) will be $F = BIl \sin \theta$. (The effective length is the length of the conductor lying within the magnetic field).

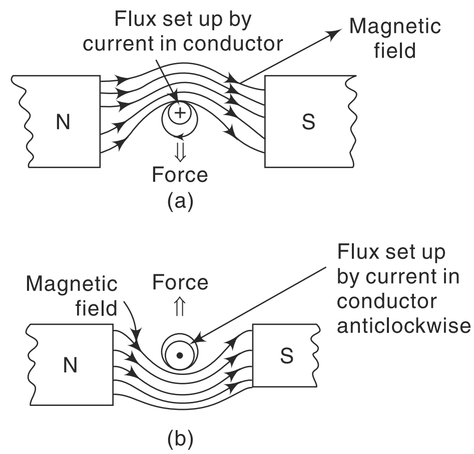



**Fig. 2.6 (a, b)** ***Force on a current-carrying conductor lying in a magnetic field***

In magnetics, there are basically two methods of calculating magnetic field at some point. One is Bio-Savart Law (or Laplace's Law) which gives the magentic field due to an infinitesimally small current-carrying conductor (wire) at some point and another is Ampere's law.

Let $dH$ be the magnetic field at a point $P$ associated with length element $dl$ carrying a steady current of $I$ amperes. Imagine a unit N-pole is present at point $P$ (Fig. 2.6(c). Then flux density $B$ due to the pole strength of 1 wb is given by

$$B = \frac{1}{4\pi r^2} \text{ wb/m}^2$$

$\therefore$ Mechanical force acting at the element ($dl$)

$$dF = BI(dl) \sin \theta = \frac{I \cdot dl \cdot \sin \theta}{4\pi r^2} \text{ N} \qquad (2.9)$$

(Since action and reaction are equal and opposite)

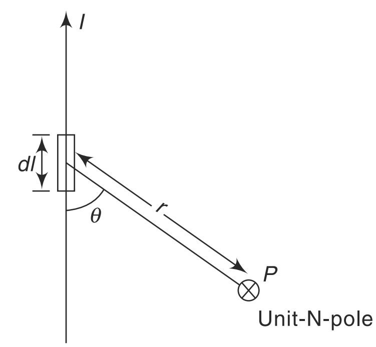



**Fig. 2.6(c)** ***A current carrying conductor in a magnetic field***

Now, as per Newton's third law the magnetic field produced by the current element ($dl$) on the unit pole at point $P$, is given by

$$dH = \frac{I \cdot dl \cdot \sin \theta}{4\pi r^2} \text{ AT/m} \qquad (2.10)$$

with the direction of ($dH$) and perpendicular to both ($dl$) and the unit vector $r$ directed from ($dl$) to $P$.

This equation is known as Biot-Savart's Law (or Laplaces' law).

The following point is worth noting regarding the Biot-Savart law.

The magnitude of $dH$ is given by

$$|dH| = \frac{I \cdot dl \cdot \sin\theta}{4\pi r^2}$$

$|dH|$ is zero at $\theta = 0°$ or $180°$

and maximum at $\theta = 90°$.

This law is employed for calculating the field strength near any system of conductors.

## 2.6 APPLICATION OF BIOT-SAVART LAWS

### 2.6.1 Determination of Magnetic Field Surrounding a Straight Long Conductor of Finite Size

Consider $XY$ (Fig. 2.7) be a straight long finite conductor carrying a current $I$ in the direction for $X$ to $Y$, and $P$ be a point at which the magnetic field $H$ is required. Let us draw a perpendicular from $P$ which meets the conductor at $Q$. Let $PQ = R$.

Let us consider the conductor to be divided into many small current elements and let $(i \cdot d\vec{l})$ be one such (vector) element, where length $EF$ (= $\delta l$). Let $(\vec{r})$ be the position vector from the element to the point $P$. Let $\angle EPQ = \phi$, $\angle FPE = \delta\phi$ and $\angle EFP = \theta$. Since the length $EF$ (= $\delta l$) is very small, $\angle QEP = \theta$. Let $EG$ be a perpendicular from $E$ to $FP$.

By Biot-Savart law, the magnetic field at $P$ due to the current-element $(i \cdot d\vec{l})$ is given by

$$\delta\vec{H} = \mu_0 \cdot \frac{I \cdot \delta l \cdot \sin\theta}{4\pi r^2} = \frac{I\,\delta l \sin\theta}{4\pi r^2} \text{ [in air]}$$

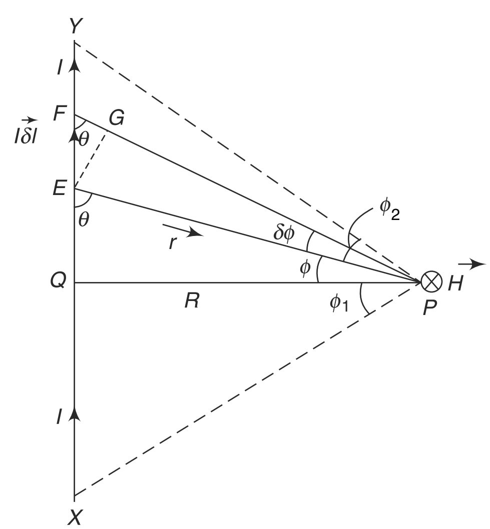



**Fig. 2.7** *Application of Biot-Savart's Law for a straight conductor*

The angle between vectors $(i \cdot \vec{dl})$ and $(\vec{r})$ is $(180 - \theta)$ and hence the magnitude of the field at $P$ is obtained as

$$\delta H = \frac{1}{4\pi} \cdot \frac{i\delta l \sin(180 - \theta)}{r^2} = \frac{1}{4\pi} \frac{i\delta l \sin\theta}{r^2} \tag{2.11}$$

By right-hand screw rule of vector product, the field $(\vec{\delta H})$ at point $P$ is perpendicular to the page directed 'downwards'.

Now from the similar triangles $\Delta EFG$ and $\Delta PEG$, we have

$$EG = EF \sin\theta = \delta l \sin\theta$$

and $$EG = EP \sin\delta\phi = r \sin\delta\phi = r \cdot \delta\phi \qquad (\delta\phi \text{ being very small})$$

$$\therefore \quad \delta l \sin\theta = r \cdot \delta\phi \tag{2.12}$$

Making this substitution in equation (2.11) we have

$$\delta H = \frac{\mu_0}{4\pi} \frac{i \cdot \delta\phi}{r}$$

From right-angled triangle $EQP$, we have

$$r = \frac{R}{\cos\phi}$$

$$\therefore \quad \delta H = \frac{1}{4\pi} \cdot \frac{i \cos\phi\, \delta\phi}{R} \tag{2.13}$$

Let us join $PX$ and $PY$. Let $\angle QPX = \phi_1$ (anticlockwise) and $\angle QPY = \phi_2$ (clockwise). Then the magnitude of the magnetic field $\vec{H}$ at point $P$ due to the whole conductor is

$$H = \int_{-\phi_1}^{\phi_2} \frac{1}{4\pi} \cdot \frac{i}{R} \cos\phi\, d\phi = \frac{1}{4\pi} \cdot \frac{i}{R} [\sin\phi]_{-\phi_1}^{\phi_2}$$

$$= \frac{1 \cdot I}{4\pi R} [\sin\phi_2 - \sin(-\phi_1)]$$

$$= \frac{1 \cdot I}{4\pi R} [\sin\phi_1 + \sin\phi_2] \tag{2.14}$$

For a conductor of infinite length, we have $\phi_1 = \phi_2 = 90°$

$$H = \frac{1}{4\pi} \cdot \frac{I}{R} (\sin 90° + \sin 90°) = \frac{1/2I}{4\pi \cdot R}$$

$$= \frac{I}{2\pi R} \tag{2.15a}$$

For a semi-infinite conductor the field is

$$H = \int_0^{\infty} \frac{1}{4\pi} \cdot \frac{i}{R} \cos\phi \cdot d\phi$$

$$= \frac{I}{4\pi R} \cdot \sin\alpha, \qquad \text{when } \angle XPY = \alpha \tag{2.15b}$$

(i.e., conductor subtends an $\alpha$ at point $P$)

## 2.6.2 Field Strength Due to Circular Loop

Consider a circular coil of length $l$ meters and radius $r$ meters having $N$ turns, and carrying a current of $I$ amperes as shown in Fig. 2.8. Let a unit N-pole be placed on the axis of the coil at $P$ at a distance $x$ meters from the centre of the coil; then force $dH$ experienced on unit N-pole, due to small are length $dl$, is given by (from Biot-Savart's law)

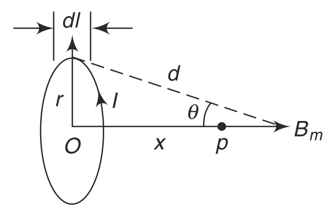



**Fig. 2.8** ***A circular coil***

$$\delta H = \frac{NIdl}{4\pi d^2}$$

Axial component of ($\delta H$) is thus $\left(\frac{NIdl}{4\pi d^2}\sin\theta\right)$ and vertical component of ($\delta H$) is $\left(\frac{NI\delta l}{4\pi d^2}\cos\theta\right)$

$\therefore$ Total force experienced on unit N-pole placed at $P$ due to entire coil is given by

$$H = \frac{NI\sin\theta}{4\pi d^2}\int_0^1 dl = \frac{NI\sin\theta}{4\pi d^2}\cdot l$$

[The vertical component of $\delta H$ is cancelled out by an equal and opposite vertical component due to element $\delta l$ at diametrically opposite side of the coil.]

$$= \frac{NI\sin\theta}{4\pi d^2}\times 2\pi r = \frac{NIr\sin\theta}{2d^2}$$

$$= \frac{NIr^2}{2(r^2+x^2)^{3/2}} \qquad \left[\therefore \frac{r}{\sin\theta} = (r^2+x^2)^{1/2}\right] \qquad (2.16)$$

Now if we want to calculate the field strength at the centre of the loop, $x$ becomes zero.

$H$ (Force experienced on a unit N-pole placed at that point) is given by $= \frac{NI}{2r}$ AT/m.

In the above expression if we want to calculate the field strength far away from the centre of the circular loop, i.e. if $x \quad r$, $r^2 + x^2 \approx x^2$,

then, $$H = \frac{1}{4\pi}\cdot\frac{2M}{x^3}$$

where $M$ = magnetic moment of the loop = $NIA = NI\pi r^2$.

## 2.6.3 Field in a Solenoid

A solenoid is a helical coil and is a very effective method of producing a magnetic field. Figure 2.9 shows a solenoid of length $l$ carrying a steady current of $I$ amperes. Let the number of turns be $N$ and the radius of the coil be $R$. Since the spacing between the turns is very small, the wire (conductor) can be considered as current sheet of width $\left(\frac{l}{N}\right)$ and negligible spacing between turns. Then the current in the solenoid causes a cylindrical current sheet having a linear current density $K$.

i.e., $K = \frac{NI}{l}$ ampere turns/meter.

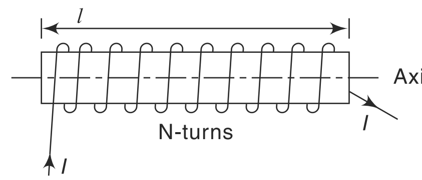



**Fig. 2.9** ***Solenoid of length l***

For an element $(dx)$, shown in Fig. 2.10,

The current density $= Kdx = NI\left(\frac{dx}{l}\right)$

Using the equation

$H = \frac{NIR^2}{2(R^2 + x^2)^{3/2}}$, the flux density $(dB)$ at the centre of the solenoid, due to element $(dx)$ is

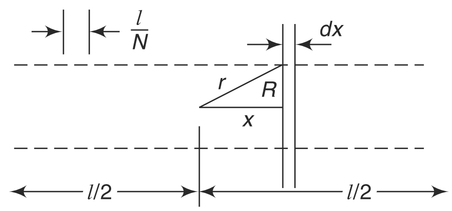



**Fig. 2.10** ***Field in an elemental portion in the solenoid***

$$dB = \frac{\mu NIR^2}{2l(R^2 + x^2)^{3/2}} dx \tag{2.17}$$

The total flux density can be formed by integrating $(dB)$ over the length of the solenoid, i.e. from $x = -\frac{l}{2}$ to $x = +\frac{l}{2}$. Thus

$$B = \frac{\mu NIR^2}{2l} \int_{-l/2}^{l/2} \frac{dx}{(R^2 + x^2)^{3/2}}$$

$$= \frac{\mu NI}{4R^2 + l^2} \tag{2.18}$$

If the length is much larger than the radius, i.e. if $l \quad R$, we have

$$B = \frac{\mu NI}{l} = \mu \cdot K$$

The obtained equations give $B$ at the centre of the solenoid. If the limit of integration is changed from 0 to $l$, we get $B$ at one end of the solenoid. This value is

$$B = \frac{\mu NI}{2(R^2 + l^2)^{1/2}} \tag{2.19}$$

If $l \quad R$, this flux density at one end is

$$B = \frac{\mu NI}{2l} = 0.5\, \mu K \qquad \left[\therefore\ K = \frac{NI}{l}\right]$$

A comparison of the above equations shows that flux density at one end of the coil is half of that at the centre.

**2.7** A circular current-carrying coil has a radius $R$. Show that the distance from the centre of the coil, on the axis, where $B$ will be $\left(\frac{1}{8}\right)$ of its value at the centre of the coil is $\sqrt{3}r$.

***Solution***

$$B_{\text{axis}} = \frac{1}{8} B_{\text{centre}} \quad \text{(given)}$$

$$\therefore \quad \frac{\mu_0 \cdot NIr^2}{2(r^2 + x^2)^{3/2}} = \frac{1}{8}\left(\frac{\mu_0 \cdot NI}{2 \cdot r}\right)$$

or $$8r^3 = (r^2 + x^2)^{3/2}$$

or $$(2r)^3 = \left\{\left(\sqrt{r^2 + x^2}\right)\right\}^3$$

or $$\sqrt{r^2 + x^2} = 2r$$

or $$r^2 + x^2 = 4r^2$$

or $$x = \sqrt{3}\, r.$$

**2.8** A current of 10 A is flowing in a flexible conductor of length 1.5 m. A force of 15 N acts on it when it is placed in a uniform field of 2 $T$. Calculate the angle between the magnetic field and the direction of the current.

***Solution***

We know $$F = BIl \sin \theta$$

$$15 = 2 \times 10\ (1.5) \sin \theta$$

$$\therefore \quad \sin \theta = \frac{15}{30} = \frac{1}{2} = \sin^{-1} (30°)$$

$$\therefore \quad \theta = 30°$$

**2.9** A straight conductor is carrying 2000 A and a point $P$ is situated in such that $\phi_1 = 60°$ and $\phi_2 = 30°$. Calculate the field strength at $P$ if its perpendicular distance from the conductor is 0.2 meter.

***Solution***

$$H = \frac{\mu_o \cdot I}{4\pi r} (\sin \phi_1 + \sin \phi_2)$$

Given $\phi_1 = 60°$, $\phi_2 = 30°$, $r = 0.2$ meter

$I = 2000$ Amps and $\mu_o = 1$ [in air]

$$\therefore \quad H = \frac{2000}{4 \times \pi \times 0.2} [\sin 60° + \sin 30°]$$

$$= 1087.05 \text{ A/m.}$$

**2.10** A solenoid of 400 turns is wound on a continuous ring of iron, the mean diameter of the ring being 10 cm. The relative permeability is 1250. What current is required in order that the flux density ($B$) in the iron shall be 12,000 maxwells/cm$^2$.

***Solution***

Given $l$ = mean circumference of ring = $10 \times \pi$ cm

$= 0.10\ \pi$ meter

and $B$ = 12,000 maxwells/cm$^2$

$= 12{,}000 \times 10^{-8} \times 10^4 = 1.2$ wb/m$^2$

We know $\quad B = \mu_r \cdot \mu_o \dfrac{NI}{l}$

$$\therefore \quad I = \frac{B \cdot l}{\mu_r \, \mu_o \cdot N} = \frac{1.2 \times 0.10\pi}{1.25 \times 10^3 \times 4\pi \times 10^{-7} \times 400}$$

$$= \frac{0.12}{2000 \times 10^{-4}} = \frac{0.12}{2 \times 10^{-1}} = \frac{1.2}{2} = 0.6 \text{ A}$$

**2.11** Prove that the magnetic field due to a current-carrying coil on the axis at a large distance $x$ from the centre of the coil varies approximately as $x^{-3}$.

***Solution***

We know $\quad B_{\text{axis}} = \dfrac{\mu_0 \cdot NIr^2}{2(r^2 + x^2)^{3/2}}$

For $\quad x \quad r$

$$B_{\text{axis}} \cong \frac{\mu_0 \cdot NIr^2}{2 \cdot x^3}$$

$\therefore \quad B_{\text{axis}} \propto \dfrac{1}{x^3}$ and hence proved.

## 2.7 AMPERE'S CIRCUITAL LAW

We know the integral of static (time independent) electric field around a closed path is zero but what about the integral of the magnetic field around a closed path? Actually, the quantity (*Hdl*) does not represent any physical quantity, like work. Although the static magnetic force does no work on a moving charge, we cannot conclude that the path integral of the magnetic field around a closed path is zero.

The line integral $\left(\oint B \cdot dl\right)$ of the resultant magnetic field along a closed plane curve is equal to $\mu_0$ times the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant.

Thus $\quad \oint B \cdot dl = \mu_0 \cdot I$

This is known as *Ampere's circuital law*. The above equation can be simplified as $B \cdot l = \mu_0 \cdot I$

But this equation can be used only under the following condition.

(a) At every point of the closed path $B$ is parallel to $dl$.

(b) Magnetic field has the same magnitude $B$ at all placed on the closed path.

If this is not the case, then the above equation can be written as

$$B_1 dl_1 \cos \theta_1 + B_2 dl_2 \cos \theta_2 + \cdots = \mu_0 \cdot I$$

## 2.8 ELECTROMAGNETIC INDUCTION

Whenever magnetic flux is linked with a circuit changes, an emf is induced in the circuit. If the circuit is closed, a current is also induced in it. The emf and current so produced lasts as long as the flux linked with the circuit changes (in direction and or in magnitude). This phenomenon is called *electromagnetic induction (EMI)*. The magnetic force can be considered as magnetic induction and its SI unit is tesla ($T$).

### 2.8.1 Laws of Electromagnetic Induction

In 1831, Michael Faraday, after performing a number of experiments, summarised a phenomenon of electromagnetic induction, which are stated as follows:

Faraday's first law states that "*whenever magnetic flux linked with a close coil changes, an induced emf is set up in the coil and the induced emf lasts as long as the change in magnetic flux continues.*"

Faraday's second law states that *the magnitude of the induced emf is proportional to the rate of change of magnetic lines of force.*

If $\phi$ is the magnetic flux linked with the current [coil of ($N$) turns] at any instant $t$, then the induced emf's expression in differential form is

$$|e| = \frac{d(N\phi)}{dt} = N \cdot \frac{d\phi}{dt}\ \text{V}.$$

Since the induced emf ($e$) sets up a current in a direction that opposes the very cause of producing magnetic field, so a minus sign is given to the induced emf.

Therefore, $$e = -N\frac{d\phi}{dt}\ \text{V} \qquad (2.20)$$

### 2.8.2 Lenz's Law

Even though Faraday's laws give no idea regarding the direction of induced emf, the direction of induced emf is, however, given by Lenz's law which is based on the law of conservation of energy and it states that "*The direction of the induced current (or emf) is such that it opposes the very cause producing this current (or emf), i.e. it opposes the change in magnetic flux.*

In view of Lenz's law, the induced emf equation takes the form $e = -N \cdot \frac{d\phi}{dt}$.

In 1834, Heinrich Lenz, a German Physicist, enunciated a simple rule, now known as Lenz's law. In fact, this law basically interpreted the law of conservation of energy which can be justified as follows.

When the north pole of a magnet is moved towards the coil, the induced current flows in a direction so as to oppose the motion of the magnet towards the coil. This is only possible when the nearer face of the coil acts as a magnetic north pole which makes an anticlockwise current to flow in the coil. Then the repulsion between the two similar poles opposes the motion of the magnet towards the coil.

Similarly, when the magnet is moved away from the coil, the direction of induced current is such as to make the nearer face of the coil as a south pole which makes a clockwise induced current to flow in the coil. Then the attraction between the opposite poles opposes the motion of the magnet away from the coil. In either case, therefore, work has to be done in moving the magnet. It is this mechanical work which appears as electrical energy in the coil. Hence the production of induced emf or induced current in the coil as in accordance with the law of conservation of energy.

### 2.8.3 Fleming's* Right-Hand Rule

The direction of the induced emf (and hence the induced current) is given by Fleming's Right Hand Rule which states that "stretch the fore finger, middle finger and the thumb of right hand in such a way that all three are mutually perpendicular to each other. If fore finger points in the direction of field, thumb points in the direction of motion of conductor, then middle finger will point along the direction of induced coventional current", as shown in Fig. 2.11.

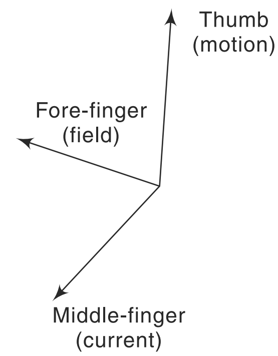



**Fig. 2.11** ***Fleming's right-hand rule.***

## 2.9 CONCEPT OF SELF-INDUCTANCE AND MUTUAL INDUCTANCE

***Self-Inductance***

When a coil carries a current it establishes a magnetic flux (Fig. 2.12). When the current in the coil changes, the magnetic flux linking with the coil also changes. It is observed that this change in the value of current or flux in the coil is opposed by the instantaneous induction of opposing emf. This property of the coil by which it opposes the change in value of current or flux through it due to the production of self induced emf is called *self-inductance*. It is measured in terms of co-efficient of self-inductance $L$. It obeys Faraday's law of electromagnetic induction like any other induced emf.

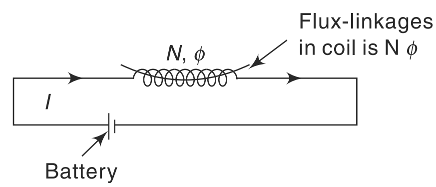



**Fig. 2.12** ***Diagramatic explanation of self-inductance***

For a given coil (provided no magnetic material such as iron is nearby) the magnetic flux linked with it will be proportional to the current, i.e.

$$\phi \propto I \text{ or } \phi = \frac{LI}{N} \qquad \text{i.e., } L = \frac{N\phi}{i} \tag{2.21a}$$

where $L$ is called the *self-inductance* (or simply inductance) of the coil and $N$ is the number of turns of the coil.

The induced emf is given by

$$E = -\frac{d\phi}{dt} = -L\frac{dI}{dt} \tag{2.21b}$$

* John Ambrose Fleming (1849-1945) was professor of Electrical Engineering at University College, London.

The S.I. unit of inductance is henry (symbol H). Henry is a big unit of inductance. Smaller units millihenry (mH) and microhenry ($\mu$H) are used

$$1 \text{ mH} = 10^{-3} \text{ H and } 1 \ \mu\text{H} = 10^{-6} \text{ H}$$

Thus, the self-inductance of a coil is 1 H if an induced emf of 1 volt is set up when the current in the coil changes at the rate of one ampere per second. Also, 1 H = 1 Wb $A^{-1}$. This is also termed as *co-efficient of self-induction* or simply *self-induction*.

The role of self-inductance in an electrical circuit is the same as that of the inertia in mechanical motion. Thus the self-inductance of a coil is a measure of its ability to oppose the change in current through it and hence is also called *electrical inertia*.

***Mutual Inductance***

Whenever a change in current occurs in a coil, an induced emf is set up in the neighbouring coil. This process is called *mutual induction*. The coil in which the emf is induced is called the *secondary coil.* If a current $I_1$ flows in the primary coil, the magnetic flux linked with the secondary coil (Fig. 2.13) will be ($N_2 \ \phi_1/I_1$) for flux $\phi$ linked with the first coil and turns $N_2$ of the second coil.

$$M = \frac{N_2 \ \phi}{I_1} \qquad (2.22a)$$

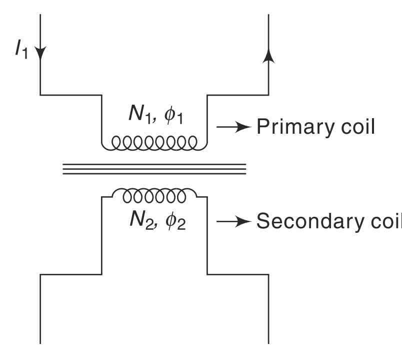



**Fig. 2.13** ***Explanation of mutual inductance***

where $M$ is called the mutual inductance between the two coils or circuits. The emf induced in the secondary coil is given by

$$E_2 = \frac{d\phi_2}{dt} = -M \frac{dI_1}{dt} \qquad (2.22b)$$

Thus the mutual inductance of a pair of circuits is 1 H if a rate of change of current of one ampere per second induces an emf of 1 V in the other circuit.

## 2.10 CONCEPT OF MAGNETIC COUPLING

The coils are said to be *magnetically coupled* if either or part of the magnetic flux produced by one links that of the other i.e., $L_1$ = self-inductance of coil 1 and $L_2$ be the self-inductance of coil 2 and $M$ be the mutual inductance of two coils, Hence, $M = K\sqrt{L_1 L_2}$ where $K$ is co-efficient of coupling. If the total flux produced by coil 1 links with the flux produced by coil 2, then $K = 1$ and $M = \sqrt{L_1 L_2}$.

If the number of turns of coil 1 is $N_1$ and number of turns of coil 2 is $N_2$ then their individual coefficients of self-induction are

$$L_1 = \frac{N_1^2}{l/\mu_0 \ \mu_r A} \quad \text{and} \quad L_2 = \frac{N_2^2}{l/\mu_0 \ \mu_r A}$$

Let the current $I_1$ flowing in coil 1 produces flux $\phi_1$ in coil 1 and $K_1$ fraction of $\phi_1$ lucks coil 2.

$$\therefore \qquad \phi_1 = \frac{N_1 \ I_1}{l/\mu_0 \ \mu_r A} \quad \text{and}$$

*contd...*

On the other hand, if there is no common flux between the two coils, then they are said to be magnetically isolated. Therefore, co-efficient of coupling $K$ between the coils $= \dfrac{\text{'actual' mutual inductance}}{\text{maximum possible value}}$

When the two coils are closely coupled magnetically through an iron core, $K$ is close to unity. On the other hand, when the two coils are loosely coupled magnetically, $K$ is equal to 0.5 or even less. In the magnetically isolated case, $K = 0$, i.e. $M = 0$.

**2.12** A magnetic flux of 400 $\mu$ Wb passing through a coil of 1500 turns is reversed in 0.1 s. Determine the average value of the emf induced in the coil.

***Solution***

The magnetic flux has to decrease from 400 $\mu$ Wb to zero and then increase to 400 $\mu$ Wb in the reverse direction, hence the increase of flux in the original direction is 800 $\mu$ Wb.

We know, average emf induced in the coil is

$$e = \frac{N\phi}{t}$$

$$= \frac{1500 \times (800 \times 10^{-6})}{0.1}$$

$$= 12 \text{ V.}$$

**2.13** A coil has a self-inductance of 40 milli-henry. Determine the emf in the coil when the current in the coil

(a) increase at the rate of 300 A

(b) raises from 0 to 10 A in 0.05 sec.

***Solution***

Given (a) Self-inductance $L = 40 \times 10^{-3}$ H

$$E = L \cdot \frac{dI}{dt} \quad \text{(only magnitude)}$$

$$= 40 \times 10^{-3} \times 300 = 12 \text{ V}$$

(b) $L = 40 \times 10^{-3}$ H

$dI = 10 - 0 = 10$ A

*contd...*

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Mutual inductance $M = \dfrac{K_1\, \phi_1\, N_2}{I_1}$ where $K_1 \le 1$

or, $$M = \frac{K_1\, I_1\, N_1\, N_2}{I_1\, l / \mu_0\, \mu_r A} = \frac{K_1\, N_1\, N_2}{l / \mu_0\, \mu_r A}$$

Similarly, if current $I_2$ in coil 2 produces flux $\phi_2$ then $\phi_2 = \dfrac{N_2\, I_2}{l / \mu_0\, \mu_r A}$ and if $K_2$ fraction of $\phi_2$ links coil 1

then $M = \dfrac{K_2\, \phi_2\, N_1}{I_2} = K_2 \dfrac{N_1\, N_2\, I_2}{I_2\, l / \mu_0\, \mu_r A} = \dfrac{K_2\, N_1\, N_2}{1 / \mu_0\, \mu_r A}$

$\therefore M^2 = K_1\, K_2 \dfrac{N_1^2}{1 / \mu_0\, \mu_r A} \dfrac{N_2^2}{l / \mu_0\, \mu_r A} = K_1\, K_2\, L_1\, L_2$

or $M = K\sqrt{L_1\, L_2}$ where $K = \sqrt{K_1\, K_2}$

$dt = 0.05$ sec

$$\therefore \quad e = L\frac{di}{dt}$$

$$= \frac{40 \times 10^{-3} \times 10}{0.05} = 8 \text{ V}$$

**2.14** Determine the emf induced in a coil of $4.19 \times 10^{-4}$ Henry when a current of 5 A is reversed in 60 milliseconds.

***Solution***

Given $L = 4.19 \times 10^{-4}$ H

Also given $dI = 5 - (-5) = 10$ A

$dt = 60$ milliseconds $= 60 \times 10^{-3}$ sec.

$$\therefore \quad \text{emf induced} = L\frac{di}{dt} \text{ (only magnitude)}$$

$$= \frac{4.19 \times 10^{-4}}{60 \times 10^{-3}} \times 10$$

$$= \frac{4.19}{60}$$

$$= 0.0698 \text{ V.}$$

**2.15** Two identical coils $X$ and $Y$ each having 1000 turns lie in parallel planes such that 60% of the flux produced by one coil links with the other. A current of 10 A in coil $X$ produces in it a flux of $10^{-4}$ wb. If the current in the coil $X$ changes from +15 A to –15 A in 0.03 seconds, what would be the magnitude of the emf induced in the coil $Y$?

***Solution***

Given $N_1 = N_2 = 1000$ turns

$I_x = 10$ A

$\phi_x = 10^{-4}$ Wb.

The amount of flux linking with second coil $= 0.6 \times 10^{-4}$ Wb.

$$\therefore \quad \frac{dI_x}{dt} = \frac{[15 - (-15)]}{0.03} = \frac{30}{0.03} = 1000 \text{ A/sec.}$$

We have

$$e_{My} = M \cdot \frac{dI_1}{dt} \text{ volts (ignoring –ve sign)}$$

where

$$M = \frac{N_2 \text{ (Amount of } \phi_1 \text{ linking with the coil } Y)}{I_1}$$

$$= \frac{1000 \times (0.6 \times 10^{-4})}{10}$$

$$= 0.6 \times 10^{-2} \text{ H}$$

$$\therefore \quad e_{My} = 0.6 \times 10^{-2} \times 1000$$

$$= 6 \text{ volts.}$$

**2.16** A square coil of 10 cm side and with 120 turns is rotated at a uniform speed of 1000 rpm about an axis at right angles to a uniform magnetic field having a flux density of 0.5 Wb/m$^2$. Determine the instantaneous value of the electromotive force if the plane of the coil is

(a) at right angles to the field
(b) at 30° to the field
(c) in the plane of the field.

***Solution***

We know

$$\phi = B.A = 0.5 \times (10 \times 10^{-2})^2 = 5 \text{ milli-Wb}$$

$$\psi = \phi \cdot N = 5 \times 10^{-3} \times 120 = 0.6$$

$$\omega = \frac{2\pi n}{60} = \frac{2 \times \pi \times 1000}{60} = 104.719 \text{ radian/sec}$$

$\therefore$ $$\psi = \psi_m \cos \omega t = 0.6 \cos 104.719t$$

$\therefore$ $$e = -\frac{d\psi}{dt} = +\, 0.6 \times 104.719 \sin 104.719t$$

$$= 62.83 \sin 104.719t \text{ volt.}$$

(a) When $\theta = 0°$

$$e = 62.83 \sin 0° = 0 \text{ V}$$

(b) When $\theta = 90° - 30° = 60°$

$$e = 62.83 \sin 60° = 62.83 \times \frac{\sqrt{3}}{2} = 54.412 \text{ V.}$$

(c) When $\theta = 90°$

$$e = 62.83 \text{ V.}$$

## 2.11 CALCULATION OF SELF-INDUCTANCE

### 2.11.1 For a Circular Coil

Consider a circular coil of radius $r$ and number of turns $N$. If current $I$ passes in the coil, then magnitude field at centre of coil

$$B = \frac{\mu_0 Nl}{2r}$$

the effective magnetic flux linked with this coil,

$$\phi = NBA = \frac{N(\mu_0 Nl)A}{2r}.$$

Since, by definition, $L = \dfrac{\phi}{I}$

$\therefore$ $$L = \frac{\mu_0 N^2 A}{2r} = \frac{\mu_0 N^2 \pi r^2}{2r} \quad [A = \pi r^2 \text{ for a circular coil}]$$

or, $$L = \frac{\mu_0 N^2 \pi r}{2r} \qquad (2.23)$$

### 2.11.2 For a Solenoid

Consider a solenoid with $n$ number of turns per meter. Let current $I$ flow in the windings of solenoid, then the magnetic field inside solenoid is given by

$$B = \mu_0 n I$$

the magnetic flux linked with its length $l$ is $\phi = NBA$, where $N$ is the total number of turns in length $l$ of solenoid.

$$\phi = (nl)BA = (nl)\,(\mu_0 \cdot n \cdot I)\,A \qquad (\because N = nl)$$

Since, $L = \dfrac{\phi}{I}$

$$\therefore \qquad L = \mu_0\, n^2\, A\, I \qquad (2.24)$$

Since $n = \dfrac{N}{l}$

$$\therefore \text{ Self-inductance, } L = \frac{\mu_0\, N^2 A}{l} \qquad (2.25)$$

## 2.12 ENERGY STORED IN AN INDUCTOR

### 2.12.1 Expression for Energy Stored

When the current in a circuit of a coil of inductance $L$ henry increases from zero to its maximum steady value of $I$ amperes, work has to be done against the opposing induced emf.

Let $dw$ be the infinitesimal work done in time $dt$, then

$dw = VI\,dt$, where $V$ is voltage across an inductor.

Since $V = L\dfrac{di}{dt}$

$$\therefore \qquad dw = \left(L\frac{di}{dt}\right) \cdot I \cdot dt = LI\,dI$$

$$\text{or} \qquad w = \int_0^1 LI\,dI = \frac{1}{2}LI^2 \text{ J.} \qquad (2.26)$$

This work done is stored in the form of energy of the magnetic field in an inductor.

Also we can write $L = \dfrac{2w}{I^2}$ if $I = 1$ Amp, $L = 2w$, using equation (2.26)

Thus the self-inductance of a circuit is numerically equal to twice the work done against the inductance emf in establishing a circuit of 1 A in the coil. Again, from the definition of self-inductance (Ref. equation number 2.25)

$$L = \frac{\mu_0\, \mu_r\, A \cdot N^2}{l}$$

$$\text{and stored energy} = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 \cdot \mu_r \cdot A \cdot N^2 \cdot I^2}{l} \text{ J}$$

But magnetic field intensity, $H = \dfrac{NI}{l}$

$$\therefore \text{ Stored energy} = \frac{1}{2}\mu_0 \cdot \mu_r \cdot A.l \cdot \frac{N^2 I^2}{l^2} = \frac{1}{2}\mu_o\, \mu_r\, A.l \cdot H^2 \text{ J}$$

Now, $Al$ = volume of the magnetic field in $m^3$

$$\text{Energy stored/m}^3 = \frac{1}{2}\mu_o \cdot \mu_r \cdot H^2 \text{ J}$$

$$= \frac{1}{2} BH\,J = \frac{1}{2} \cdot B \cdot \frac{B}{\mu_0 \cdot \mu_r}\,\text{J} \qquad [\;B = \mu_o\,\mu_r\,H]$$

$$= \frac{B^2}{2\mu_0} \qquad [\;\mu_r = 1 \text{ in air}]. \tag{2.26a}$$

### 2.12.2 Expression of Magnetic Energy Density ($U_M$)

Since, $W = \frac{1}{2} LI^2$, for a solenoid

$L = \mu_o n^2 Al$ and $B = \mu_o nI$

$$\therefore \quad W = \frac{1}{2} (\mu_o \cdot n^2 Al) \left(\frac{B}{\mu_o\, n}\right)^2$$

$$= \frac{B^2}{2\mu_o} (Al) \tag{2.27a}$$

Again we know $\frac{W}{Al} = \frac{\text{Energy}}{\text{Volume}}$ = Magnetic energy density (say $U_M$)

$$\therefore \quad U_M = \frac{B^2}{2\mu_o}\,\text{J}. \tag{2.27b}$$

Thus we can conclude that the energy stored in magnetic field in terms of per unit volume of the magnetic material used is called the *magnetic energy density*.

## 2.13 COMBINATION OF INDUCTANCES

### 2.13.1 Dot Covention

The emf induced due to mutual inductance may either aid or oppose the emf induced due to self-inductance in a magnetic coupling circuit. It depends on the relative

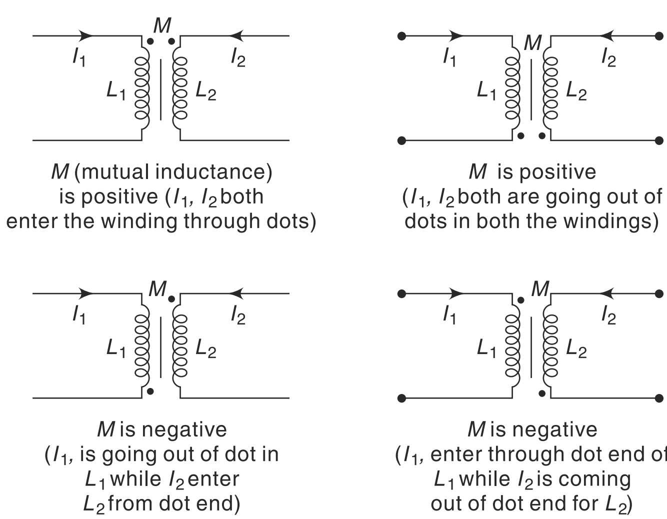



**Fig. 2.14** ***Dot convention to determine sign of M (A dot represents the +ve polarity of the winding at any instant).***

direction of currents, the relative modes of windings of the coils as well as the physical location, i.e. either far away or very close with respect to the other. Figure 2.14 clearly explains the sign of mutually induced emf.

### 2.13.2 Inductances in Series and Parallel

***Series Connection***

When two inductors are coupled in series, a mutual inductance exists between them.

Figure 2.15(a) shows the connection of two inductive coils in series aiding. The flux produced by the two coils are additive in nature as per dot convention. Let $L_{12}$ be is the self-inductance of the coil 1 and $L_2$ is the self-inductance of coil 2 and $M$ is the mutual inductance between coil 1 and coil 2.

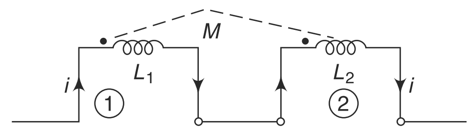


**Fig. 2.15(a)** ***Inductance in series (Cumulative coupling) (flux-aiding)***

For coil 1, the self-induced emf $e_1 = -L_1 \dfrac{di}{dt}$ and the mutual induced emf = $-M\dfrac{di}{dt}$ due to change of current in coil 2.

For coil 2, self-induced emf $e_2 = -L_2 \dfrac{di}{dt}$ and mutually induced emf = $-M\dfrac{di}{dt}$ due to change of current in coil 1.

Therefore the total induced emf of the above connection can be written as

$$e = -L_1 \frac{di}{dt} - L_2 \frac{di}{dt} - 2M\frac{di}{dt}$$

$$= -(L_1 + L_2 + 2M)\frac{di}{dt}. \qquad (2.28)$$

If ($L_s$) is the equivalent inductance of the coil in series then it can be expressed as

$$e = -L_s \frac{di}{dt} \qquad (2.29)$$

Comparing the equations (2.28 and 2.29), we have

$$\therefore \quad -L_s \frac{di}{dt} = -(L_1 + L_2 + 2M)\frac{di}{dt}$$

$$\therefore \quad L_s = L_1 + L_2 + 2M \qquad \left[\therefore \quad \frac{di}{dt} \neq 0\right]. \qquad (2.30a)$$

Similarly, for the series opposing, from the Fig. 2.15(b) we can write

$$L_s = L_1 + L_2 - 2M \qquad (2.30b)$$

If the two coils of self-inductances $L_1$ and $L_2$ having mutual inductance $M$ are in series and are far away from each other, so that the mutual inductance between them is negligible, then the net self-inductance is

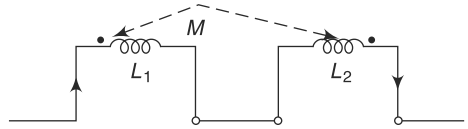


**Fig. 2.15(b)** ***Inductance in series (flux opposing) (Differential coupling)***

$$L_s = L_1 + L_2 \quad [\ M = 0].$$

### *Parallel Connection*

When two coils are coupled in parallel and mutual inductance $M$ exists, the equivalent inductance can be calculated as follows:

**For Parallel Aiding** The Fig. 2.16 shows the connection of parallel connection of two coils where the flux is additive as per dot convention. Using Kirchoff's voltage law, we have

$$V = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad (2.31a)$$

also,

$$V = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \qquad (2.31b)$$

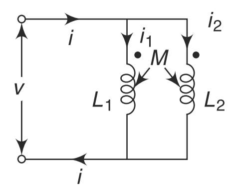



**Fig. 2.16** ***Inductances in parallel flux aiding***

From the above two equations, we can write

$$L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Here $i = i_1 + i_2$ $\therefore\ i_2 = i - i_1.$

Thus,

$$L_1 \frac{di_1}{dt} + M \frac{d}{dt}(i - i_1) = L_2 \frac{d}{dt}(i - i_1) + M \frac{di_1}{dt}$$

or

$$L_1 \frac{di_1}{dt} + M \frac{di}{dt} - M \frac{di_1}{dt} = L_2 \frac{di}{dt} - L_2 \frac{di_1}{dt} + M \frac{di_1}{dt}$$

i.e.

$$(L_1 + L_2 - 2M) \frac{di_1}{dt} = (L_2 - M) \frac{di}{dt}$$

$$\therefore \quad \frac{di_1}{dt} = \frac{L_2 - M}{L_1 + L_2 - 2M} \cdot \frac{di}{dt}. \qquad (2.32a)$$

Similarly

$$\frac{di_2}{dt} = \frac{L_1 - M}{L_1 + L_2 - 2M} \cdot \frac{di}{dt}. \qquad (2.32b)$$

Using equations (2.32a) and (2.32b), we can write,

$$V = L_1 \left( \frac{L_2 - M}{L_1 + L_2 - 2M} \right) \frac{di}{dt} + M \left( \frac{L_1 - M}{L_1 + L_2 - 2M} \right) \frac{di}{dt}$$

$$= \frac{L_1 L_2 - L_1 M + L_1 M - M^2}{L_1 + L_2 - 2M} \cdot \frac{di}{dt}$$

$$= \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \frac{di}{dt}. \qquad (2.32c)$$

If $L_P$ be the equivalent inductance of the parallel combination, it can be written as

$$V = L_P \frac{di}{dt} \qquad (2.32d)$$

Comparing equation (2.32c) and (2.32d) we have

$$L_P \frac{di}{dt} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \left( \frac{di}{dt} \right)$$

$$\therefore \qquad L_P = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} \qquad (2.33a)$$

Equation (2.33a) gives the required equivalent inductance for parallel connections when inductances are in the flux aiding mode.

Similarly, for inductances in parallel opposing, we can write

$$L_P = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} \qquad (2.33b)$$

This is the required equivalent inductance in parallel for the flux opposing mode.

Again, if the two coils are far away from each otheR, $M \cong 0$ and hence

$$L_P = \frac{L_1L_2}{L_1 + L_2} \qquad (2.33c)$$

Therefore by combining parallel aiding and parallel oppositing, the final expression for the *equivalent inductance* is

$$L_P = \frac{L_1L_2 - M^2}{L_1 + L_2 \pm 2M} \qquad (2.33d)$$

**2.17** Two coils are connected in parallel as shown in Fig. 2.17. Calculate the net inductance of the connection.

***Solution***

The net inductance in the given circuit

$$= \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

$$= \frac{0.2 \times 0.3 - (0.1)^2}{0.2 + 0.3 + 2 \times 0.1}$$

$$= \frac{0.06 - .01}{0.7} = \frac{0.05}{0.7} = 0.0714 \text{ H.}$$

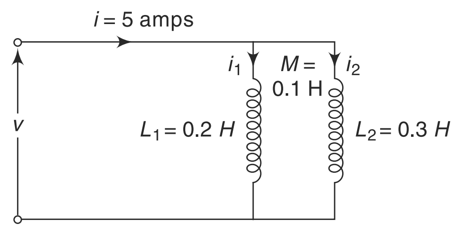



**Fig. 2.17**

**2.18** The combined inductance of the two coils connected in series is 0.60 H and 0.40 H, depending on the relative directions of currents in the coils. If one of the coils, when isolated, has a self-inductance of 0.15 H, then find: (a) the mutual inductance, and (b) the co-efficient of coupling $K$.

***Solution***

$$L_{\text{additive}} = L_1 + L_2 + 2M$$

$$0.60 = 0.15 + L_2 + 2M \qquad \text{(i)}$$

$$L_{\text{subtractive}} = L_1 + L_2 - 2M$$

$$0.40 = 0.15 + L_2 - 2M \qquad \text{(ii)}$$

adding equations (i) and (ii), we have

$$1.0 = 0.\ 3 + 2L_2.$$

$$\therefore \qquad L_2 = \frac{(1.0 - 0.3)}{2} = 0.35 \text{ H}$$

Substituting this value of $L_2$ in equation (i),

$$0.60 = 0.15 + 0.35 + 2M$$

or $\qquad M = 0.05$ H

(b) Co-efficient of coupling, $K = \dfrac{M}{\sqrt{L_1 L_2}}$

$$= \frac{0.05}{\sqrt{0.15 \times 0.35}} = 0.218 = 0.22.$$

**2.19** Pure inductors each of inductance 3 H are connected as shown in Fig. 2.18. Find the equivalent inductance of the circuit.

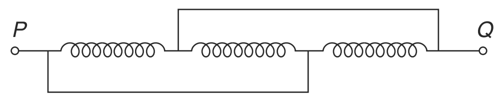



**Fig. 2.18** ***The equivalent inductance of the circuit***

***Solution***

Since all three are in parallel. Hence the equivalent inductance is $L/3 = 3/3 = 1$ H.

**2.20** A current of 10 A when flowing through a coil of 2000 turns establishes a flux of 0.6 milliwebers. Calculate the inductance $L$ of the coil.

***Solution***

Given $I = 10$ A

$N = 2000$ turns

$\phi = 0.6 \times 10^{-3}$ wb

$L$ = to be calculated.

We have $L = \dfrac{N\phi}{I} = \dfrac{2000 \times 0.6 \times 10^{-3}}{10}$

$= 0.12$ H.

**2.21** Determine the inductance $L$ of a coil of 500 turns wound on an air cored torroidal ring having a mean diameter of 300 mm. The ring has a circular cross section of diameter 50 mm.

***Solution***

Given $N = 500$ turns

Mean diameter, $D = 300 \text{ mm} = 300 \times 10^{-3}$ m

$l = \pi D = \pi \times 300 \times 10^{-3}$ m

$= 0.942$ m

Cross-sectional diameter $d = 50$ mm

$= 50 \times 10^{-3}$ m

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (50 \times 10^{-3})^2}{4}$$

$= 1.963 \times 10^{-3}$ m$^2$.

For air cored torroidal ring, $\mu_r = 1$ and $L$ = is to be calculated.

We have, inductance $L = \dfrac{N^2}{\text{Reluctance}}$

$$\left[\because \quad L = \frac{\mu_o \mu_r AN^2}{l} = \frac{N^2}{l/\mu_o\mu_r A} = \frac{N^2}{\text{Reluctance}}\right.$$

where $\quad \text{Reluctance} = \left.\frac{l}{\mu_o \cdot \mu_r \cdot A}.\right]$

(The concept of reluctance is explained in article 2.18 and 2.24)

Here $\quad \text{Reluctance} = \dfrac{\pi \times 300 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 1.963 \times 10^{-4}}$

$= 3.818 \times 10^8$ AT/Wb

$$\therefore \quad L = \frac{N^2}{\text{Reluctance}} = \frac{500 \times 500}{3.818 \times 10^8}$$

$= 0.000654$ H

$= 6.54 \times 10^{-4}$ H.

**2.22** Two coils having 80 and 350 turns respectively are wound side by side on a closed iron circuit of mean length 2.5 m with a cross-sectional area of 200 $cm^2$. Calculate the mutual inductance between the coils. Consider relative permeability of iron as 2700.

***Solution***

Given
$N_1 = 80$ turns
$N_2 = 350$ turns
$l = 2.5$ m
$A = 200 \text{ cm}^2 = 200 \times 10^{-4} \text{ m}^2$
$\mu_r = 2700$
$\mu_0 = 4\pi \times 10^{-7}$ H/m
$M$ = to be calculated.

We have $\quad M = \dfrac{N_1 \cdot N_2}{\text{Reluctance}}$ [for two coils of turns $N_1$ and $N_2$]

where reluctance $= \dfrac{l}{\mu_o \cdot \mu_r \cdot A}$ (Ref. article 2.18)

$= \dfrac{2.5}{4\pi \times 10^{-7} \times 2700 \times 200 \times 10^{-4}}$

$= 36860$ AT/Wb

$\therefore$ Mutual inductance $(M) = \dfrac{80 \times 350}{36860}$

$= 0.760$ H.

**2.23** A solenoid 60 cm long and 24 cm in radius is wound with 1500 turns. Calculate:
(a) the inductance
(b) the energy stored in the magnetic field when a current of 5 A flows in the solenoid.

***Solution***

*Given:* $l = 60 \text{ cm} = 0.6 \text{ m}$, $N = 1500$ turns, $A = \pi (0.24)^2 \text{ m}^2$
$\mu = \mu_o\mu_r = 4\pi \times 10^{-7} \times 1$, $I = 5$ A.

(a) *Inductance:*

We know $$L = \frac{\mu \cdot N^2 \cdot A}{l}$$

$$= \frac{4\pi \times 10^{-7} \times (1500)^2 \times \pi\,(0.24)^2}{0.6}$$

$$= 0.8534 \text{ H}$$

(b) *Energy stored:*

We have $$W = \frac{1}{2} LI^2$$

$$= \frac{1}{2}(0.8534)\,(5)^2 = 10.67 \text{ J.}$$

**2.24** Two coils of inductance 8 H and 10 H are connected in parallel. If their mutual inductance is 4 H, determine the equivalent inductance of the combination if (a) mutual inductance assists the self-inductance, (b) mutual inductance opposes the self-inductance.

***Solution***

It is given that

$$L_1 = 8 \text{ H}, L_2 = 10 \text{ H}, M = 4 \text{ H}$$

(a) $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{8 \times 10 - 4^2}{8 + 10 - 2 \times 4} = \frac{80 - 16}{18 - 8} = 6.4 \text{ H.}$$

(b) $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{8 \times 10 - 4^2}{8 + 10 + 2 \times 4} = \frac{80 - 16}{26} = 2.46 \text{ H.}$$

**2.25** Three coils are connected in series. Their self-inductances are $L_1$, $L_2$ and $L_3$. Each coil has a mutual inductance $M$ with respect to the other coil. Determine the equivalent inductance of the connection. If $L_1 = L_2 = L_3 = 0.3$ H and $M = 0.1$ H, calculate the equivalent inductance. Consider that the fluxes of the coil are additive in nature.

***Solution***

Let the current $i$ and $v_1$, $v_2$, $v_3$ be the voltage across the three coils.

$$\therefore \quad v_1 = L_1 \frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt}$$

$$v_2 = L_2 \frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt}$$

$$v_3 = L_3 \frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt}$$

$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3 + 6M)\frac{di}{dt}$$

$\because$ Equivalent inductance $= L_1 + L_2 + L_3 + 6M$.

Now putting the values of $L_1$, $L_2$, $L_3$ and $M$, we have

$$\text{Equivalent inductance} = 0.3 + 0.3 + 0.3 + 6 \times 0.1$$

$$= 0.9 + 0.6$$

$$= 1.5 \text{ H.}$$

**2.26** In a telephone receiver, the cross-section of the two poles is 10 cm × 0.2 cm. The flux between the poles and the diaphragm is $6 \times 10^{-6}$ wb. With what force is the diaphragm attracted to the poles? Assume $\mu_0 = 4\pi \times 10^{-7}$.

***Solution***

Here $\phi = 6 \times 10^{-6}$ wb

$A = 1.0 \text{ cm} \times 0.2 \text{ cm} = 0.2 \text{ cm}^2 = 2 \times 10^{-5} \text{ m}^2$

$$B = \frac{\phi}{A} = \frac{6 \times 10^{-6}}{2 \times 10^{-5}} = 0.3 \text{ wb/m}^2.$$

∴ Force acting on the diaphragm

$$F = \frac{B^2 A}{2\mu_o} \text{ N}$$

$$= \frac{(0.3)^2 \times 2 \times 10^{-5}}{2 \times 4\pi \times 10^{-7}} = 0.7159 \text{ N}.$$

**2.27** Determine the force in kg necessary to separate two surfaces with 200 cm$^2$ of contact area, when the flux density perpendicular to the surfaces is 1.2 wb/m$^2$.

***Solution***

Given $A = 200 \text{ cm}^2 = 2 \times 10^{-2} \text{ m}^2$

and $B = 1.2 \text{ wb/m}^2$

$$\therefore \quad F = \frac{B^2 A}{2\mu_o \times 9.81} \text{ kg} = \frac{(1.2)^2 \times 2 \times 10^{-2}}{2 \times 4 \times \pi \times 10^{-7} \times 9.81}$$

$$= 1167.64 \text{ kg}.$$

## 2.14 LIFTING POWER OF A MAGNET

Magnetic force is utilised in lifting magnets, operation of brakes, relays, circuit breakers, etc. All such devices have electromagnets and a steel armature. An air gap exists between the forces of the electromagnet and the armature and energy is stored in the air gap.

Let us consider two poles, north and south, as shown in Fig. 2.19.

Here, $A$ = Area of cross-section (in m$^2$) of each pole

$F$ = Force in Newtons between the poles.

Suppose one of the poles (say N-pole) is pulled apart against this attractive force through a small distance of $dx$ meters. Then, work done in moving the S-pole against the force of attraction is $F\,dx$ joules. However in moving the N-pole through a small distance $dx$, the volume of the path is increased to $(A\,dx)$ m$^3$.

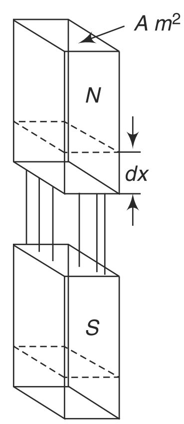



**Fig. 2.19** ***Lifting power of magnet***

Also, energy stored $E$ in the magnetic field (air) is given by $E = \dfrac{B^2}{2\mu_o}$ joules/m$^3$

i.e., Increase in stored energy $= \dfrac{B^2}{2\mu_o} \cdot A \cdot dx$ joules.

As we know, the increase in stored energy = work done, hence we can write $\dfrac{B^2}{2\mu_o} \cdot A \cdot dx = F \cdot dx$, when $F$ is the force of attraction or lifting power of the magnet.

$$\therefore \qquad F = \frac{B^2 A}{2\mu_o} \text{ Newtons} = \frac{B^2 A}{2\mu_o \times 9.81} \text{ kg}$$

and lifting power per unit area = $\frac{B^2}{2\mu_o}$ Newton/meter$^2$

where $B$ is the flux density in wb/m$^2$.

## 2.15 CONCEPT OF MAGNETIC CIRCUIT

*The path of a magnetic flux is known as magnetic circuit.* Similarly, the flow of magnetic flux is almost analogus to the flow of electric current in an electric circuit. In fact, the laws of magnetic circuit are almost similar (but not exactly same) to those electric circuits. It is known to us that to carry electric current in an electric circuit, usually aluminium or copper wires are used because the resistance of these materials is comparatively much lower than other materials. Similarly, to carry magnetic flux, iron or soft steel circuits are used as "opposition" of these materials to flux is low in comparison with other materials.

The study of magnetic circuit concepts is essential in the design analysis and application of electromagnetic devices like transformers, electromagnetic relays, electrical machines, etc.

## 2.16 CONCEPT OF MAGNETO-MOTIVE FORCE (MMF)

It is the amount of work done (in joules) required to carry a unit magnetic pole once through the entire magnetic field. In fact it is a kind of magnetic flux through a magnetic circuit and is called the *magneto motive force*. It's unit being ampere-turns (AT), it is actually measured by the product of number of turns $N$ in the coil of a magnetic circuit, and the current $I$ in amperes required to produce the magneto motive force.

Thus, $\qquad$ F(MMF) = $NI$ AT/m

Where $N$ = number of turns in the coil and $I$ = current through the coil (in Amps). It should be noted that through the unit of MMF is ampere turn (AT), it's dimension is taken as ampere since $N$ is dimensionless.

## 2.17 MAGNETIC FIELD INTENSITY

The MMF for unit length (along the path of magnetic flux) is defined as the *magnetic field intensity* and it is designated by the symbol $H$. The magnetic field intensity thus can be expressed as

$$H = \frac{\text{Magneto motive force}}{\text{Mean length of the magnetic path}}$$

i.e.

$$H = \frac{F}{l} = \frac{NI}{l} \text{ AT/m.} \qquad (2.34)$$

where $l$ is the *mean length* of the magnetic circuit in meters. Magnetic field intensity is also termed as *magnetising force* or magnetic *field strength*.

## 2.18 CONCEPT OF RELUCTANCE

It is designated by the symbol "$S$" and is analogus to resistance of an electric circuit. Flux in a magnetic circuit is limited by *reluctance.* Thus, reluctance $S$ is a measure of the opposition offered by a magnetic circuit to the establishment of magnetic flux.

It is directly proportional to the length and inversely proportional to the area of cross section of the magnetic path.

$$S \propto \frac{l}{a}$$

$$\therefore \quad S = \frac{l}{\mu_o \cdot \mu_r \cdot a} \quad (2.35)$$

The unit of reluctance is AT/Wb.

For air, vaccum and non-magnetic materials $\mu_r = 1$

$$\therefore \quad S = \frac{l}{\mu_o \cdot a} \quad (2.36)$$

the reciprocal of reluctance is called the "permeance". It is designated by the symbol $A$.

$$\therefore \quad \text{Permeance } (= A) = \frac{1}{S} \text{ wb/A [or henry (H)]} \quad (2.37)$$

Concept of magnetic reluctance is based on the following assumptions:

(a) B-H curve of the magnetic core is linear

(b) Leakage flux is of negligible order.

## 2.19 PERMEABILITY AND RELATIVE PERMEABILITY

*Permeability of a material* $\mu$ is defined as *its conducting power for magnetic lines of force.* It is the ratio of the flux density $B$ produced in a material to the magnetic field strength $H$, i.e. $\mu = \frac{B}{H}$.

Permeability of free space or vaccum is minimum and its value in SI units is $4\pi \times 10^{-7}$ henry/meter.

It may be noted here that flux density ($B$) is usually expressed as flux (Wb) per unit area (sq. m) and its unit in SI system is Tesla (T).

$$\therefore \quad 1 \text{ T} = 1 \text{ Wb/m}^2$$

### *Relative Permeability*

When the magneto motive force is applied to a ferromagnetic material, the flux produced is very large compared with that in air, free space (vaccum) or a non-magnetic material. *The ratio of the flux density produced in material to the flux density produced in air or a non-negative material by the same magnetic field* *intensity is called the relative permeability of that material.* It is designated by the symbol $\mu_r$. The relative permeability of vaccum is taken as unity.

Thus, permeability of any medium,

$$\mu = \text{Absolute permeability of air} \times \text{Relative permeability}$$
$$= \mu_o \cdot \mu_r = 4\pi \times 10^{-7} \times \mu_r \text{ Wb/AT.}$$

(with special nickel-iron alloys the value of $\mu_r$ may be as high as $2 \times 10^5$ whereas for most commonly used magnetic materials the values of $\mu_r$ is much smaller).

**2.28** A coil of 600 turns and of resistance of 20 Ω is wound uniformly over a steel ring of mean circumference 30 cm and cross-sectional area 9 cm². It is connected to a supply of 20 V (DC). If the relative permeability of the ring is 1,600 find (a) the reluctance, (b) the magnetic field intensity, (c) the mmf, and (d) the flux.

***Solution***

Here $N$ = 600 turns, resistance of the coil is 20 Ω, $l$ = 30 cm = 0.3 m, $A$ = 9 cm² = 9 × $10^{-4}$ m², relative permeability $\mu_r$ = 1600 and $\mu_o = 4\pi \times 10^{-7}$.

(a) Reluctance $S = \dfrac{l}{\mu_o \cdot \mu_r \cdot a} = \dfrac{0.3}{4\pi \times 10^{-7} \times 1600 \times 9 \times 10^{-4}}$

$$= 1.657 \times 10^5 \text{ At/wb}$$

(b) The magnetic field intensity

$$H = \left(\frac{NI}{l}\right) = \frac{600 \times 1}{0.3} = 2000 \text{ AT}$$

$$\left(\text{where } I = \frac{V}{\text{resistance of the coil}} = \frac{20}{20} = 1 \text{ Amp}\right)$$

(c) MMF = ($NI$) = 600 × 1 = 600 AT

(d) Flux $(\phi) = \dfrac{\text{mmf}}{\text{reluctance}} = \dfrac{NI}{S} = \dfrac{600 \times 1}{1.657 \times 10^5} = \dfrac{600}{1.65 \times 10^5}$

$$= 3.62 \times 10^{-3} \text{ wb} = 3.62 \text{ m Wb.}$$

**2.29** What is the value of the net mmf acting in the magnetic circuit shown in Fig. 2.20.

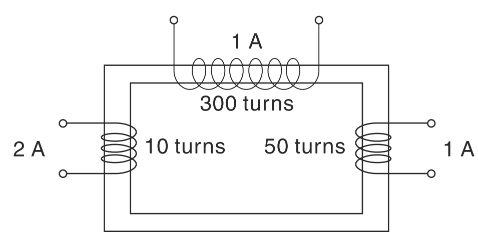



**Fig. 2.20** ***Magnetic Circuit of Ex. 2.29***

***Solution***

Net mmf acting in the magnetic circuit = mmf of all the three coils = $N_1I_1 + N_2I_2 + N_3I_3$ = (10 × 2 + 50 × 1 + 300 × 1)AT = 370 AT.

[It may be observed here that since the flux in all the coils are additive within the core, hence AT's are additive.]

**2.30** A mild-steel ring having a cross-sectional area of 400 $mm^2$ and a mean circumference of 400 mm has a coil of 200 turns wound uniformly around it. [given $\mu_r = 300$]. Determine:

(a) the reluctance $S$ of the ring,

(b) the current required to produce a flux of 800 $\mu$ Wb in the ring.

***Solution***

(a) Flux density $B$ in the ring is $\dfrac{800 \times 10^{-6}}{400 \times 10^{-6}} = 2 \text{ Wb/m}^2$.

$\therefore$ The reluctance $S$ of the ring is $\dfrac{0.4}{300 \times 4\pi \times 10^{-7} \times 0.4 \times 10^{-3}} = 2.65 \times 10^6$ A/Wb

Again we know $$\phi = \frac{\text{mmf}}{\text{Reluctance}}$$

$$800 \times 10^{-6} = \frac{\text{mmf}}{\text{Reluctance}}$$

$$\therefore \quad \text{mmf} = 800 \times 10^{-6} \times 2.65 \times 10^6$$
$$= 2.122 \times 10^3 \text{ AT}$$

and magnetizing current is $\dfrac{\text{mmf}}{\text{No of turns}} = \dfrac{2.122 \times 10^3}{200}$

$$= 10.6 \text{ A.}$$

**2.31** A magnetic circuit having 150 turns coils and the cross-sectional area and length of the magnetic circuit are $5 \times 10^{-4}$ $m^2$ and $25 \times 10^{-2}$ m respectively. Determine $H$ and the relative permeability $\mu_r$ of the core when the current is 2 A and the total flux is $0.3 \times 10^{-3}$ Wb.

***Solution***

When $I = 2$ A

$$\text{mmf} = NI = 150 \times 2 = 300 \text{ A/T}$$

$$H = \frac{NI}{l} = \frac{300}{25 \times 10^{-2}} = 1200 \text{ A/m}$$

$$B = \frac{\text{flux } (\phi)}{\text{area } (a)} = \frac{0.3 \times 10^{-3}}{5 \times 10^{-4}} = 0.6 \text{ T}$$

$$\mu = \frac{B}{H} = \frac{0.6}{1200} = 500.00 \times 10^{-6} \text{ H/m}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{500.00 \times 10^{-6}}{4\pi \times 10^{-7}} = 3.9788 \times 10^2 = 397.88.$$

**2.32** An air cored coil has 500 turns. The mean length of magnetic flux path is 50 cm and the area of cross-section is $5 \times 10^{-4}$ $m^2$. If the exciting current is 5 A, determine (a) $H$ (b) the flux density and (c) the flux ($\phi$).

***Solution***

$$\text{mmf} = NI = 500 \times 5 = 2500 \text{ A}$$

Given, $l = 50$ cm $= 0.5$ m

$a = 5 \times 10^{-4}$ $m^2$

(a) $\therefore\ H = \frac{Ni}{l} = \frac{2500}{0.5} = 5000.00$ A/m.

(b) $B$ = flux density $= \mu \cdot H = \mu_r \cdot \mu_0 \cdot H$

$= \mu_o \cdot H \quad [\mu_r = 1]$

$= 4\pi \times 10^{-7} \times 5000.00$

$= 6.283 \times 10^{-3}$ T.

(c) Flux $(\phi) = B \times a \quad = 6.283 \times 10^{-3} \times 5 \times 10^{-4}$

$= 3.1415 \times 10^{-6}$ Wb.

**2.33** Two identical co-axial circular loops carry a current $I$ each circulating in the same direction. If the loops approach each other, the current in each decreases. Justify the statement.

***Solution***

When the loops approach each other, the field becomes strong, which should not be allowed in accordance with Lenz's law. So the current in both should be in such a way that the field decreases and hence $I$ decreases.

**2.34** An iron ring 10 cm mean circumference is made from a round iron of cross-section $10^{-3}$ m$^2$. It's relative permeability is 500. If it is wound with 250 turns, what current will be required to produce a flux of $2 \times 10^{-3}$ Wb?

***Solution***

The lines of magnetic flux follow the circular path of the iron so that

$l = 100$ cm $= 1$ m

$a$(area) $= 10^{-3}$ m$^2$

$$\therefore \quad \text{Reluctance } S = \frac{1}{\mu_r\,\mu_o\,a} = \frac{1}{(500 \times 4\pi \times 10^{-7} \times 10^{-3})}$$

$$= 1.59 \times 10^6 \text{ A/Wb.}$$

Given Flux $(\phi) = 2 \times 10^{-3}$ Wb

$$\therefore \quad H = \phi \cdot S = 2 \times 10^{-3} \times 1.59 \times 10^6$$

$$= 3.1847 \times 10^3 \text{ AT.}$$

As we know $H = NI$

$$\therefore \quad I = \frac{H}{N} = \frac{3.1847 \times 10^3}{250}$$

$$= 12.738 \text{ A.}$$

**2.35** An air gap 1.1 mm long and 40 sq. cm in cross-section exists in a magnetic circuit. Determine (a) Reluctance $S$ of the air-gap, and (b) mmf required to create a flux of $10 \times 10^{-4}$ Wb in the air gap.

***Solution***

(a) Reluctance $(S)$ $= \frac{l}{\mu_r \cdot \mu_o\, a} = \frac{l}{m_o \cdot a} \qquad [\because \mu_r = 1]$

$$= \frac{1.1 \times 10^{-3}}{4\pi \times 10^{-7} \times 40 \times 10^{-4}} = 2.1885 \times 10^5 \text{ A/m}$$

(b) Since $\phi = 10 \times 10^{-4}$ Wb (given)

$\therefore$ mmf = flux × reluctance

$= 10 \times 10^{-4} \times 2.1885 \times 10^{5}$

$= 218.85$ AT.

**2.36** An iron ring of mean length 110 cm with an air gap of 1.5 mm has a winding of 600 turns. The relative permeability of iron is 600. When a current of 4 A flows in the winding, calculate the flux density $B$. Do not consider fringing.

***Solution***

Given that $l_i = 110$ cm – 0.15 cm

$= 109.85$ cm

$= 1.0985$ m

$l_g = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m.

$N = 600$ turns, $\mu_r = 600$ (given)

$I = 4$ A

$$\therefore \text{ Flux density } \quad B = \frac{\mu_o \cdot NI \cdot \mu_r}{l_i} + \frac{m_o NI \cdot 1}{l_g}$$

$$= \mu_o NI \left( \frac{\mu_r}{l_i} + \frac{1}{l_g} \right)$$

$$= 4\pi \times 10^{-7} \times 600 \times 4 \times \left[ \frac{600}{1.0985} + \frac{1}{1.5 \times 10^{-3}} \right] \text{Wb/m}^2$$

$$= 3.0159 \times 10^{-3} \times (5.46 \times 10^{2} + 6.666 \times 10^{2})$$

$$= 3.657 \text{ Wb/m}^2.$$

## 2.20 DEFINITION OF AMPERE

Consider two parallel wires separated by 1 m in space carrying a current of 1 A each, then $I_1 = I_2 = 1$ A and $d = 1$ m. From the expression of magneto motive force developed between two parallel conductors, we have $dF/dl = 2 \times 10^{-7}$ Newton per meter.

This is used to formally define the unit 'ampere' of electric current.

Therefore, ampere is defined as the current, which when flowing in each of the two infinitely long parallel wires (conductors) of negligible cross-section and placed 1 meter apart in vaccum produces on each wire a force of $2 \times 10^{-7}$ Newton per meter length.

## 2.21 *B-H* CHARACTERISTICS

The graph between the flux density $B$ and field intensity $H$ of a magentic material is called $B$-$H$ or familiar magnetization curve. Figure 2.21 shows a typical $B$-$H$ curve for an iron specimen. As seen from the curve it can be divided into four distinct regions $OA$, $AB_1$, $B_1C$ and the region beyond $C$.

The slope of the *B-H* curve for iron (Fig. 2.21) can be explained as follows:

(a) In the region *OA* (in step region), magnetic field strength *H* (another name is magnetizing force) is too weak to cause any appreciable alignment of domains (or elementary magnets). Consequently, the increase in flux density *B* is small. In the neighbourhood of the origin the graph is a straight line through the origin, and the slope gives the initial permeability.

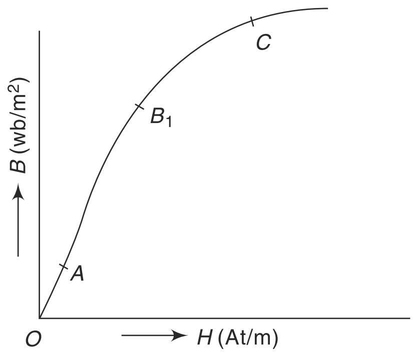



**Fig. 2.21** ***B-H curve for an iron sample***

(b) In the region $AB_1$, more and more domains get aligned as *H* increases, consequently, *B* increases almost linearly with *H*.

(c) In the region $B_1C$ only a fewer domains are left unaligned, consequently, the increase in *B* with *H* is very small.

(d) Beyond *C,* i.e. beyond the knee zone, no more domains are left unaligned and the iron material is said to be magnetically saturated. This upper portion of the curve is represented with fair accuracy by Frohlich's equation

$$B_S = B - \mu_o H = \frac{H}{a + bH}$$

$$\therefore \qquad \frac{1}{B_S} = \frac{a + bH}{H} = \frac{a}{H} + b$$

where (*a*) is a hardness constant while $\left(b = \frac{1}{B_S}\right)$. Here $B_S$ is the saturation flux density.

A magnetic material is said to be magnetically saturated when it fails to attain a still higher degree of magnetization, even when the magnetizing power is increased enormously. The slope of the curve beyond *C* shows that the increase in *B* with increase in *H* is nominal. This point *C* is called the *point of saturation* for the material.

The slope of the *B-H* curve at any point *E* is given by tan $\theta = B/H$, again $B = \mu_o \cdot \mu_r \cdot H$; $\mu_r = 1/\mu_o \cdot \tan\theta \left[\text{since } \frac{B}{H} = \tan\theta\right]$

Thus, ($\mu_r$) is proportional to the slope of the B-H curve at any point. Starting from as definite value at the origin, the slope increases as *B* increases until it becomes a maximum. It then gradually decreases as *B* increases further. The slope becomes almost zero in the saturation region when the curve becomes almost horizontal. The *B-H* curve shows that the permeability $\mu_r$ of a magnetic material changes with the flux density *B*. It is important to note that the lesser the impurities like carbon, sulphur and phosphorus in iron (the magnetic material) higher is its permeability.

***Importance of B-H curve***

(a) It helps in resulting the magnetic material for a specific application.

(b) It helps in making practical magnetic calculations in the design or analysis of the magnetic circuits.

## 2.22 FERROMAGNETIC MATERIALS

*Ferromagnetism*, characterised by the strong attraction or repulsion of one magnetised body by another. In fact, in some materials, the permanent atomic magnetic moments have a strong tendency to align themselves even without any external field. These materials are called *ferromagnetic materials* and permanent magnets are made from them. The force between the neighbouring atoms, responsible for their alignment, and it can only be explained on the basis of Bohr's theory of atomic structure.

According to this theory, the electrons revolve around the nucleus in fixed orbits. Since these electrons are in motion they constitute electric currents. These currents produce magnetic fields called *orbital magnetic* fields. In addition to this, each electron spins on is axis as it revolves in an orbit. (the spin of an electron is similar to the spin of the earth). A spinning electron has a charge in motion and, therefore, constitutes an electric current. This current produces a magnetic field called the *spin magnetic* field.

For an individual atom these fields are very weak. Strong magnetic fields can be produced if the atoms are grouped in a material in such a way that their orbital and spin fields reinforce one another. In ferromagnetic materials there is an appreciable interaction between neighbouring atoms. Atoms do not act singly but in groups called *domains*. Each domain contains between $10^9$ and $10^{20}$ atoms. If a magnetic field is applied, the domains which are aligned along the direction of the field grow in size and those opposite to it get reduced. Also, domains may orient themselves in favour of the applied field. Consequently the resultant magnetic flux density inside the material is much greater than the flux density of the applied field. Examples of ferromagnetic materials are iron, cobalt and nickel but there many ferromagnetic alloys, some of which do not even possess iron as one of their components. The ferromagnetic materials are all solids.

## 2.23 TYPES OF MAGNETIC MATERIALS

From an engineering point of view, magnetic materials can be broadly classified into two groups, namely:

***(a) Soft Magnetic Materials*** i.e. soft iron (3.5 to 4.5% silicon content) and are used in transformers, electric machines and taperecorder tapes. Their magnetization can be changed rapidly. Its susceptibility, permeability and retentivity are greater while coercivity and hysteresis loss per cycle are smaller than those of steel. These possess a uniform structure (i.e. well-aligned crystal grains).

***(b) Hard Magnetic Materials*** like steel or alloy alnico (Al + Ni + Co) are used for permanent magnets, coercivity and curie temperature for these materials are high and their retentivity is low. Their hysteresis loops are generally characterized by a broad hysteresis loop of large area compared to soft magnetic materials. Its demagnetization takes place with difficulty.

## 2.24 MAGNETIC CIRCUIT LAWS

The path of the magnetic flux is called the magnetic circuit. Just as the flow of electric current in an electric circuit necessiates the presence of an emf, so the establishment of a magnetic flux requires the presence of a mmf. In fact there is a close mathematical analogy between magnetic and d.c. resistive circuits. For a d.c. resistive circuits, Ohm's law relationship is

$$I = \frac{\text{e.m.f}}{\text{resistance}} = \frac{V}{R}, \quad \text{also} \quad R = \rho\frac{l}{a}; \quad \rho = \frac{1}{\sigma}$$

where $\sigma$ is the conductivity.

For a magnetic circuit, magentic flux $\phi$ is equal to mmf divided by reluctance, i.e.

$$\phi = \frac{\text{mmf}}{\text{reluctance}} = \frac{F}{S} \text{ (in symbols)} \tag{2.38}$$

Again we know for electric circuits $R = \rho \cdot \frac{l}{a}$ and for magnetic circuits

$$S = \frac{1}{\mu_o \cdot \mu_r} \cdot \frac{l}{a}$$

Therefore the definition of reluctance $S$ is obtained from the above analogy. Reluctivity is *specific reluctance* and is comparable to resistivity $\rho$ which is *specific resistance*.

Again $\phi$ can be written as from the equation (2.38)

$$\text{Flux} = \phi = \frac{\text{mmf}}{\text{Reluctance}} = \frac{\text{AT}}{\frac{1}{\mu_o \cdot \mu_r} \cdot \frac{l}{a}}$$

$$\therefore \quad \text{AT} = \frac{\phi}{a} \cdot \frac{1}{\mu_o \cdot \mu_r} \cdot \frac{B \cdot l}{\mu_o \cdot \mu_r} = H \times l \tag{2.39}$$

= field strength of the particular magnetic path × length of this particular magnetic path.

Therefore by analogy, $\sigma$ the conductivity of the conductor is equal to the $(\mu_o \cdot \mu_r)$ in the magnetic circuit.

No name is in common use for the unit of reluctance; it is evidently measured in ampere-turns per weber of magnetic flux in the circuit.

By analogy, the laws of resistance in series and parallel also hold good for reluctances in a composite magnetic circuit.

In case of a composite electric circuit we have for series connected conductors

$$R_{\text{Total}} = R_1 + R_2 + R_3 + \cdots + R_n$$

Similarly, with a composite magnetic circuit, we have to substitute $S$ for $R$.

i.e.
$$S_{\text{Total}} = S_1 + S_2 + S_3 + \cdots + S_n \tag{2.40}$$

Again in case of parallel magnetic circuit, the same mmf is applied to each of the parallel paths and the total flux divides between paths in inverse proportion to their reluctances.

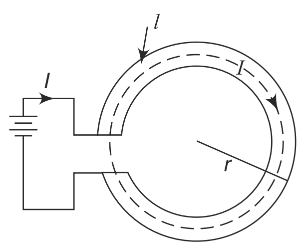
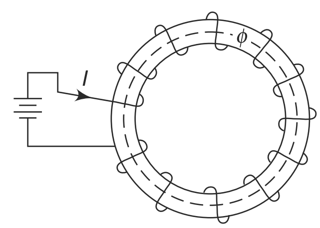
$$\phi = \phi_1 + \phi_2 + \phi_3 + \cdots + \phi_n$$

The total reluctance $S_{\text{Total}}$ of a number of reluctancs in parallel is given by

$$\frac{1}{S_{\text{Total}}} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} + \ldots + \frac{1}{S_n} \tag{2.41}$$

Similarly, *permeances* (reciprocal of reluctances) in series and parallel obey the same rules as electrical conductances.

## 2.25 COMPARISON BETWEEN ELECTRIC AND MAGNETIC CIRCUITS

| Electric circuit | Magnetic circuit |
|---|---|
| **1. Basic Model** | |
| The toroidal copper ring (Fig. 2.22) is assumed open by an infinitesmal amount with the ends connected to a battery.<br><br><br>**Fig. 2.22 (a)** | The toroidal iron ring is assumed wound with N turns of wire with a current $i$ flowing through it. The magneto motive force creates the flux $\phi$.<br><br><br>**Fig. 2.22 (b)** |
| **2. Driving Forces** | |
| Applied battery voltage is $E$ or $V$ | Applied Ampere turns is $AT$ or $NI$ |
| **3. Response** | |
| Current $= \dfrac{\text{Driving force}}{\text{Electric resistance}}$<br>$\therefore \quad I = \dfrac{E}{R}$ | Flux $= \dfrac{\text{Driving force}}{\text{Magnetic resistance}}$<br>$= \dfrac{AT}{R}$<br>where $R = \dfrac{l}{a \cdot \mu_o \cdot \mu_r}$ AT/Wb<br>where $a$ = area of magnetic path |

**4. Impedance**

Resistance $R = \rho \times \frac{l}{a}$, here $l = 2\pi r$ = mean length of turn of the toroid and $a$ is the toroidal cross-section

Reluctance (= $R$) = $\frac{l}{\mu_o\ \mu_r\ a} = \frac{l}{\mu \cdot a}$ where $l = 2\pi r$ = mean length of turn of the toroid and $a$ is the toroidal cross-sectional area.

**5. Equivalent Circuit**

Here electromotive force $E$ is the driving force and electric current flows through the circuit.

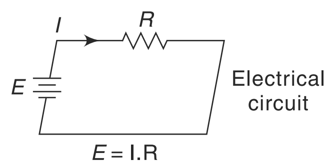



**Fig. 2.23 (a)**

Here electromotive force is the driving force and magnetic flux does not flows and it only links with the coil.

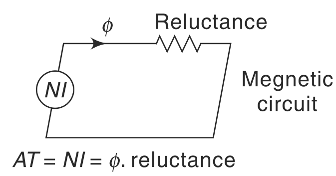



**Fig. 2.23 (b)**

**6. Field Intensity**

Electric field intensity: with the application of the voltage $E$ to the homogeneous copper toroid, an electric potential gradient $\varepsilon$ produced within the material and is given by

$$\varepsilon = \frac{E}{l} = \frac{E}{2\pi n}\ \text{V/m}$$

This electric field must occur in a closed loop path if it is to be maintained. If then follows the closed line integral of $\varepsilon$ is equal to the battery voltage $E$. Thus

$$\oint \varepsilon \times dl = E$$

Magnetic field intensity: when a magnetomotive force is applied to the homogeneous iron toroid, there is produced within the material a magnetic potential gradient given by

$$H = \frac{NI}{l} = \frac{NI}{2\pi r}\ \text{AT/m}$$

As already pointed out in connection with Ampere's circuited law, the closed line integral of $H$ equals the enclosed magnetomotive force. Thus

$$\oint H \times dl = NI$$

**7. Voltage drop | mmf drop**

Voltage drop is given $V = I \times R$ where $R$ is the resistance of the copper toroid between the two points.

$NI = \phi$. Reluctance where $R$ is the reluctance of the iron toroid between the points.

**8. Current density | Flux density**

Current density is the amount of ampere per unit area = $\frac{\text{Amp}}{m^2} = \frac{I}{a}$

Flux density is expressed as webers per unit area i.e. $\frac{\text{Wb}}{\text{m}^2} = \frac{\phi}{a}$.

## 2.26 DISTINCTION BETWEEN MAGNETIC AND ELECTRIC CIRCUITS

1. In an electric circuit current does not flow in air, unless the dielectric strength between the curent-carrying conductor and nearest earth fails but flux in a magnetic circuit can flow in air.
2. Current flows (actual flow of electrons) in an electric circuit but flux does not flow because magnetic flux lines are imaginary.
3. Based on property, some materials act as insulators and some as conductors but there is no such material as 'insulators' to magnetic circuits.
4. The resistance in electric circuit changes with temperature but at constant ampere turns (AT), reluctance does not change with temperature.
5. The electric current can be confined to flow in an accurately defined path but there is no good magnetic insulator to confine all the magnetic flux to one prescribed path in a magnetic circuit. There is always some leakage flux.

**2.37** An iron ring has a mean circumference of 80 cm and having cross-sectional area of 5 cm$^2$ and having coil of 150 turns. Using the following data, calculate the existing current for a flux of $6.4 \times 10^{-4}$ Wb. Also calculate the relative permeability ($\mu_r$).

| | | | | | |
|---|---|---|---|---|---|
| $B$ (Wb/m$^2$) | : | 0.9 | 1.1 | 1.2 | 1.3 |
| H (A/m) | : | 260 | 450 | 600 | 820 |

***Solution***

It is given that $\phi = 6.4 \times 10^{-4}$ Wb

$$\therefore \text{ flux density } B = \frac{\phi}{\text{area}} = \frac{6.4 \times 10^{-4}}{5 \times 10^{-4}} = 1.28 \text{ Wb/m}^2$$

Assuming B-H curve to be linear in the range from 1.2 to 1.3 Wb/m$^2$,

$$H = 600 + \frac{820 - 600}{1.3 - 1.2} \times (1.28 - 1.2)$$

$$= 776 \text{ A/m.}$$

$$\therefore \quad \text{mmf} = 776 \times \frac{80}{100} = 620.8 \text{ AT}$$

$$\therefore \quad I = \frac{620.8}{150.0} = 4.138 \text{ A}$$

We know $B = \mu \cdot H = \mu_o \cdot \mu_r \cdot H$

$$\therefore \quad \mu_r = \frac{B}{\mu_o \cdot H} = \frac{1.28}{4\pi \times 10^{-7} \times 776} = 1312.089$$

**2.38** In the magnetic circuit shown in Fig. 2.24, the cross-sectional area of limbs $Q$ and $R$ are 0.01 m$^2$ and 0.02 m$^2$ respectively and the lengths of air-gap are 1.1 mm and 2.1 mm respectively, are cut in the limbs $Q$ and $R$. If the magnetic medium can be assumed to have infinite permeability and the flux in the limb is 1.5 Wb, calculate the flux in the limb $P$.

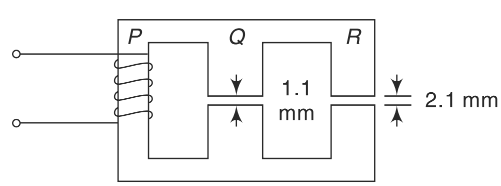



**Fig. 2.24** *Magnetic circuit of Ex. 2.38*

***Solution***

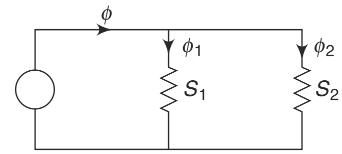



**Fig. 2.25** ***Electrical equivalent of Fig. 2.24***

It is given that

Area of cross-section of limb $Q = 0.01$ m$^2$

Area of cross-section of limb $R = 0.02$ m$^2$

Length of air-gap = 1.1 mm for limb $Q$ and 2.1 mm for limb $R$

Flux in limb $Q = 1.5$ Wb

As because the permeability of the magnetic medium is infinity, reluctance of the given iron path is zero. The electrical equivalent is shown in Fig. 2.25.

Now if we assume $S_1$ is the reluctance of air-gap of limb $Q$ and $S_2$ is the reluctance of the air-gap of limb $R$ respectively. Let $\phi_1$ is the flux across the air-gap of limb $Q$ and $\phi_2$ is the flux across the air-gap of limb $R$.

$$\therefore \quad S_1 \times \phi_1 = S_2 \times \phi_2$$

$$\frac{l_1}{\mu_o \times a_1} \times \phi_1 = \frac{l_2}{\mu_o \times a_2} \times \phi_2$$

$$\therefore \quad \phi_2 = \frac{a_2}{a_1} \times \frac{l_1}{l_2} \times \phi_1$$

$$= \frac{0.02}{0.01} \times \frac{1.1}{2.1} \times 1.5$$

$$= 1.5714 \text{ Wb.}$$

$\therefore$ Flux in the limb

$$P = \phi_1 + \phi_2$$

$$= 1.5 + 1.5714 = 3.0714 \text{ Wb.}$$

**2.39** A ring, made of steel has a rectangular cross-sectional area. The outer diameter of the ring is 25 cm while the inner diameter is 20 cm, the thickness being 2 cm. The ring has a winding of 500 turns and when carrying a current of 3A, produces a flux density of 1.2 T in the air gap produced when the ring is cut to have an air gap of 1 mm length (Fig. 2.26). Find (a) the magnetic field intensity of the steel ring and in the air gap, (b) relative permeability of the magnetic material, (c) total reluctance of the magnetic circuit, (d) inductance of the coil and (e) emf induced in the coil when the coil carries a current of $i_{(ac)} = 5 \sin 314\, t$.

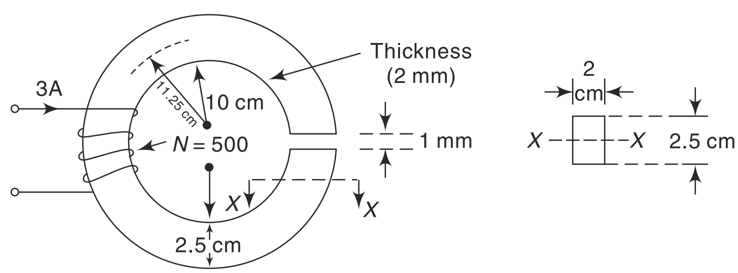



**Fig. 2.26** ***Given Circuit of Ex. 2.39***

***Solution***

$$NI = 500 \times 3 = 1500 \text{ AT}$$

$$B_{\text{steel}} = B_{\text{gap}} = 1.2 \text{ T}$$

(a) $\therefore$ $$H_{\text{gap}} = \frac{1.2}{4\pi \times 10^{-7}} = 9.55 \times 10^5 \text{ AT/m}$$

Since $$NI = H_{\text{gap}} \times l_g + H_{\text{core}} \times l_{\text{core}},$$

where $l_{\text{gap}}$(mean length of gap) = 1 mm = $1 \times 10^{-3}$ m,

$$l_{\text{core}} = 2\pi \times \left(10 + \frac{2.5}{2}\right) = (2\pi \times 11.25) \text{ cm}$$

$$= 2\pi \times 11.25 \times 10^{-2} \text{ m.}$$

We can write,

$$1500 = 9.55 \times 10^5 \times 10^{-3} + H_{\text{core}} \times 2\pi \times 11.25 \times 10^{-2}$$

i.e. $$H_{\text{core}} = 771.20 \text{ AT/m.}$$

(b) Also, $H_{\text{core}} = \dfrac{B_{\text{core}}}{\mu_o \mu_r}$; $\therefore$ $\mu_r = \dfrac{1.2}{4\pi \times 10^{-7} \times 771.20} = 1238.2$

(c) $$S = S_1 + S_2 = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 2 \times 2.5 \times 10^{-4}} + \frac{2\pi \times 11.25 \times 10^{-2}}{4\pi \times 10^{-7} \times 1238.2 \times 2 \times 2.5 \times 10^{-4}}$$

$$= 2.5 \times 10^6 \text{ AT/Wb}$$

(d) $$L = \frac{N\phi}{I} = \frac{500 \times (B \times a)}{3} = \frac{500 \times (1.2 \times 2.5 \times 2 \times 10^{-4})}{3} = 0.1 \text{ H}$$

(e) $$E = L\frac{di}{dt} = 0.1 \times \frac{d}{dt} 5 \sin 314\, t) = 0.5 \frac{d}{dt} \sin (314\, t)$$

$$= 157 \cos 314\, t.$$

## 2.27 LEAKAGE FLUX IN MAGNETIC CIRCUIT AND FRINGING AND STAKING

In a magnetic circuit, it is never possible to confine all the fluxes in the direction of designated path, since a portion of the total flux will follow different paths from the intended path (generally through air). The shapes of these paths, and the amount of flux in them, depend on the geometry of the magnetic circuit and also in the value of the relative permeability $\mu_r$. Therefore the part of the magnetic flux that has its path within the magnetic circuit is known as the useful flux or main flux and that taking other paths is called *leakage flux*. This phenomenon of wastage of some flux is called *magnetic leakage*. Sum of the two parts is called the *total flux produced*.

The ratio of the total flux produced by the magnet to the main flux is called *leakage co-efficient* or *leakage factor*.

Mathematically, leakage factor $= \dfrac{\phi_T}{\phi_m} = \dfrac{\text{Total useful flux}}{\text{Main flux}} = \dfrac{\phi_l + \phi_m}{\phi_m}$

where $\phi_l$ = leakage flux

$\phi_m$ = main flux

$\phi_T$ = total flux

This leakage co-efficient is generally designated by $\lambda$ and its value ranges from 1.12 to 1.25, i.e. is always greater than unity.

Magnetic leakage in magnets is undesirable since it increases their weight as well as cost of manufacturer.

### *Fringing*

Figure 2.27 shows a ring provided with an air gap. The flux lines crossing this air-gap tend to repel each other and therefore buldge out across the edges of the air gap. This phenomenon is known as *fringing*. Due to this fringing, the effective gap area is larger than that of the ring. Longer the air-gap, greater is the fringing. Generally, the increase in cross-sectional area of air-gap due to fringing is assumed to be about 9 to 10%.

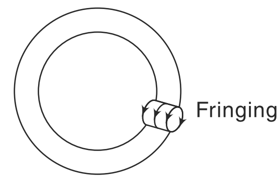



**Fig. 2.27** ***Concept of fringing***

### *Stacking*

Magnetic circuits are generally laminated to reduce eddy current loss. These laminations are coated with insulating varnish. Therefore, a small space is present between the successive laminations. So the effective magnetic cross-sectional area is less than the overall area of the stack. *Stacking factor* is defined as the ratio of the effective area to the total area. This factor plays an important role during calculation of flux densities in magnetic parts. This factor is usually less than unity.

## 2.28 MAGNETIC HYSTERESIS

When a bar of ferromagnet material is magnetised by a varying magnetic field and the intensity of magnetizaton $B$ is measured for different values of magnetizing field $H$, the graph of $B$ versus $H$ is shown in Fig. 2.28 and it is called $B$-$H$ curve or magnetization curve. From graph, it is observed that

(a) When the magnetizing field is increased from 0, the intensity of magnetization ($H$) increases and becomes maximum. This maximum value is called the saturation value.

The state of magnetic material in which the value of $H$ becomes maximum and does not increase further on increasing the value of $H$ is called the *state of magnetic saturation.*

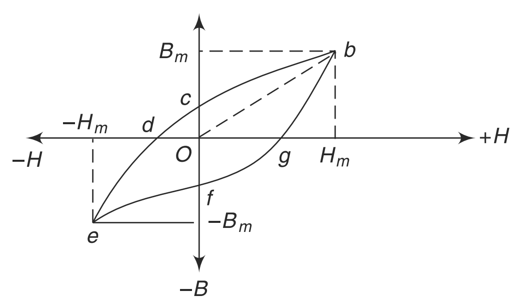



**Fig. 2.28** ***Magnetic hysteresis***

(b) When $H$ is reduced, $B$ reduces but is not zero when $H = 0$. The remainder value $OC$ of magnetization, when $H = 0$, is called the *residual magnetism* or *retentivity*. The property by virtue of which the magnetism $B$ remains in material even on the removal of magnetizing field is called *retentivity* or *residual magnetism* or *ramnant magnetism*.

(c) When magnetic field $H$ is reversed, the magnetization decreases and for a particular value of $H$, denoted by $H_C$, it becomes zero, i.e. $H_C = od$ when $I = 0$. This value of $H$ is called *coercivity*.

So, the process of demagnetizing a material completely by applying magnetizing field in a negative direction is defined as coercivity. Coercivity assesses the softness or hardness of a magnetic material. If the coercivity of a magnetic material is low then it is magnetically soft and when its value is high then the material is magnetically hard.

(d) When the field $H$ is further increased in reverse direction the intensity of magnetization attains saturation value in reverse direction (i.e., point $e$).

(e) When $H$ is decreased to zero and changed direction in steps, we get the part *efgb*.

Thus complete cycle of magentization and demagnetization is represented by *bcdefgb*. In the complete cycle the intensity of magnetization $H$ is lagging behind the applied magnetizing field. This is called *hysteresis* and the closed loop *bcdefgb* is called *hysteresis cycle*.

The energy loss in magnetizing and demagnetizing a specimen is proportional to the area of hysteresis loop.

The selection of a material for a specific purpose depends on its hysteresis loop. When the magnet is to operate on ac voltage it undergoes a large number of reversals every second. The material for such application should have a low hysteresis loss and therefore, the hysteresis loop should enclose a small area. Soft iron is one such example.

In recent times some development has been made in Ni-Fe alloys. They are called *square loop* materials produced by maintaining the alloy for a time in a magnetic field at a temperature of 400°C to 590°C. The ultimate of this development is to make the knee point of magnetizing curve very sharp, the coercive force becomes small and the permeability is very high. The hysteresis curve showing variation of $B$ with $H$ of in "square loop" material is shown in Fig. 2.29. In fact these properties are essential in devices like magnetic storage of information like in computers.

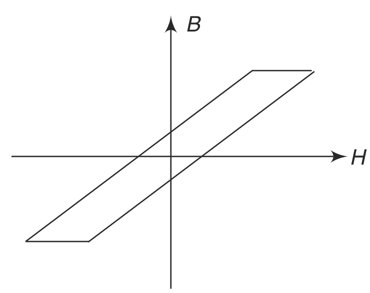



**Fig. 2.29** ***B-H curve for square loop material***

## 2.29 HYSTERESIS LOSS

This loss occurs due to the *B-H* magnetization curve which swings to positive and negative maximum $B_m$ before returning to zero. Ideally the energy absorbed during the positive swing should be returned to the source during reversal of the magnetizing cycle. But in actuality there is only a partial return to source, the rest being dissipated as heat.

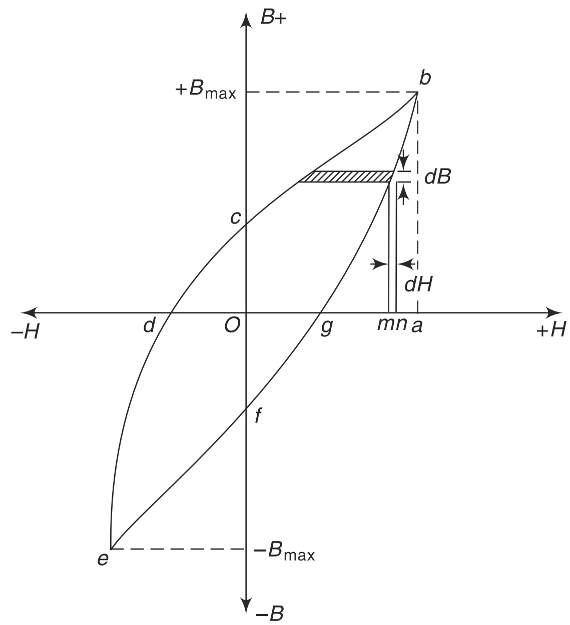



**Fig. 2.30** ***Hysteresis loop for steel ring***

Figure 2.30 represents the hysteresis loop obtained of a steel ring of mean circumference ($l$) meters and cross-sectional area ($a$) square meters. Let ($N$) be the number of turns on the magnetizing coil.

Let ($dB$) = increase of flux density when the magnetic field intensity is increased by a very small amount $dH$ (say) in $dt$ seconds, and $i$ = current in amperes corresponding to $om$, i.e., $om = \frac{Ni}{l}$.

Instantaneous emf induced in the winding is $a \times dB \times \frac{N}{dt}$ V.

and component of applied voltage to neutralize this emf equals

$$\left(an \times \frac{dB}{dt}\right) \text{V.}$$

Therefore instantaneous power supplied to the magnetic field is

$$\left(i \times an \times \frac{dB}{dt}\right) \text{W.}$$

and energy supplied to the magnetic field in time $dt$ second is

$$(i \times an \times dB) \text{ J}$$

Since, $$om = \frac{Ni}{l}$$

$$i = l \times \frac{om}{N}$$

hence energy supplied to magnetic field in time $dt$ is

$$l \times \frac{om}{N} \times an \times dB \text{ J} = (om \times dB \times lA) \text{ J}$$

$$= \text{area of shaded strip, J/m}^3.$$

Thus energy supplied to the magnetic field when $H$ is increased from zero to $oa$ is equal to area $f\ g\ b\ B_m\ f$ J/m$^3$. Similarly, energy returned from the magnetic field when $H$ is reduced from $oa$ to zero is area $b\ B_m cb$ J/m$^3$. Then net energy absorbed by the magnetic field is

Area $f\ g\ b\ B_m f$ J/m$^3$.

Hence, hysteresis loss for a complete cycle is

area of $e\ f\ b\ c\ e$ jouled per m$^3$.

If we define hysteresis loss as $P_n$,

$$\therefore \quad P_n = v.f.\,(K_h \cdot B_m^n)\text{ W} \tag{2.42}$$

where $v$ is volume of core material and $f$ is the frequency of variation of $H$ in Hz.

The value of $n$ is between 1.5 and 2.5 ($1.5 \le n \le 2.5$) but mostly it is 1.6.

$B_m$ is the maximum flux density in Tesla and $K_H$ is a constant and $n$ is the exponent which depends on the material.

The constant $K_H$ (depending on the chemical properties of the material and the heat treatment and mechanical treatment the material has been subjected to) may have value as low as $5 \times 10^{-7}$ for permalloys and as high as $6 \times 10^{-5}$ for cast iron. However $K_H$ for electrical sheet steel is generally $4 \times 10^{-5}$.

The exponent $n$ has been found by Steinmetz as 1.6 and does not have any theoretical basis. This value suits most material at flux densities generally not exceeding 1 wb/m$^2$. However, for higher value of flux densities, the value may be as great as 2.5.

## 2.30 EDDY CURRENTS (OR FOUCAULT'S CURRENTS) AND EDDY CURRENT LOSS

When a metallic body is moved in a magnetic field in such a way that the flux through it changes or is placed in a changing magnetic field, induced currents circulate throughout the volume of the body. These are called *eddy currents*. If the resistance of the said conductor is small, then the magnitudes of the eddy currents are large and the metal gets heated up. This heating effect is a source of power loss in iron-cored devices such as dynamos, motors and transformers,

The eddy current loss is given by

$$P_e = K_e \cdot f \cdot B_m^2\, t^2\, v^2 \text{ W} \tag{2.43}$$

where $K_e$ is a constant, $f$ is frequency, $B_m$ is the maximum flux density, $t$ is the thickness of the core material and $v$ is the total volume of the core material.

If the core is made of laminations insulated from one another, the eddy currents are confined to their respective sheets, the eddy current loss is thereby reduced. Thus, if the core is split up into five laminations, the emf per lamination is only a fifth of that generated in the solid core. Also, the cross-sectional area per path is reduced to about a fifth, so that the resistance per path is roughly five times that of the solid core. Consequently the current per path is about one-twenty-fifth of that in the solid core. Hence:

$$\frac{I^2R \text{ loss per lamination}}{I^2R \text{ loss in solid core}} = \left(\frac{1}{25}\right)^2 \times 5 = \frac{1}{125} \text{ (approx.)}$$

Since there are five laminations,

$$\frac{\text{Total eddy current loss per laminated core}}{\text{Total eddy current loss in solid core}} = \frac{1}{125} = \left(\frac{1}{5}\right)^2$$

It follows that the eddy current loss is approximately proportional to the square of the thickness of the laminations.

Hence the eddy current loss can be reduced to any desired value, but if the thickness of the laminations is made less than about 0.4 mm, the reduction in the loss does not justify the extra cost of construction.

Since the emfs induced in the core are proportional to the frequency and the flux, therefore the eddy current loss is proportional to (frequency × flux)$^2$.

Eddy current loss can also be reduced considerably by the use of silicon-iron alloy and employing conducting material of high resistivity.

The hysteresis and eddy current losses are together known as *core losses* or *iron losses*. For any particular material $B_m$ and $f$ are also nearly constant and does not vary with current. Therefore the core losses are also known as *constant losses* and is independent of the load current.

## 2.31 RISE AND DECAY OF CURRENT IN INDUCTIVE CIRCUIT

Let us consider a circuit (Fig. 2.31) consisting of a battery of emf $E$, a coil of self-inductance $L$ and a resistor $R$. The resistor $R$ may be a separate circuit element or it may be the resistance of the inductor windings. Growth of current by closing switch $S_1$, we connect $R$ and $L$ in series with constant emf $E$. Let $i$ be the current at some time $t$ after switch $S_1$ is closed and $\left(\frac{di}{dt}\right)$ be its rate of change at that time. Applying Kirchoff's law starting at the negative terminal and proceeding counter clockwise around the loop,

$$E - V_{ab} - V_{bc} = 0$$

$$\therefore \quad E - iR - L\frac{di}{dt} = 0$$

or $$E - iR = L\frac{di}{dt}$$

or $$\int_0^t \frac{dt}{L} = \int_0^i \frac{di}{E - iR}$$

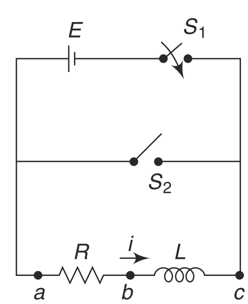



**Fig. 2.31** ***Charging and discharging in inductive circuit***

or $$I = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \tag{2.44a}$$

By letting $\frac{E}{R} = i_0$ and $\frac{L}{R} = \Upsilon$, the above expression reduces to

$$i = i_0\left(1 - e^{\frac{t}{\Upsilon}}\right) \tag{2.44b}$$

and $\Upsilon = L/R$ is called the *time constant* of the L-R circuit.

If $t \to \infty$, then the current $i = i_0 = E/R$. It is also called the *steady state current* or the *maximum current* in the circuit.

At a time equal to one time constant the current has risen to $(1 - e^{-1})$ or about 63% of its final value $i_0$.

The *i-t* graph is as shown in Fig. 2.32.

Note that the final current $i_0$ does not depend on the inductance $L$, it is the same as it would be if the resistance $R$ alone were connected to the source with emf $E$. Let us have an insight into the behaviour of an L-R circuit from energy considerations.

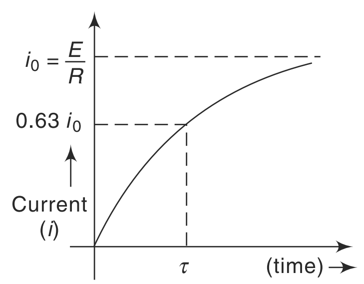


**Fig. 2.32** ***i-t charging characteristic***

The instantaneous rate at which the source delivers energy to the current $P = Ei$ is equal to the instantaneous rate at which energy is dissipated in the resistor ($= i^2R$) plus the rate at which energy is stored in the inductor

$$\frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt} \tag{2.45}$$

Thus $$E \cdot i = i^2R + Li\frac{di}{dt} \tag{2.46}$$

### *Decay of Current*

Now, let us suppose switch $S_1$ in the circuit shown in Fig. 2.33 has been closed for a long time and that the current has reached its steady state value $i_0$. Resetting our stopwatch to reduce the initial value the close switch $S_2$ at time $t = 0$ and at the same time we should open the switch $S_1$ to by-pass the battery. The current through $L$ and $R$ does not instantaneously go to zero but decays exponentially. Applying Kirchhoff's law to find current in the circuit (Fig. 2.33) at time $t$, we can write.

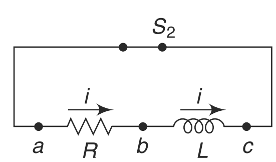


**Fig. 2.33** ***L-R circuit***

$$(V_a - V_b) + (V_b - V_c) = 0$$

or $$i \cdot R + L\frac{di}{dt} = 0 \qquad [\text{as } V_a = V_c]$$

$$\frac{di}{i} = -\frac{R}{L}dt$$

or $$\int_0^0 \frac{di}{i} = -\frac{R}{L}\int_0^t dt$$

or $$i = i_o\, e^{-\frac{t}{\Upsilon}} \tag{2.47}$$

where $\Upsilon \left(= \frac{L}{R}\right)$ is the time for current to decrease to $\left(\frac{1}{e}\right)$ or about 37% of its original value. The current ($i$)—time ($t$) graph for the decaying condition is as shown in Fig. 2.34

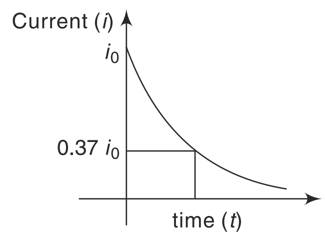



**Fig. 2.34** ***Current decay in an inductive circuit***

The energy that is needed to maintain the current during this decay is provided by the energy stored in the magnetic field. Thus, the rate at which energy is dissipated in the resistor is equal to the rate at which the stored energy decreases in the magnetic field of the inductor.

$\therefore$ $$i^2R = -\frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\left(-\frac{di}{dt}\right)$$

or $$i^2R + Li\frac{di}{dt} = 0$$

or $iR + L\dfrac{di}{dt} = 0$, which confirms when $R$-$L$ circuit is short-circuited the current does not cease to flow immediately (i.e. at $t = 0$) but reduced to zero gradually.

**2.40** The hysteresis loop of a specimen of $5 \times 10^{-4}$ m$^3$ of iron is 12 cm$^2$. The scale is 1 cm = 0.4 Wb/m$^2$ and 1 cm = 400 AT/m. Find out the hysteresis loss, when subjected to an alternating flux density of 50 c/sec.

***Solution***

$$\begin{aligned}\text{Hysteresis loss} &= 12\text{ cm}^2 \times (400\text{ AT/m})\text{ cm}^{-1} \times (0.4\text{ Wb/m}^2)\text{ cm}^{-1}\\ &= 1.92 \times 10^3\text{ Wb/m}^3\\ &= 1.92 \times 10^3 \times 50\text{ J/m}^3\text{/sec}\\ &= 9.6 \times 10^4\text{ J/m}^3\text{/sec.}\end{aligned}$$

It given that volume of the specimen is $5 \times 10^{-4}$ m$^3$.

$$\therefore\quad \begin{aligned}\text{Hysteresis loss} &= 9.6 \times 10^4 \times 5 \times 10^{-4}\text{ J/sec.}\\ &= 48\text{ J/s} = 48\text{ W.}\end{aligned}$$

**2.41** The flux in a magnetic core is varying sinusoidally at a frequency of 600 c/s. The maximum flux density $B_{max}$ is 0.6 Wb/m$^2$. The eddy current loss then is 16 W. Find the eddy current loss in this core, when the frequency is 800 c/sec, and the flux density is 0.5 Wb/m$^2$ (Tesla).

***Solution***

We know, eddy current loss $\propto B_{max}^2 \times f$

$$\text{at 600 c/sec: } P_{e_1} \propto (0.6)^2 \times 600 \tag{i}$$

$$\text{at 800 c/sec: } P_{e_2}\text{(say)} \propto (0.5)^2 \times 800 \tag{ii}$$

Dividing equation (ii) by equation (i) gives:

$$\frac{P_{e_2}}{16} = \frac{(0.5)^2 \times 800}{(0.6)^2 \times 600} \qquad [\because P_{e_1} \text{ is } 16 \text{ W}]$$

$$= 9.259 \times 10^{-1}$$

$$\therefore \quad = 16 \times 9.259 \times 10^{-1} \text{ W}$$

$$= 14.8148 \text{ W.}$$

**2.42** A coil having a resistance of 10 Ω and inductance of 15 H is connected across a d.c. voltage of 150 V. Calculate: (i) The value of current at 0.4 sec after switching on the supply. (ii) With the current having reached the final value the time it would take for the current to reach a value of 9 A after switching off the supply.

***Solution***

It is given that

$$V(d.c) = 150 \text{ V}$$
$$R = 10\ \Omega$$
$$L = 15 \text{ H}$$

(i) ∴ The value of the current

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{150}{10}\left(1 - e^{-\frac{10}{15}\times 0.4}\right)$$

$$= 15\left(1 - e^{-\frac{4}{15}}\right)$$

$$= 3.51 \text{ A}$$

(ii) Let us assume that at $t = t_1$, $i = 9$ A

$$\therefore \quad 9 = 15 \times e^{-\frac{t_1}{1.5}} \qquad \text{(for decaying)}$$

$$e^{-\frac{t_1}{1.5}} = \frac{9}{15}$$

taking $\log_e$ in both sides,

$$-\frac{t_1}{1.5} = \log_e \frac{9}{15}$$

$$\therefore \quad t_1 = 0.7662 \text{ sec.}$$

**2.43** For the network shown in Fig. 2.35

(a) Find the mathematical expression for the variation of the current in the inductor following the closure of the switch at $t = 0$ on to position '*a*';

(b) The switch is closed on to position '*b*' when $t = 100$ ms, calculate the new expression for the inductor current and also for the voltage across *R*;

(c) Plot the current waveforms for $t = 0$ to $t = 200$, variant being over by 50 ms.

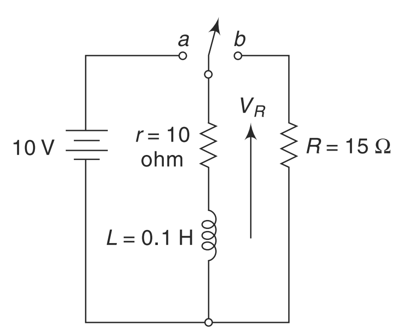



**Fig. 2.35** ***Network of Ex. 2.43***

***Solution***

(a) For the switch in position '*a*', the time constant is

$$\Upsilon_a = \frac{L}{r} = \frac{0.1}{10} = 10 \text{ ms}$$

$$\therefore \qquad i_a = \frac{V}{r}\left(1 - e^{-\frac{t}{\Upsilon_b}}\right) = \frac{10}{10}\left(1 - e^{-\frac{t}{10 \times 10^{-3}}}\right)$$

$$= \left(1 - e^{-\frac{t}{10^{-2}}}\right) \text{A.}$$

(b) For the switch in position '*b*' the time constant

$$\Upsilon_b = \frac{L}{R + r} = \frac{0.1}{15 + 10} = 4 \text{ ms}$$

$$\therefore \qquad i_b = \frac{V}{R} e^{-\frac{t}{\Upsilon_b}} \qquad \text{(for decaying)}$$

$$= \frac{10}{10} e^{-\frac{t}{4 \times 10^{-3}}} = e^{-\frac{t}{4 \times 10^{-3}}} \text{A.}$$

The current continues to flow in the same direction as before, therefore the voltage drop across *R* is negative to the direction of the arrow shown in Fig. 2.35. $v_R = i_b \cdot R = -15 \times e^{-t/4 \times 10^{-3}}$ V.

It will be noted that in the first switched period, five times the time constant is 50 ms. The transient has virtually vanished at the end of this time and it would not have mattered whether the second switching took place then or later. During the second period the transient took only 25 ms.

(c) The profile current waveform has been plotted in Fig. 2.36.

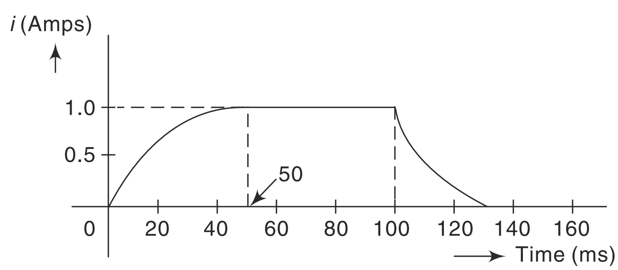



**Fig. 2.36** ***Current profile of circuit in Fig. 2.35***

**2.44** A d.c. voltage of 150 V is applied to a coil whose resistance is 10 Ω and inductance is 15 H. Find: (i) the value of the current 0.3 sec after switching on the supply; (ii) with the current having reached the final value, how much time it would take for the current to reach a value of 6 A after switching off the supply.

***Solution***

(a) It is given that

$V = 150$ V, $R = 10\ \Omega$, $L = 15$ H

∴ The value of the current 0.3 sec after switching on is

$$i = \frac{150}{10}\left(1 - e^{-\frac{10}{15} \times 0.3}\right) = 2.72 \text{ A.}$$

(b) After switching off the supply, the current will be decaying and is given by

$$i = \frac{V}{R} e^{-\frac{R}{L}t} \quad \therefore \quad 6 = \frac{150}{10} e^{-\frac{10}{15} \times t}$$

$\therefore$ $t = 1.375$ sec.

## ADDITIONAL PROBLEMS

**A2.1** Calculate the self-inductance of an air-cored solenoid, 40 cm long, having an area of cross-section 20 cm$^2$ and 800 turns.

*Hints:* $$L = \frac{\mu_o \cdot N^2 \cdot A}{l}$$ [here we assume $\mu_0 = 4\pi \times 10^{-7}$ Hm$^{-1}$].

$$\therefore \quad L = \frac{4\pi \times 10^{-7} \times 800^2 \times 20 \times 10^{-4}}{40 \times 10^{-2}} = 4.022 \times 10^{-3} \text{ H}$$

**A2.2** A horizontal power-line carries a current of 90 A from east to west. Compute the magnetic field generation at a point 1.5 meter below the line.

***Solution***

The magnitude of the magnetic field $\vec{B}$ due to a long current carrying conductor at a distance $R$ is given by

$$B = \frac{\mu_o}{2\pi} \cdot \frac{i}{R}$$

where $$\frac{\mu_o}{2\pi} = 2 \times 10^{-7} \text{ Tm A}^{-1}$$

Putting, $i = 90$ A, $R = 1.5$ m, we have

$$B = (2 \times 10^{-7}) \times \frac{90}{1.5} = 1.2 \times 10^{-5} \text{ T.}$$

Applying Right Hand Rule, we find $\vec{B}$ is directed towards the south.

**A2.3** If the vertical component of the earth's magnetic field be $4.0 \times 10^{-5}$ Wb m$^{-2}$, then what will be the induced potential difference produced between the rails of a meter-gauge running north-south when a train is running on them with a speed of 36 km h$^{-1}$?

***Solution***

When a train is on the rails, it cuts the magnetic flux lines of the vertical component of the earth's magnetic field. Hence, a potential dfifference is induced between the ends of its axle.

Distance between the rails = 1 m; speed of train ($v$) = 36 km/hr = 10 m/sec. Magnetic field $B_V = 4.0 \times 10^{-5}$ Wb/m. $\therefore$ The induced potential difference in $e = Bvl = (4.0 \times 10^{-5}) \times 10 \times 1$ $= 4.0 \times 10^{-9}$ V.

**A2.4** The current in the coil of a large electromagnet falls from 6 A to 2 A in 10 ms. The induced emf across the coil is 100 V. Find the self-inductance of the coil.

***Solution***

The self-induced emf is given by

$$e = -L\frac{di}{dt}$$

Here $di = 2 - 6 = -4$ A

$dt = 10$ ms $= 10^{-2}$ sec

and $e = 100$ V

$$\therefore \quad L = -e\frac{dt}{di} = -100 \times \frac{10^{-2}}{-4} = 0.25 \text{ H.}$$

**A2.5** The current (in ampere) in an inductor is given by $i = 5 + 16t$, where $t$ is in seconds. The self-induced emf in it is 10 mV. Find (a) the self-inductance, and (b) the energy stored in the inductor and the power supplied to it at $t = 1$.

***Solution***

The induced emf in the inductor due to current change is

$$|e| = L\frac{di}{dt}$$

$$\therefore \quad L = \frac{|e|}{di/dt}$$

Hence $i = 5 + 16t$, from this, we have

$$\frac{di}{dt} = 0 + 16 = 16 \text{ A sec}^{-1}, \text{ and } e = 10 \text{ mV} = 10 \times 10^{-3} \text{ V}$$

$$\therefore \quad L = \frac{10 \times 10^{-3} \text{ V}}{15 \text{ A sec}^{-1}} = 0.666 \times 10^{-3} \text{ H} = 0.666 \text{ mH}$$

(b) The current at $t = 1$ sec is

$$i = 5 + 16t = 5 + 16 \times 1$$

$$= 21 \text{ A}$$

$\therefore$ Energy stored in the inductor is

$$\frac{1}{2}Li^2 = \frac{1}{2} \times (0.666 \times 10^{-3}) \times (21)^2$$

$$= 137.8 \times 10^{-3} = 137.8 \text{ mJ}$$

Power supplied to the inductor at $t = 1$ sec is

$$P = ei = (10 \times 10^{-3} \text{ V}) \times 21 = 0.21 \text{ W.}$$

**A2.6** Show that the time for attaining half the value of the final steady current in an $L$-$R$ series circuit is $0.6931L/R$.

***Solution***

The instantaneous current during its growth in an $L$-$R$ series circuit is given by $i = i_0\left(1 - e^{-\frac{R}{L}\cdot t}\right)$

where $i_0$ = final steady current for $i/i_0 = 1/2$, we have

$$\frac{1}{2} = 1 - e^{-\frac{R}{L}\cdot t}$$

$$\therefore \quad e^{-\frac{R}{L}\cdot t} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore \quad e^{-\frac{R}{L}\cdot t} = 2$$

$$\frac{R}{L}\cdot t = \log_e 2 = 0.6931$$

$$\therefore \quad t = 0.6931 \times \frac{L}{R}.$$

**A2.7** An aeroplane with a 20 m wingspread is flying at 250 m/s parallel to the earth's surface at a plane where the horizontal component of the earth's magnetic field is $2 \times 10^{-5}$ Tesla and angle of dip 60°. Calculate the magnitude of the induced emf between the tips of the wings.

***Solution***

As the aeroplane is flying horizontally parallel to the earth's surface the flux linked with it will be due to the vertical component $B_V$ on the earth's field.

$$\therefore \quad B_V = B_H \tan\theta = 2 \times 10^{-5} \times \tan 60°$$

$$= 2\sqrt{3} \times 10^{-5} \text{ Wb/m}^2.$$

$\therefore$ Induced emf is $|e| = B_V\, lv \sin\theta$

$$= 2\sqrt{3} \times 10^{-5} \times 20 \times 250 \times \sin 90°$$

$$\text{or} \quad l = \frac{\sqrt{3}}{10} \text{ V} = 0.173 \text{ V}.$$

**A2.8** Determine the cross-sectional area of the horseshoe electromagnet having flux density of 0.08 Wb/m$^2$ and lifting power of 500 Kg.

***Solution***

Flux density $B = 0.08$ Wb/m$^2$

Lifting power of each pole of electromagnet $F = \frac{500}{2}$ Kg. $= 250 \times 9.81 = 2452.5$ N

$$\mu_0 = 4\pi \times 10^{-7}$$

Now $F = \frac{B^2 A}{2\mu_0}$ where $A$ is the cross

Sectional area of each pole

$$\therefore \quad A = \frac{2\mu_0 F}{B^2} = \frac{2 \times 4\pi \times 10^{-7} \times 2452.5}{(0.08)^2} \text{ sq. m}$$

$$= 0.963 \text{ sq. m}$$

Hence the cross-sectional area of each pole of the electromagnet is 0.963 sq.m

**A2.9** A coil has a resistance of 5 Ω and an inductance of 1 H. At $t = 0$ it is connected to a 2 V battery. Find (a) the rate of rise of current at $t = 0$; (b) the rate of rise of current when $i = 0.2$ Amps and (c) the stored energy when $i = 0$ and $i = 0.3$ A.

***Solution***

$$\Upsilon = \frac{L}{R} = \frac{1}{5} = 0.2 \text{ sec.}$$

(a) $\frac{di}{dt} = \frac{E}{L} e^{-\frac{t}{\Upsilon}} = 2e^{-5t}$

at $t = 0$, $\frac{di}{dt} = 2$ A/sec.

(b) $i = \frac{E}{R}(1 - e^{-5t})$

$= 0.4(1 - e^{-5t})$

time $t_1$ when $i = 0.2$ A is

$$0.2 = 0.4\,(1 - e^{-5t_1})$$

$$\text{or} \quad t_1 = 0.1386 \text{ sec.}$$

$$\frac{di}{dt} = 2e^{-5(0.1386)}$$
$$= 1 \text{ A/sec}$$

(c) At $i = 0$, stored energy = 0

when $i = 0.3$ A, stored energy

$$= \frac{1}{2} Li^2 = \frac{1}{2} \times 1 \times (0.3)^2 = 0.045 \text{ J.}$$

**A2.10** An iron ring of circular cross-section of $5 \times 10^{-4}$ m$^2$ has a mean circumference of 2 m. It has a saw-cut of $2 \times 10^{-3}$ m length and is wound with 800 turns of wire. Determine the exciting current when the flux in the air gap is $0.5 \times 10^{-3}$ Wb. [given: $\mu_r$ of iron = 600 and leakage factor is 1.2] Assume areas of air gap and iron are same.

***Solution***

The flux linking with the iron ring is

$$\phi_{\text{iron}} = \phi_{\text{air-gap}} \times \text{Leakage factor}$$
$$= 0.5 \times 10^{-3} \times 1.2 \qquad \text{[As the leakage factor is given as 1.2]}$$
$$= 0.6 \times 10^{-3} \text{ Wb.}$$

Again we know,

Ampere turns required = $NI$

$$= \left[\frac{\phi_{\text{iron}} \times l_{\text{iron}}}{\mu_r \times \mu_o \text{ of iron} \times \text{area}} + \frac{\phi_{\text{air-gap}} \times l_{\text{air}}}{\mu_o \text{ of air} \times \text{area}}\right] \qquad [\because \mu_r \text{ for air} = 1]$$

$$\therefore \quad I = \frac{1}{800}\left[\frac{0.6 \times 10^{-3} \times 2}{4 \times \pi \times 10^{-7} \times 600 \times 5 \times 10^{-4}} + \frac{0.5 \times 10^{-3} \times 2 \times 10^{-3}}{4\pi \times 10^{-7} \times 5 \times 10^{-4}}\right]$$

$$= 5.95 \text{ A.}$$

**A2.11** For the network shown in Fig. 2.35 (Ex. No. 2.43) the switch is closed on the position '$a$'. Next, it is closed on to position '$b$' when $\tau = 10$ ms. Again, find the expression of current and hence draw the current waveforms.

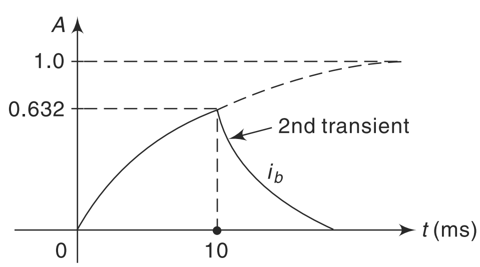



**Fig. 2.37** ***Current decay profile***

***Solution***

For the switch in position '$a$', the time constant ϒ is 10 ms as in Ex. No. 2.43, and the current expression is as before. However, the switch is moved to position '$b$' while the transient is proceeding. When $t = 10$ ms

$$i = \left(1 - e^{-\frac{t}{10 \times 10^{-3}}}\right) = \left(1.0 - e^{\frac{-10 \times 10^{-3}}{10 \times 10^{-3}}}\right)$$

$= (1 - e^{-1}) = 0.632$ A.

i.e., the second transient commences with an initial current in $R$ of 0.632 A.

$\therefore$ The current decay is, $i_b = 0.632 \times \dfrac{t}{4 \times 10^{-3}}$ A. which is shown in Fig. 2.37.

**A2.12** A coil of resistance 24 Ω and having inductor 36 H is suddenly connected to a d.c. of 60 V supply. Determine

(a) the initial rate of change of current $\left(\dfrac{di}{dt}\right)$

(b) the time-constant

(c) the current after 3 sec.

(d) the enrgy stored in the magnetic field at $t = 3$ sec.

(e) the energy lost as heat energy at $t = 3$ sec.

***Solution***

It is given that: $V = 60$ V, $R = 24\ \Omega$, $L = 36$ H

(a) Initial rate of change of current:

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}\cdot t}\right)$$

$$\therefore \quad \frac{di}{dt} = -\frac{V}{R}\cdot\left(-\frac{R}{L}\right)\cdot - e^{-\frac{R}{L}\cdot t}$$

$$= \frac{V}{L}e^{-\frac{R}{L}\cdot t}:$$

When $t = 0$,

$$\frac{di}{dt} = \frac{V}{L}\cdot e^{o} = \frac{V}{L} = \frac{60}{36} = 1.67 \text{ A/sec:}$$

(b) Time constant (ϒ):

$$\Upsilon = \frac{L}{R} = \frac{36}{24} = 1.5 \text{ sec.}$$

(c) Current; the current at $t = 3$ sec is

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}\cdot t}\right) = \frac{60}{24}\left(1 - e^{-\frac{24}{36}\times 3}\right) = 2.16 \text{ A.}$$

(d) Energy stored:

at $t = 3$ sec, the energy stored in the magnetic field is $\frac{1}{2}Li^2$

$$= \frac{1}{2}\times 36 \times (2.16)^2 = 84 \text{ J.}$$

(e) Energy lost as heat energy:

at $t = 3$ sec, the energy lost as heat energy is

$$i^2 \times R = (2.16)^2 \times 24 \cong 112 \text{ J.}$$

**A2.13** A horse shoe electromagnet of 50 cm length and cross sectional area 10 cm$^2$ is wound with 500 turns of exciting coil. Assuming that a load of 80 Kg. is making close contact with the magnet determine the exciting current necessary to lift the load. Relative permeability of iron is 700.

***Solution***

Cross-sectional area of each pole $A = 10\text{ cm}^2 = 0.001$ sq.m

Force of attraction of each pole $F = \frac{80}{2}$ kg. $= 40 \times 9.81$ N $= 392.4$ N

Now $F = \frac{B^2 A}{2\mu_0}$

or, $$B = \sqrt{\frac{2F\mu_0}{A}} = \sqrt{\frac{2 \times 39.2 \times 4\pi \times 10^{-7}}{0.001}} = 0.314 \text{ Wb/m}^2$$

Magnetizing force $H = \frac{B}{\mu} = \frac{0.314}{4\pi \times 10^{-7} \times 700} = 357.14$ AT/m

Required m mf is $Hl = 357.14 \times 50 \times 10^{-2} = 178.57$ AT

Hence necessary exciting current

$$I = \frac{178.57}{500} \text{ A} = 0.357 \text{ A.}$$

**A2.14** A horseshoe type relay requires an excitation of 3000 AT to raise an armature when the air gap between the relay and the armature is 3 mm. Determine the force on the armature if each pole shoe has cross sectional area of 5 cm$^2$, relative permeability of iron is 300 and length of the iron path is 80 cm.

***Solution***

Total mmf = 3000 AT

$\therefore$ 3000 = AT for iron part + AT for air gap

or, $$3000 = \frac{B}{\mu_0\,\mu_r} l_i + \frac{B}{\mu_0} l_g = \frac{B}{4\pi \times 10^{-7}} \left\{ \frac{0.8}{300} + \frac{0.003}{1} \right\}$$

$$= 4511.677 \text{ B}$$

or, $$B = \frac{3000}{4511.677} = 0.665 \text{ Wb/m}^2$$

Force of attraction of each pole = Force on armature due to each pole

$$F = \frac{B^2 A}{2\mu_0} = \frac{(0.665)^2 \times 5 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} = 88 \text{ N}$$

$\therefore$ Total force on the armature = 2 × 88 = 176 N

## HARDER PROBLEMS

**H2.1** Determine the mutual inductance and co-efficient of coupling if two coils with 10, 000 turns and 15,000 turns lie in parallel planes in such a way that 70% of the flux produced in coil 1 links coil 2. If is given that a current of 3 A in coil 1 produces a flux of 0.8 m Wb in it while a current of 5 A in coil 2 produces flux of 1 m Wb.

***Solution***

Flux per ampere in coil 1 $\frac{\phi_1}{I_1} = \frac{0.8 \times 10^{-3}}{3}$ Wb/A

$= 0.267$ m Wb/A

Flux linked with coil 2 is $\frac{70}{100} \times 0.267 = 0.187$ m Wb

∴ mutual inductance $M = 15000 \times 0.187 \times 10^{-3}$

$= 2.805$ H

Now self-inductance of coil 1

$$L_1 = \frac{10{,}000 \times 0.8 \times 10^{-3}}{3} \text{ H} = 2.67 \text{ H}$$

Self-inductance in coil 2

$$L_2 = \frac{15000 \times 1 \times 10^{-3}}{5} = 3 \text{ H}$$

∴ Coefficient of coupling $K = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.805}{\sqrt{2.67 \times 3}}$

$= 0.99$

**H2.2** An iron ring made up of three parts, $l_1$ = 12 cm, $a_1$ = 6 cm$^2$; $l_2$ = 10 cm, $a_2$ = 5 cm$^2$, $l_3$ = 8 cm and $a_3$ = 4 cm$^2$. It is surrounded by a coil of 200 turns. Determine the exciting current required to create a flux of 0.5 m Wb in the iron ring. [Given $\mu_1$ = 2670, $\mu_2$ = 1055, $\mu_3$ = 680.]

***Solution***

Total reluctance $S = S_1 + S_2 + S_3$

$$= \sum_{\mu_r \frac{l}{a} = 1}^{3} \frac{l}{\mu_o \mu_r a} = \frac{l_1}{\mu_o \mu_r a_1} + \frac{l_2}{\mu_o \mu_r a_2} + \frac{l_3}{\mu_o \mu_r a_3}$$

$$= \frac{1}{4\pi \times 10^{-7}} \left[ \frac{0.12}{2670 \times 6 \times 10^{-4}} + \frac{0.1}{1055 \times 5 \times 10^{-4}} + \frac{0.08}{680 \times 4 \times 10^{-4}} \right]$$

$$= \frac{1}{4\pi \times 10^{-7}} [0.074906 + .189573 + 0.294117]$$

$= 4.445 \times 10^5$ AT/Wb.

∴ $\text{Flux } (\phi) = \frac{\text{mmf}}{\text{reluctance}} = \frac{NI}{4.445 \times 10^5}$

∴ $I = \frac{\text{flux} \times 4.45 \times 10^5}{N}$

$$= \frac{0.5 \times 10^{-3} \times 4.45 \times 10^5}{200}$$

$= 1.11125$ A

$= 1111.25 \times 10^{-3}$ Amps = 1111.25 mA.

**H2.3** An iron ring of 100 cm mean circumference is made from round iron of cross-section 10 cm$^2$. It's relative permeability is 500. Now a saw-cut of 2 mm wide has been made on it. It is wound with 200 turns. Determine the new current required to produce a flux of $0.12 \times 10^{-2}$ Wb in the air-gap, given that the leakage factor $x$ is 1.24, and that the relative permeability of the iron under the new condition is 350.

***Solution***

Given:

$$\phi_{\text{air-gap}} = 1.2 \times 10^{-3} \text{ Wb}$$
$$l_{\text{air-gap}} = 0.2 \times 10^{-2} = 2 \times 10^{-3} \text{ m}$$
$$a_{\text{air-gap}} = 10^{-3} \text{ m}^2$$
$$\mu_{\text{air}} = 1.$$

∴ Reluctance in the air-gap is

$$\frac{2 \times 10^{-3}}{(4\pi \times 10^{-7} \times 1.2 \times 10^{-3})} = 1.325 \times 10^6 \text{ AT/Wb}$$

$$\phi_{\text{iron-path}} = 1.25 \times 1.2 \times 10^{-3} \text{ Wb}$$
$$= 1.5 \times 10^{-3} \text{ Wb}$$
$$l_{\text{iron-path}} = 0.998 \text{ m}$$
$$a_{\text{iron-path}} = 10^{-3} \text{ m}^2$$
$$\mu_{\text{iron-path}} \text{ (new)} = 350.$$

∴ Reluctance in the iron-path $= \dfrac{998 \times 10^{-3}}{(350 \times 4\pi \times 10^{-7} \times 1.2 \times 10^{-3})}$

$$= 1890912.3 \text{ AT/Wb} = 1.89 \times 10^6 \text{ AT/Wb.}$$

As we cannot add up the values of air-gap reluctance and iron-path reluctance to get the total reluctance, we therefore calculate in this way.

$$H = H_{\text{air-gap}} + H_{\text{iron-path}}$$
$$= (\text{Reluctance of air-gap} \times \text{Flux in this path})$$
$$+ (\text{Reluctance of iron-path} \times \text{Flux in this path})$$
$$= (1.325 \times 10^6 \times 1.2 \times 10^{-3}) + (1.89 \times 10^6 \times 1.5 \times 10^{-3})$$
$$= 4.425 \times 10^3 = 4425$$

∴ $$\text{Current } (= I) = \frac{4{,}425}{200} = 22.125 \text{ A.}$$

**H2.4** Two coils with terminals $T_1$, $T_2$ and $T_3$, $T_4$ respectively are placed side by side. Measured separately, the inductance of the first is 1200 $\mu$H and that of the second coil is 800 $\mu$H. With $T_2$ joined with $T_3$ (Fig. 2.38), the total inductance between the two coils is 2500 $\mu$H. What is the mutual inductance? If $T_2$ is joined with $T_4$ instead of $T_3$, what would be the value of equivalent inductance of the two coils?

***Solution***

Given $L_1 = 1200\ \mu\text{H}$, $L_2 = 800\ \mu\text{H}$, $T_{14} = 2500\ \mu\text{H}$.

Let the mutual inductance between the two coils be $M$, then total inductance $L_1 + L_2 + 2M$. In the first case (refer (Fig. 2.38)

$$T_{14} = L_1 + L_2 + 2M$$
$$2500 = 1200 + 800 + 2M$$

∴ $$M = \frac{500}{2} = 250\ \mu\text{H.}$$

If $T_{13}$ is the total inductance in the second case, then

$$T_{13} = L_1 + L_2 - 2M \qquad \text{(See Fig. 2.39)}$$
$$= 1200 + 800 - 2 \times 250$$
$$= 1500\ \mu\text{H}.$$

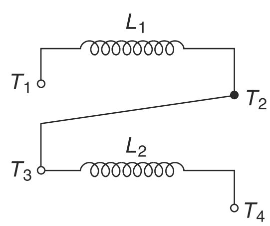



**Fig. 2.38** ***Connection of two coils, 1st case***

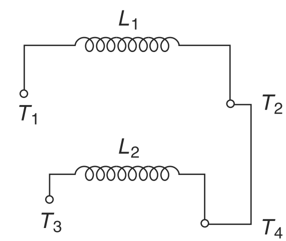



**Fig. 2.39** ***Connection of two coils, 2nd case***

**H2.5** A solenoid of inductance $L$ and resistance $R$ is connected to a battery. Prove that the time ta.ken for the magnetic energy to reach 1/4 of its maximum value is $L/R \log_e(2)$.

***Solution***

The growth of current in an $LR$ circuit is given by

$$I = I_0\left(1 - e^{-\frac{R}{L}\cdot t}\right) \qquad \text{(i)}$$

where $I_0$ is the maximum current. The energy stored at time $t$ is

$$u = \frac{1}{2}LI^2$$

We are required to find the time at which the energy stored is 1/4 the maximum value, i.e., when $u = \frac{\mu_o}{4}$ where $u_o = \frac{1}{2}LI^2$.

i.e., $$\frac{1}{2}LI^2 = \frac{1}{4}\left(\frac{1}{2}LI_0^2\right) \quad \text{or} \quad I = \frac{I_0}{2}$$

$\therefore$ Using the equation 1, we have

$$\frac{I_0}{2} = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\frac{1}{2} = 1 - e^{-\frac{R}{L}\cdot t}$$

or $$e^{-\frac{R}{L}t} = \frac{1}{2}$$

or $$-\frac{R}{L}t = \log_e\left(\frac{1}{2}\right) = -\log_e(2)$$

$\therefore$ $$t = \frac{L}{R}\log_e(2)$$

**H2.6** The resistors of 100 Ω and 200 Ω and an ideal inductance of 10 H are connected to a 3-V battery through a key $K$, as shown in the Fig. 2.40.

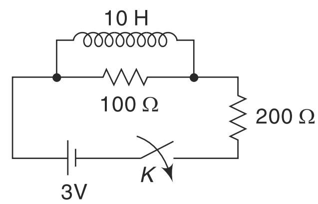



**Fig. 2.40** ***Circuit of Ex. H2.6***

If $K$ is closed at $t = 0$, calculate

(a) the initial current drawn from the battery
(b) the initial potential drop across the inductance
(c) the final current drawn from the battery
(d) the final current through 100 Ω resistance.

***Solution***

(a) 'Immediately' after closing $K$, there is almost no current in the inductance due to self-induction, the current is only in resistance. Thus, the initial current is

$$i = \frac{3V}{(100 + 200)\ \Omega} = 0.01\text{ A}$$

(b) The p.d. across the inductor is same as across the 100 Ω resistance, that is $0.01 \times 100 = 1$ V.

(c) When the current has become steady the opposing emf in the inductance is zero and it short circuits the 100 Ω resistance. The resistance of the circuit is now only 200 Ω and so the current drawn from the cell is $3V/200\ \Omega = 0.015$ A.

This is the final current in the 200 Ω resistance.

(d) The final current through the 100 Ω is zero.

**H2.7** A small flat coil of area $2.0 \times 10^{-4}$ m$^2$ with 25 closely wound turns is placed with its plane perpendicular to a magnetic field. When the coil is suddenly withdrawn from the field, a charge of 7.5 mc flows through the coil. The resistance of the coil is 0.50 Ω. Estimate the strength of the magnetic field.

***Solution***

The magnetic flux passing through each turn of coil of area $A$, perpendicular to a magnetic field $B$ is given by

$$\phi_B = BA$$

when the coil is withdrawn from the field, the flux through it vanishes. Therefore, the change in flux is $d\phi_B = 0 - BA = -BA$.

By Faraday's law, the emf induced in the coil is

$$e = -N \cdot \frac{d\phi_B}{dt} = \frac{NBA}{dt}$$

where $dt$ is the time taken in withdrawal. The induced current in the coil of resistance (say $R$) is

$$i = \frac{e}{R}.$$

This current persists only during the time interval $dt$. Hence the charge flowed through the coil is $q = i \times dt = \frac{e}{R} \cdot dt = \frac{NBA}{R}$.

$$\therefore \qquad B = \frac{qR}{NA}.$$

Substituting the given values, we have

$$B = \frac{(7.5 \times 10^{-3}) \times 0.50}{25 \times (2.0 \times 10^{-4})} = 0.75 \text{ Wb/m}^2.$$

**H2.8** A rectangular iron-core is shown below in Fig. 2.41. It has a mean length of magnetic path of 100 cm, cross-section of (2 cm × 2 cm), relative permeability of 1400 and an air-gap of 5 mm is cut in the core. The three coils carried by the core have number of turns $N_1 = 335$, $N_2 = 600$ and $N_3 = 600$, and the respective currents are $I_1 = 1.6$ A, $I_2 = 4.0$ A and $I_3 = 3.0$ A. The directions of the currents are as shown. Calculate the flux in the air-gap.

***Solution***

The mmf acting in the magnetic circuit (current considering in the clockwise direction)

$$= \Sigma NI = -335 \times 1.6 + 600 \times 4 - 600 \times 3 = 64 \text{ AT}$$

$$\therefore \qquad 64 \text{ AT} = \frac{\phi}{\mu_o A}\left[\frac{l_i}{\mu_r} + l_g\right],$$

where $l_i$ = mean length, $\mu_r$ = relative permeability and

$l_g$ = air-gap cut length.

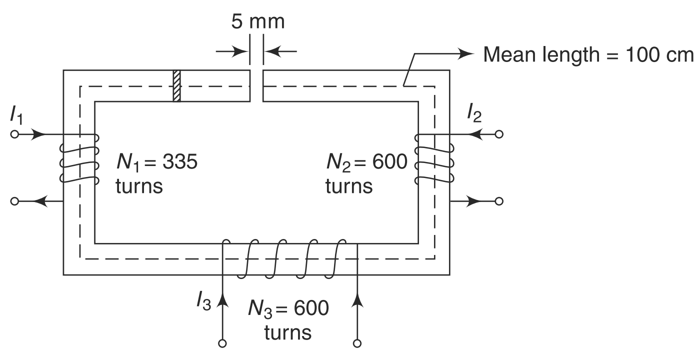



**Fig. 2.41** ***A rectangular iron-cone (Ex. H2.8)***

$$\therefore \qquad 64 = \frac{\phi}{4\pi \times 10^{-7} \times (2 \times 2) \times 10^{-4}}\left[\frac{1}{1400} + 5 \times 10^{-3}\right]$$

$$= 1.136 \times 10^7 \, \phi$$

$$\therefore \qquad \text{Flux, } \phi = \frac{64}{1.136 \times 10^7} = 5.63 \times 10^{-6} \text{ Wb}$$

$$= 5.63 \, \mu\text{Wb}$$

**H2.9** Determine the mutual inductance if the current changing at the rate of 1000 A/s in coil 1 induces an emf of 10 V in coil 2, both coils being identical with 800 turns and lying in parallel planes. If the self-inductance of each coil is 0.02 H find the flux in coil 1 per ampere and percentage of this flux linking coil 2.

***Solution***

The emf induced in the secondary coil, i.e., coil 2

$$e_M = M\frac{dI_1}{dt} = 10 \text{ V}$$

∴ mutual inductance

$$M = \frac{10}{\frac{dI_1}{dt}} = \frac{10}{1000} = 0.01 \text{ H}$$

Self-inductance of each coil

$$L = L_1 = L_2 = \frac{N_1\,\phi_1}{I_1} = 0.02 \text{ H}$$

∴ Flux in coil 1 per ampere

$$\frac{\phi_1}{I_1} = \frac{0.02}{800} = 2.5 \times 10^{-5} \text{ Wb/A}$$

Percentage of flux of coil 1 linking coil 2 is

$$\frac{M}{\sqrt{L_1\,L_2}} = \frac{0.01}{\sqrt{L_2}} = \frac{0.01}{0.02} = 0.5 \text{ or } 50\%$$

**H2.10** A magnetic circuit shown in Fig. 2.42 is constructed of wrought iron: The cross-section of the central limb is 6 cm$^2$, and each outer limb is 4 cm$^2$. If the coil is wound with 500 turns, determine the exciting current required to set up a flux of 1.0 m Wb in the central limb.

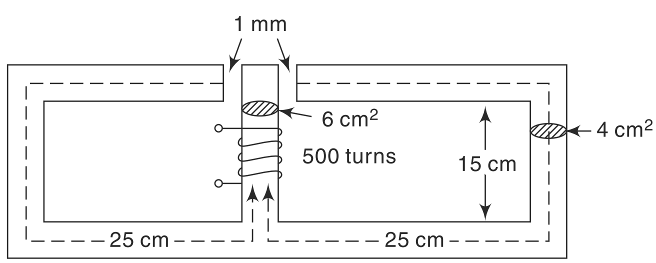



**Fig. 2.42** ***Magnetic circuit of Ex. H2.10***

B-H curve of wrought iron are:

| | | |
|---|---|---|
| $B$ (Wb/m$^2$) | 1.25 | 1.67 |
| $H$ (AT/m) | 600 | 2,100 |

***Solution***

Given that Flux ($\phi_1$) in the central limb = $1.0 \times 10^{-3}$ Wb

Area ($a_1$) of the central limb = $6 \times 10^{-4}$ m$^2$

$l_1$ = 15 cm = 0.15 m

$$B_1 = \frac{\phi_1}{a_1} = \frac{1.0 \times 10^{-3}}{6.0 \times 10^{-4}} = 1.667 \text{ Wb/m}^2$$

$\therefore$ AT required $= H_1 l_1 = 2100 \times 0.15$

$= 315$ AT

For outer limb flux $(\phi_2) = \frac{1}{2} \times 1.0 \times 10^{-3}$ Wb

Area $(a_2) = 4 \times 10^{-4}$ m$^2$,

Length $l_2 = 25$ cm $= 0.25$ m

$$\therefore \quad B_2 = \frac{1/2 \times 1.0 \times 10^{-3}}{4 \times 10^{-4}} = 1.25 \text{ Wb/m}^2$$

From B-H curve, $H_2 = 600$ AT/m

$\therefore$ AT required $= H_2 l_2 = 600 \times 0.25$

$= 150$ AT

Air-gap $A_g = B_g = 1.25$ Wb/m$^2$

$l_g = 1 \times 10^{-3}$ m

$$\therefore \text{ AT required} = \frac{B_g \cdot l_g}{\mu_o} = \frac{1.25 \times 1 \times 10^{-3}}{4\pi \times 10^{-7}} = 994.45 \text{ AT}$$

$\therefore$ Total AT required

$= 315 + 150 + 994.45$

$= 1,459.45$ AT.

$\therefore$ The exciting current $I$

$$= \frac{NI}{N} = \frac{1459.45}{500} = 2.92 \text{ A}$$

## EXERCISES

1. Define mmf, reluctance and permeability. Deduce an expression for the force between two parallel conductors. Explain the significance of reluctance in a magnetic circuit.
2. Compare magnetic and electric circuits.
3. What are the different types of magnetic losses? What is the eddy-current loss? What are the considerable effects of eddy currents? How can they be minimised? Mention some application of eddy currents. How is the different types of losses to be minimised?
4. Explain the terms: magnetic leakage and flux fringing. Derive an expression for the weight which can be lifted by a horse-shoe magnet.
5. Why do magnetic circuits usually have air-gaps? How does the presence of air-gaps affect the magnetic circuit calculations which has higher reluctance an air-gap or an iron path? And why? Prove that $B = \mu$H.
6. Draw and explain the *B-H* curves for air and a magnetic material.
7. Explain with the aid of a typical *B-H* curve the meaning of the following terms:

Relative permeability, coercivity, and remanence.

What information can be derived from the *B-H* loop?

What is meant by magnetic hysteresis?

8. Explain briefly under what conditions it is advantageous to use in a magnetic circuit:
   (a) a granulated iron core
   (b) a laminated iron core.
9. An air-cooled solenoid has a diameter of 30 cm and a length of 5070 cm and is wound with 3000 turns. If a current of 6 A flow in the solenoid find the energy stored in its magnetic field.

[Hints: Calculate $A = \frac{\pi D^2}{4} = \frac{\pi \times 9 \times 10^{-4}}{9}$

and $L = \frac{\mu_o \cdot \mu_r \cdot N^2 A}{l}$,

have $\mu_o = 4\pi \times 10^{-7}$ $\mu_r = 1$

$\therefore$ Energy stored $= E = \frac{1}{2} LI^2$ J.] [*Ans:* 2875 J]

10. An air cored torodial coil has 450 turns and a mean diameter of 300 mm and a cross-sectional area of 300 mm$^2$. Determine the self-inductance of the coil and the average voltage induced in it when a current of 2 A is reversed in 40 m.sec. [*Ans:* 8.1 mH; 8.1 mV]

[Hint: $L = \frac{\mu_o \mu_r AN^2}{l} = \frac{4\pi \times 10^{-7} \times 1 \times 300 \times 10^{-6} \times (450)2}{\pi \times 300 \times 10^{-3}}$ H

$= 0.81 \times 10^{-4}$ H.

$\text{emf} = L\frac{di}{dt} = 0.81 \times 10^{-4} \times \frac{2+2}{40 \times 10^{-3}} = 0.0081$ V.]

11. Two identical coils, having 1000 turns each, lie in parallel planes such that 90% flux produced by one coil links with the other. If a current of 4 A flowing in one coil produces a flux of 0.05 m Wb in it, find the magnitude of mutual inductance between the two coils. [*Ans:* 11.25 mH]
12. A solenoid of length 1 meter, and diameter 10 cm has 5000 turns. Calculate: (i) the approximate inductance, and (ii) the energy stored in a magnetic field when a current of 2 A flows in the solenoid.

[*Ans:* $L = 0.247$ H; Energy stored = 0.493 J]

13. The coils having 150 and 200 turns respectively are wound side by side on a closed magnetic circuit of cross-section $1.5 \times 10^{-2}$ m$^3$ and mean length 3 m. The relative pearmeability of the magnetic circuit is 2000. Calculate (a) the mutual inductance between the coils; (b) the voltage induced in the second coil if the current changes from 0 to 10 A in the first coils is 20 m.sec.

[*Hints:* $N_1 = 150$, $N_2 = 200$

$a = 1.5 \times 10^{-2}$, $l = 3$ m, $\mu_r = 2000$

(a) $M = \mu_o \cdot \mu_r \cdot N_1 \cdot N_2 \cdot \dfrac{a}{l} = 0.377$ H

(b) $di_1 = 10 - 0 = 10$ A.

$dt = 20$ ms $= 20 \times 10^{-3}$ sec.

$\therefore \quad e_2 = M \cdot \dfrac{di_1}{dt} = 188.5$ V]

14. Two coils of negligible resistance and of self-inductances 0.2 H and 0.1 H, are connected in series. If the mutual inductance is 0.1 H, calculate the effective inductance of the combination. [*Ans:* 0.5 H or 0.1 H]

15. Two coils *A* and *B*, each with 100 turns, are mounted so that part of the flux set up by one links the other. When the current through coil *A* is changed from +2*A* to –2*A* in 0.5 second, an emf of 8 mV is induced in coil *B*, calculate: (i) the mutual inductance between the coils, and (ii) the flux produced in coil *B* to 2*A* in coil *A*. [*Ans:* $M = 1$ mH; $\phi = 200$ m wb]

[*Hints:* $N_A = N_B = 100$ turns

$I_A = 2A$, $dI_A = 2 - (-2) = 4$ A

$dt = 0.5$ sec, $e_M = 8 \times 10^{-3}$ V

(i) Now $e_M = M \cdot \dfrac{di_A}{dt}$, $M = 1 \times 10^{-3}$ H

(ii) Flux induced in $B = \phi_B = \dfrac{MI_A}{N_B} = 2 \times 10^{-5}$ Wb.

16. A coil has 100 turns of wire, and a flux of 5 m Wb linkage with this coil changes to zero in 0.05 second. Determine the self-induced emf in the coil. [*Ans:* 10 V]

17. Two long single-layer solenoids have the same length and the same number of turns but are placed coaxially one within the other. The diameter of the inner coil is 60 mm and that of the outer coil is 75 mm. Determine the coefficient of coupling between the coils. [*Ans:* 0.8]

18. A coil has a self-inductance of 1 H. If a current of 25 mA is reduced to zero in a time of 12 ms, find the average value of the induced voltage across the terminals of the coil. [*Ans:* 2.08 V]

19. A conductor of length 300 cm moves at an angle of 30° to the direction of uniform magnetic field of strength 2.0 Wb/$m^2$ with a velocity of 100 m/sec. Calculate the emf induced. What will be the emf induced if the conductor moves at right angles to the field?

[*Hints:* (i) $e = Blv \sin 30° = 300$ V;

(ii) $e = Blv \sin 90° = 600$ V.]

20. A conductor having a length of 80 cm is placed in a uniform magnetic field of 2 Wb/$m^2$ (Tesla). If the conductor moves with a velocity of 50 m/sec, find the induced emf when it is (i) at right angle (ii) at an angle of 30° and (iii) parallel to the magnetic field. [*Ans:* 80 V, 40 V and 0 V]

[*Hint:* $B = 2$ Wb/$m^2$, l = 0.8 m, v = 50 m/sec.

(i) $e = Blv = 2 \times 0.8 \times 50 = 80$ V [$\because \sin\theta = \sin 90° = 1$]

(ii) $e = 80 \sin 30° = 40$ V

(iii) $e = 80 \sin 0° = 0$.]

21. Calculate the mmf required to produce a flux of 0.01 Wb across an air-gap of 2 mm. of length having an effective area of 200 $cm^2$ of a wrought iron ring of mean iron path of 0.5 m and cross-sectional area of 125 $cm^2$. Assume a leakage co-efficient of 1.25. The magnetization curve of the wrought iron is given below:

| | | | | | |
|---|---|---|---|---|---|
| B ($Wb/m^2$): | 0.6 | 0.8 | 1 | 1.2 | 1.4 |
| H (AT/m): | 75 | 125 | 250 | 500 | 1000 |

[*Ans:* 921.18 AT]

[*Hint:* $\phi = 0.01$ Wb

*Air-gap:* $$H = \frac{B}{\mu_o} = \frac{\phi}{A \cdot \mu_o} = \frac{0.01}{200 \times 10^{-4} \times 4\pi \times 10^{-7}} \text{ AT/m}$$

$$= 3.98 \times 10^5 \text{ AT/m.}$$

Total AT required = $3.98 \times 10^5 \times 2 \times 10^{-3} = 796.178$

*Iron path:* $$B = \frac{0.01 \times 1.25}{125 \times 10^{-4}} \text{ Wb/m}^2 = 1 \text{ Wb/m}^2$$

$\therefore$ $H = 250$ AT/m

Total AT required = $250 \times 0.5 = 125$

Total mmf = 796.178 + 125 = 921.178]

22. (a) An iron ring, having a mean diameter of 75 cm and a cross-sectional area of 5 $cm^2$ is wound with a magnetizing coil of 120 turns. Using the following data, calculate the current required to set-up a magnetic flux of 630 $\mu$ Wb in the ring.

| | | | | |
|---|---|---|---|---|
| Flux density ($T$) | 0.9 | 1.1 | 1.2 | 1.3 |
| A/m | 260 | 450 | 600 | 820 |

[*Ans:* 14.4 A]

(b) The air gap in a magnetic circuit is 1.1 m long and 20 $cm^2$ in cross-section. Calculate (i) the reluctance of the air-gap and (ii) the ampere turns required to send a flux of 700 $\mu$ Wb across the air-gap.

[*Ans:* (b) $4.375 \times 10^5$ A/Wb, 306.25 AT]

23. A mild steel having a cross-sectional area of 10 $cm^2$ and a mean circumference of 60 cm has a coil of 300 turns wound around it. Determine

(i) reluctance of the steel ring,

(ii) current required to produce a flux of 1 m.Wb. in the ring. Relative permeability of the given steel is 400 at the flux density developed in the core.

(iii) if a slit of 1 mm. is cut in the ring, what will be the new value of current? Assume no fringing effect.

[*Ans:* $119.43 \times 10^4$ AT/Wb; 3.98 A; 6.635 A]

[*Hint:* (i) Reluctance $= \dfrac{l}{\mu_o \, \mu_r \, a}$

$$= \frac{0.6}{4\pi \times 10^{-7} \times 400 \times 10 \times 10^{-4}} \text{ AT/Wb}$$

$$= 119.426 \times 10^4 \text{ AT/Wb}$$

(ii) AT = $1 \times 10^{-3} \times 119.426 \times 10^4 = 1194.26$

$$I = \frac{1194.26}{300} \text{ A} = 3.98 \text{ A}$$

(iii) Reluctance of iron path $= \dfrac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 10 \times 10^{-4}}$ AT/Wb

$$= 79.62 \times 10^4 \text{ A/Wb}$$

AT for air gap = $79.62 \times 100 \times 1 \times 10^{-3} = 796.2$.

$$\therefore \quad I = \frac{1194.26 + 796.2}{300} = 6.635 \text{ A.}]$$

24. A magnetic circuit has an area of 10 cm$^2$ and length of 0.2 m. If $\mu_r = 2000$, find the reluctance. [*Ans:* $0.79 \times 10^5$ A/Wb]

25. Find out the inductance and energy stored in the magnetic field of an air-cored solenoid of 100 cm long, 10 cm in diameter and some wound with 900 turns and a current of 7.5 A is passing through the solenoid.

[*Ans:* $L = 8$ mH; $E = 0.225$ J]

[*Hints:* Inductance of solenoid

$$L = \frac{N^2 \cdot A \cdot \mu_o \cdot \mu_r}{l} = 0.008\ H$$

and energy stored = $E = \dfrac{1}{2} LI^2 = 0.225$ J]

26. A circuit has 2000 turns enclosing a magnetic circuit of 30 cm$^2$ sectional area. When with 5 A, the flux density is 1.2 Wb/m$^2$ and 10 A it is 1.7 Wb/m$^2$, find the mean value of the inductance between these current limits and the induced emf if the current falls from 10 A to 4 A in 0.08 sec.

[*Hints:* $L = N \cdot \dfrac{d\phi}{dt} = N \cdot \dfrac{d(BA)}{dt} = NA\, \dfrac{dB}{dt}$

here $dB = 1.7 - 1.2$ and $dt = 10 - 5$

$\therefore$ $L = 0.6$ H

again $dI = 10 - 4 = 6$ A and $dt = 0.08$

$\therefore$ $e_L = L \cdot \dfrac{dI}{dt} = 45$ V]

27. A conductor 1.5 m long carries a current of 50 A at right angles to a magnetic field of density 1.2 $T$. Calculate the force on the conductor.

[*Ans:* $F = 90$ N]

[*Hints:* $F = BIl \sin \theta$]

28. A horseshoe electromagnet is required to lift a 200 kg weight. Find the exciting current requried if the electromagnet is wound with 500 turns. The magnetic length of the electromagnet is 60 cm and is of permeability 500. The reluctance of the load can be neglected. The pole face has a cross-section 20 sq. cm. [*Ans:* 2.35 A]

[*Hint:* $F = 2 \times \frac{B^2 A}{2\mu_o} = \frac{B^2 A}{\mu_o}$

$\therefore$ $B^2 = \frac{\mu_o F}{A} = \frac{4\pi \times 10^{-7} \times 200 \times 9.81}{20 \times 10^{-4}}$

or $B = 1.232 \text{ Wb/m}^2$

$H = \frac{B}{\mu_o \mu_r} = \frac{1.232}{4\pi \times 10^{-7} \times 500} = 1961.78 \text{ AT/m}$

Total AT = $1961.78 \times 0.6 = 1177$

$I = \frac{1117}{500} = 2.35$ A.]

## MULTIPLE CHOICE QUESTIONS

1. In electrical machine, laminated cores are used with a view to reduce
   (a) copper loss (b) hysteresis loss
   (c) eddy current loss (d) all of the above
2. The unit of reluctance is
   (a) dimension less (b) ampere turn
   (c) ampere turn/meter (d) ampere turn/weber
3. The permeability of all non-magnetic materials including air is
   (a) $2\pi \times 10^{-7}$ H/m (b) $4\pi \times 10^{-7}$ H/m
   (c) $\pi \times 10^{-7}$ H/m (d) $6\pi \times 10^{-7}$ H/m
4. A coil of 400 turns has a flux of 0.5 mWb linking with it when carrying a current of 2A. What is the value of inductance if the coil?
   (a) 100 H (b) 10 H
   (c) 0.001 H (d) 0.1 H
5. The magnetism left in the iron after exciting field has been removed is known as
   (a) reluctance (b) performace
   (c) susceptance (d) residual magnetism
6. The initial permeability of an iron rod is
   (a) the permeability almost in non-magnetised state
   (b) the towest permeability of the iron rod
   (c) the highest permability of iron rod
   (d) the permeability at the end of the rod

7. A crack in the magnetic path of the inductor will
   (a) not affect the inductance of the coil
   (b) increase the inductance value
   (c) decrease the inductance value
8. Magnetic moment is the
   (a) pole strength (b) vector quantity
   (c) scalar quantity (d) universal constant
9. A conductor of length 1 m moves at right angles to a magnetic field of the density 1 wb/m$^2$ with a velocity of 20 m/s. The induced emf in conductor will be
   (a) 100 V (b) 20 V
   (c) 2 V (d) 40 V
10. The tubes of force within the magnetic material are known as
    (a) tubes of induction (b) electric flux
    (c) lines of force (d) none of the above
11. A magnetic field exists around
    (a) copper (b) iron
    (c) moving charges (d) aluminium
12. Reciprocal of permeability is
    (a) conductivity (b) reluctivity
    (c) susceptibility (d) permittivity
13. When the current in a circuit is constant, what will be the value of induced voltage?
    (a) same value as current (b) can't say
    (c) half of the current value (d) zero
14. Presence of magnetic flux in a magnetic circuit is due to
    (a) mmf (b) emf
    (c) low reluctance path (d) none of the above
15. In the left hand rule, forefinger always represents
    (a) voltage (b) current
    (c) magnetic field (d) direction of force on the conductor
16. At radio frequencies, the iron core material of inductors
    (a) has a low permeability
    (b) is laminated
    (c) is called ferrite
    (d) reduces inductance as well as losses
17. Suceptibility is positive for
    (a) ferromagnetic substances (b) Paramagnetic substances
    (c) non-magnetic substances (d) none of these

18. The area of the face of a pole is 1.5 $m^2$ and the total flux is 0.18 webers. The flux density in the air gap
    (a) 0.12 tesla (b) 120 tesla
    (c) 1.2 tesla (d) $1.2 \times 10^{-2}$ tesla
19. A conductor of 0.2 m long carries a current of 3 A at right angle to a magnetic field of 0.5 tesla. The force acting on the conductor will be
    (a) 30 N (b) 3.0 N
    (c) 1.0 N (d) 0.3 N
20. Reluctivity is analogous to
    (a) resistivity (b) permeability
    (c) conductivity (d) none of these
21. Which of the following magnetic path will require the largest mmf?
    (a) iron core (b) air-gap
    (c) filament (d) inductance coil
22. Two parallel long conductors will carry 100 A. If the conductors are separated by 20 mm, the force per meter of the length of the each conductor will be
    (a) 100 N (b) 10 N
    (c) 1 N (d) 0.1 N
23. The reciprocal of reluctance is
    (a) permeance (b) conductance
    (c) suceptance (d) admittance
24. The area of hysteresis loop is the measure of
    (a) permitivity (b) permeance
    (c) energy loss per cycle (d) magnetic flux
25. Sparking occurs when a load is switched off because the circuit has
    (a) high capacitance (b) high impedance
    (c) high inductance (d) high resistance
26. Two coils of 300 turns and 2000 turns are placed such that 60% of the flux produced by the first coil links the second coil. A current of 1A in the first coil produces 0.1 mWb flux. The mutual inductance between the coils is
    (a) 0.06 H (b) 1.12 H
    (c) 0.08 H (d) 0.12H
27. The force experienced by a current carrying conductor in a magnetic field is
    (a) directly proportional to the current flowing through the conductor
    (b) directly proportional to flux density
    (c) directly proportional to length of the conductor
    (d) all of the above

28. The magnetic field due to a current carrying coil on the axis at a large distance $x$ from the centre of the coil varies approximately as
    (a) $x^2$ (b) $x^3$
    (c) $x^{-2}$ (d) $x^{-3}$
29. Only transformer emf is generated in
    (a) a moving loop in a time varying magnetic field
    (b) a moving loop in a static magnetic field
    (c) a stationary loop in a static magnetic field
    (d) a stationary loop in a moving magnetic field
30. The flux density at a point 60 mm in air from a long straight conductor carrying current of 500 A is
    (a) $0.97*10^{-3}$ T (b) $1.67*10^{-3}$ T
    (c) $1.01*10^{-3}$ T (d) $2.30*10^{-3}$ T
31. A coil is connected across a 12 V supply. The current in the coil increases at the rate of 300 A. The self inductance of the coil is
    (a) 36 mH (b) 40 mH
    (c) 4 mH (d) 12 mH
32. When two inductive coils are connected in series, opposing form, then the equivalent inductance of the coil will be
    (a) $L_1 + L_2 + M$ (b) $L_1 + L_2 - 2M$
    (c) $L_1 + L_2 + 2M$ (d) $L_1 + L_2 - M$
33. Reluctance of a magnetic circuit is given by by
    (a) flux * mmf (b) flux/mmf
    (c) mmf/flux (d) none of these
34. Area of B-H curve is equal to the
    (a) net work done / cycle / $m^3$
    (b) net work done / cycle / $m^2$
    (c) net work done / cycle / m (d) none of these
35. A coil of 12 Ω resistance and having an inductor of 18 H is connected to a dc supply of 100 Volt. The time constant of the coil is
    (a) 0.66 seconds (b) 216 seconds
    (c) 30 seconds (d) 1.5 seconds
36. Two coils of 2 mH and 8 mH self inductance are placed so close together that the effective flux in one coil is completely linked with the other. The mutual inductance between these coils is
    (a) 16 mH (b) 10 mH
    (c) 6 mH (d) 4 mH
37. The force developed per meter length between two current-carrying conductors 10 cm apart and carrying 1000 A and 1200 A currents respectively is
    (a) 0.6 N (b) 1.2 N
    (c) 2.4 N (d) 3 N

38. Which one of the following statements is true?
    (a) Both light and sound waves can travel in vacuum.
    (b) Both light and sound waves in air are transverse.
    (c) The sound waves in air are longitudinal while the light waves are transverse.
    (d) Both light and sound waves in air are longitudinal.
39. If $\lambda_v$, $\lambda_x$ and $\lambda_m$ represent the wavelengths of visible light, *X*-rays and microwaves respectively, then
    (a) $\lambda_m > \lambda_x > \lambda_v$ (b) $\lambda_m > \lambda_v > \lambda_x$
    (c) $\lambda_v > \lambda_x > \lambda_m$ (d) $\lambda_v > \lambda_m > \lambda_x$
40. Which of the following rays are not electromagnetic waves?
    (a) X-rays (b) $\gamma$-rays
    (c) $\beta$-rays (d) Heat rays
41. A conducting circular loop is placed in a uniform magnetic field of 0.04 T with its plane perpendicular to the magnetic field. The radius of the loop starts shrinking at 2 mm/s. The induced emf in the loop when the radius is 2 cm is
    (a) $1.6\pi$ μV (b) $3.2\pi$ μV
    (c) $4.8\pi$ μV (d) $0.8\pi$ μV
42. A metal ring is held horizontally and a bar magnet is dropped through the ring with its length along the axis of the ring. The acceleration of the falling magnet is
    (a) more than g (b) equal to g
    (c) less than g (d) either (a) or (c)
43. A straight line conductor of 0.4 m length is moved with a speed of 7 m/s perpendicular to a magnetic field of 0.9 Wb/m$^2$ intensity. The induced emf across the conductor is
    (a) 5.04 V (b) 25.2 V
    (c) 1.26 V (d) 2.52 V
44. The current *I* in an inductance coil varies with time according to the graph given in Fig. 2.43. Which one of the following graphs gives the variation of voltage with time?

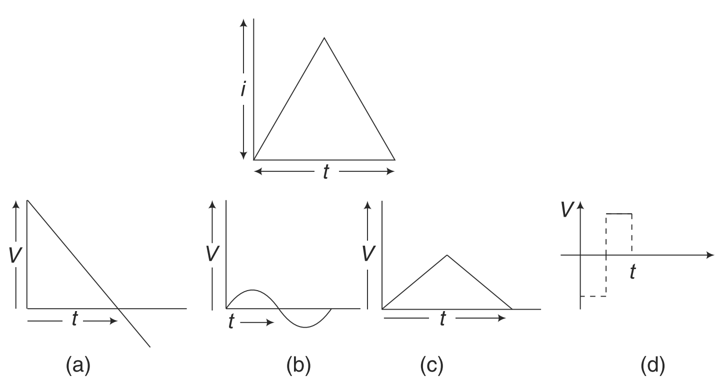



**Fig. 2.43**

45. Under the influence of a uniform magnetic field, a charged particle is moving in a circle of radius $R$ with constant speed v. The time period of the motion
   (a) depends on both $R$ and $v$
   (b) is independent of both $R$ and $v$
   (c) depends on $R$ and not on $v$
   (d) depends on $v$ and not on $R$
46. Two circular coils 1 and 2 are made from the same wire but the radius of the first coil is twice that of the second coil. What is the ratio of potential difference in volt to be applied across them so that the magnetic field at their centres is the same?
   (a) 2 (b) 3
   (c) 4 (d) 6
47. A very long straight wire carries a current $I$. At the instant when a charge $+Q$ at the point $P$ has velocity $v$, as shown in Fig. 2.44, the force on the charge is

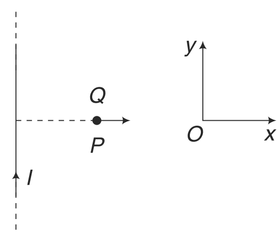



**Fig. 2.44**

   (a) along $Oy$ (b) opposite to $Oy$
   (c) along $Ox$ (d) opposite to $Ox$
48. If the number of turns, area and current through a coil is given by $n$, $A$ and $i$ respectively then its magnetic moment will be
   (a) $niA$ (b) $n^2iA$
   (c) $niA^2$ (d) $ni/\sqrt{(A)}$
49. The magnetic field at the centre P in Fig. 2.45 will be

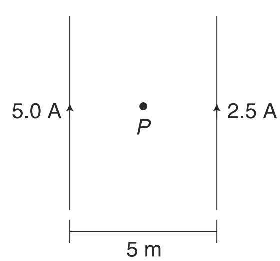



**Fig. 2.45**

   (a) $\mu_0/4\pi$ (b) $\mu_0/\pi$
   (c) $\mu_0/2\pi$ (d) $4\mu_0\pi$

50. The magnetic field due to 0.1 A current flowing through a circular coil of 0.1 m radius and 1000 turns at the centre of the coil is
    (a) $6.28 \times 10^{-4}$ T (b) $4.31 \times 10^{-2}$ T
    (c) $2 \times 10^{-1}$ T (d) $9.81 \times 10^{-4}$ T
51. If the iron core of an iron-cored coil is removed to get an air-cored coil, the inductance of the coil
    (a) decreases (b) increases
    (c) remains same (d) cannot be predicted
52. Two long parallel wires are at a distance of 1 m. Both of them carry one ampere of current. The force of attraction per unit length between the two wires is
    (a) $5 \times 10^{-8}$N/m (b) $2 \times 10^{-8}$ N/m
    (c) $2 \times 10^{-7}$N/m (d) $10^{-7}$ N/m
53. A wire of certain material is stretched slowly by ten percent. The new resistance and specific resistance become respectively
    (a) 1.1 times, 1.1 times (b) 1.2 times, 1.1 times
    (c) 1.21 times, same (d) both remain the same
54. The magnetic field at the centre of a coil of 50 turns, 0.5 m radius and carrying a current of 2 A is
    (a) $3 \times 10^{5}$ T (b) $1.25 \times 10^{-4}$ T
    (c) $0.5 \times 10^{-5}$ T (d) $4 \times 10^{6}$ T
55. A straight wire of 0.5. mm diameter carrying a current of 1 A is replaced by another wire of 1 mm diameter carrying the same current. The strength of the magnetic field far away is
    (a) one quarter of the earlier value
    (b) no change
    (c) twice the earlier value
    (d) one half of the earlier value
56. Electromagnetic field is employed as a medium for operation of
    (a) motors (b) generators
    (c) transformer (d) all of these
57. When a conductor is moved inside a uniform magnetic field,
    (a) it experience a force (b) an emf is induced
    (c) a current flows through it (d) a flux is produced
58. High-permeability magnetic field material helps
    (a) to confine the flux within the magnetic circuit
    (b) to allow the flux to leak
    (c) in producing more current
    (d) none of these

59. The unit of magnetic field intensity is
    (a) ampere turns (b) weber
    (c) ampere per metre (d) weber/metre$^2$
60. The magnetomotive force set up in a magnetic circuit
    (a) is another name for electromotive force
    (b) is analogous to potential difference
    (c) is analogous to resistance of an electric circuit
    (d) results in a flow of current in the magnetic circuit
61. In a magnetic circuit,
    (a) flux is analogous to current
    (b) reluctance is analogous to resistance
    (c) magnetomotive force is analogous to potential difference
    (d) all of these
62. Which of the following are not representative of the magnetic field intensity?
    (a) H
    (b) E
    (c) Magnetomotive force per metre
    (d) Ampere turns per metre
63. The circuital law which states that an electric current flowing through a number of turns of a conductor produces a magnetic field in its vicinity was enunciated by
    (a) Faraday (b) Lenz
    (c) Ohm (d) None of these
64. The unit of mmf is normally taken as
    (a) weber (b) weber/metre$^2$
    (c) ampere turns (d) ampere turns per metre
65. If the direction of current flowing in a conductor is in the plane of paper, the magnetic flux lines produced by it are
    (a) concentric circles in the clockwise direction
    (b) concentric circles in the anticlockwise direction
    (c) straight lines parallel to the conductor and in the opposite direction of the current flow
    (d) straight lines parallel to the conductor and in the direction of the current flow
66. When a conductor of length $l$ m is moved perpendicular to a magnetic field of intensity $B$ tesla, at a velocity of $v$ m/s, the magnitude of the induced voltage is given by
    (a) $Blv$ (b) $BIl$
    (c) $Blv \sin \theta$ (d) none of the above

67. The permeability of a non-magnetic medium is
    (a) $6 \times 10^{-12}$ (b) $4\pi \times 10^{-7}$
    (c) $8.85 \times 10^{-12}$ (d) 1.0
68. Which of the following is the representative unit of permeability?
    (a) $\frac{\text{netwons}}{(\text{ampere})^2}$ (b) $\frac{\text{volt} \times \text{second}}{\text{ampere}} \times \text{meter}$
    (c) henry/metre (d) All of these
69. The relative permeability for air is
    (a) more than 10,000 (b) between 10,000 and 40000
    (c) equal to unity (d) none of these
70. Which of the following does not apply to the reluctance $R$ of a magnetic circuit?
    (a) $R \propto \frac{1}{\mu_0}$ (b) $R \propto \frac{1}{\mu_r}$
    (c) $R \propto 1/l$ (d) $R \propto \frac{1}{A}$
71. When compared with an electric circuit, which of the following parameters in a magnetic circuit are analogous?
    (a) Current and flux
    (b) Electric field strength and magnetic field strength
    (c) Voltage drop and mmf drop
    (d) All of these
72. In a magnetic circuit, once a flux is set up,
    (a) no further energy is required
    (b) energy is continuously required to maintain the flux
    (c) energy is released in the form of heat
    (d) none of these
73. Magnetic circuits obey
    (a) Kirchhoff's law (b) Thevenin's law
    (c) Norton's law (d) none of these
74. Saturation of a magnetic circuit implies that with an increase in field intensity, the flux density
    (a) decreases (b) increases proportionally
    (c) increases marginally (d) none of these
75. The *word hysteresis* means
    (a) to lead (b) to be in step
    (c) to lag (d) none of these
76. 'Retentivity' and 'coercive force' are used in connection with
    (a) hysteresis loop (b) loop current
    (c) current cycle (d) none of the above

77. The unit of H × B is
    (a) joule (b) joule/metre
    (c) joule/metre$^2$ (d) joule/metre$^3$
78. A magnetic circuit consists of an iron core and an air gap. Most of the energy is stored in the air gap because the air gap has
    (a) a much higher reluctance
    (b) a much lower reluctance
    (c) zero reluctance
    (d) none of these
79. Self-inductance of a coil is represented by
    (a) $\dfrac{0}{\dfrac{dl}{dt}}$ (b) $\dfrac{\psi}{i}$
    (c) $\dfrac{N^2}{R}$ (d) all of these
80. Mutual inductance is defined as the ratio of change
    (a) of flux in coil 1 to the change of flux in coil 2
    (b) of current in coil 1 to the change of current in coil 2
    (c) of flux in coil 1 to the change of current in coil 1
    (d) of flux in coil 2 to the change of current in coil 1
81. When two coils are placed together and there is no flux leakage, the coupling coefficient $k$ is
    (a) zero (b) unity
    (c) infinity (d) none of these
82. The law that states that when two coils are placed together in close proximity to each other, a change in flux in one coil induces an emf in the second coil was enunciated by
    (a) Lenz (b) Faraday
    (c) Hertz (d) Ohm
83. The self-inductance of a coil is dependent on
    (a) the number of turns in the coil only
    (b) the geometry of the core only
    (c) the number of turns in the coil and geometry of the core
    (d) none of these
84. Two coils of $N_1$ and $N_2$ turns are wound on a common core. If the current in coil 1 changes at 1A/sec and it induces an emf of 1 V in coil 2,the magnitude of mutual inductance is
    (a) $\dfrac{N_1}{N_2}$ henry (b) $\dfrac{N_2}{N_1}$ henry
    (c) $N_1N_2$ henry (d) 1 henry

85. The dot convention is used to define the sign of
    (a) self-inductance of the coils
    (b) mutual inductance of the coils
    (c) direction of current flow in the coils
    (d) none of these
86. The energy stored in a linear magnetic circuit is given by
    (a) $\frac{1}{2}\text{Li}^2$ (b) $\frac{1}{2}\text{BHV}$
    (c) $\frac{1}{2}\theta^2 \times R$ (d) all of these
87. The expression for energy stored in two mutually coupled and excited coils is by given
    (a) $\frac{1}{2}L_1 i_2^{\,2} + \frac{1}{2}L_2 i_1^{\,2} + Mi_1 i_2$
    (b) $\frac{1}{2}L_1 i_1^{\,2} - \frac{1}{2}L_2 i_2^{\,2} + Mi_1 i_2$
    (c) $\frac{1}{2}L_1 i_1^{\,2} + \frac{1}{2}L_2 i_2^{\,2} + Mi_1 i_2$
    (d) $\frac{1}{2}L_2 i_1^{\,2} - \frac{1}{2}L_1 i_2^{\,2} + Mi_1 i_2$
88. Two coils having self inductance of $L_1$ and $L_2$ henry are differentially coupled. If the mutual inductance between the coils is $M$, the total inductance of the coils is
    (a) $L_1 + L_2 + 2M$ (b) $L_1 + L_2 - 2M$
    (c) $L_1 - L_2 + 2M$ (d) $L_1 - L_2 - 2M$
89. The force of attraction in an electromagnet is due largely to the
    (a) energy stored in the air gap between the magnet and the non-magnetic material
    (b) energy stored in the ferromagnetic material
    (c) energy stored in the nonmagnetic material
    (d) none of these
90. The force of attraction produced by an electromagnet is a function of
    (a) $B^2$ (b) A
    (c) $\mu_0$ (d) all of these
91. What is the effect of fringing on the area of an air gap?
    (a) Increases it (b) Decreases it
    (c) Keeps it same (d) None of these
92. How does the flux vary when a steel ring is replaced by a wooden ring of the same dimension?
    (a) No change (b) Increases
    (c) Decreases (d) None of these

93. The mutual inductance between two closely coupled coils is 0.5 H. The coils are rewound to reduce the number of turns in one coil to 1/3 and to increase in the other by three times. Which of the following represents the mutual inductance of the coils?
(a) 0.17 H (b) 0.5 H
(c) 0.75 H (d) 1.5 H

94. Which of the following compares well with the field intensity set up in the air gap and the ferromagnetic medium of an electromagnet?
(a) Much higher (b) Equal
(c) Much less (d) None of these

95. How should the current in a magnetic circuit be changed to produce the same flux, if the reluctance is halved?
(a) Doubled (b) No change
(c) Halved (d) One-fourth

96. The energy stored in the air gap of an electromagnet is proportional to
(a) $A$ (b) $B^2$
(c) $l$ (d) All of these

97. The magnitude of the coefficient of coupling is dependent on
(a) $L_1$ (b) $L_2$
(c) proximity of the coils (d) all of these

98. For which of the following arrangements in Fig. 2.46, the self-inductance and mutual inductance will have the same sign?

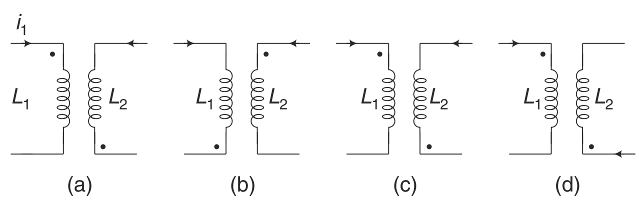



**Fig. 2.46**

99. The desirable properties for making permanent magnets are _________ resistivity and ________ coercive force.
(a) high, low (b) high, high
(c) low, low (d) low, high

100. A magnetic material loses its magnetic properties when
(a) dipped in insulating oil (b) brought near an iron piece
(c) dipped in cold water (d) strongly heated

## ANSWERS

| | | | | | |
|---|---|---|---|---|---|
| 1. (c) | 2. (d) | 3. (b) | 4. (d) | 5. (d) | 6. (a) |
| 7. (c) | 8. (b) | 9. (b) | 10. (a) | 11. (c) | 12. (b) |

| | | | | | |
|---|---|---|---|---|---|
| 13. (d) | 14. (a) | 15. (c) | 16. (c) | 17. (a) | 18. (a) |
| 19. (d) | 20. (a) | 21. (b) | 22. (d) | 23. (a) | 24. (c) |
| 25. (a) | 26. (d) | 27. (d) | 28. (d) | 29. (d) | 30. (b) |
| 31. (b) | 32. (b) | 33. (c) | 34. (a) | 35. (d) | 36. (d) |
| 37. (c) | 38. (c) | 39. (b) | 40. (c) | 41. (b) | 42. (c) |
| 43. (d) | 44. (d) | 45. (b) | 46. (c) | 47. (a) | 48. (a) |
| 49. (c) | 50. (a) | 51. (a) | 52. (c) | 53. (c) | 54. (b) |
| 55. (b) | 56. (d), | 57. (b), | 58. (a) | 59. (c) | 60. (b), |
| 61. (d), | 62. (b), | 63. (d), | 64. (c), | 65. (a), | 66. (a), |
| 67. (b), | 68. (d), | 69. (c), | 70. (c), | 71. (d), | 72. (a), |
| 73. (a), | 74. (c), | 75. (c), | 76. (a), | 77. (d), | 78. (a), |
| 79. (d), | 80. (d), | 81. (b), | 82. (b), | 83. (c), | 84. (d), |
| 85. (b), | 86. (d), | 87. (c), | 88. (b), | 89. (a), | 90. (d), |
| 91. (a), | 92. (c), | 93. (b), | 94. (a), | 95. (c), | 96. (d), |
| 97. (d), | 98. (d), | 99. (b), | 100. (d) | | |

## UNIVERSITY QUESTIONS WITH ANSWERS

1. (a) State and explain Farady's law of electromagnetic induction.

[2004, 2014]

*Answer*: Refer Article 2.8

(b) What is the difference between statically induced emf and dynamically induced emf?

*Answer:* In statically induced emf the conductors or coils remains stationary and the magnetic flux linked with them changes. Thus emf is induced in the coils.

In dynamically induced emf the magnetic flux remains stationary and conductors or coils rotate and cut the magnetic flux inducing emf in the conductors or coils.

Emf is induced in the transformers statically and in dc generators dynamically.

(c) A conductor having a length of 80 cm is placed in a uniform magnetic field of 2 wb/m$^2$ (Tesla). If the conductor moves with a velocity of 50 m/sec, find the induced emf when it is (i) at right angle (ii) at an angle of 30° and (iii) parallel to the magnetic field.

*Answer:* $l = 80$ cm $= 0.8$ m

$B = 2$ wb/m$^2$

$v = 50$ m/sec

(i) When the conductor is at right angle to the magnetic field $\theta = 90°$

$\therefore$ Induced emf $= Blv \sin\theta = 2 \times 0.8 \times 50 \times \sin 90° = 80$ volts

(ii) When the conductor is at an angle of 30° to the magnetic field $\theta = 30°$

$\therefore$ Induced emf = $2 \times 0.8 \times 50 \sin 30° = 40$ volts

(iii) When the conductor is parallel to the magnetic field $\theta = 0°$

$\therefore$ Induced emf = $2 \times 0.8 \times 50 \sin 0° = 0$ volts

2. (a) Compare magnetic circuit with electric circuit. [2004]

*Answer*: Article 2.25

(b) A mild steel ring having a cross sectional area of 10 cm$^2$ and a mean circumference of 60 cm has a coil of 300 turns wound around it. Determine:

(i) reluctance of the steel ring.

(ii) current required to produce a flux of 1 mwb in the ring. Relative permeability of the given steel is 400 at the flux density developed in the core.

(iii) If a slit of 1 mm is cut in the ring, what will be the new value of current? Assume no fringing effect.

*Answer*: Given

$$A = 10 \text{ cm}^2 = 0.001 \text{ m}^2$$
$$l = 60 \text{ cm} = 0.6 \text{ m}$$
$$N = 300$$
$$\mu_r = 400.$$

(i) Reluctance of the steel ring $= \dfrac{l}{\mu_o \mu_r A} = \dfrac{0.6}{4\pi \times 10^{-7} \times 400 \times 0.001}$

$$= 1.19 \times 10^6 \text{ AT/wb}$$

(ii) $\phi = 0.001$ wb

Mmf = reluctance × flux = $1.19 \times 10^6 \times 0.001 = 1.19 \times 10^3$ AT

If current be $I$ then

$$NI = 1.19 \times 10^3$$

$$\therefore \quad I = \frac{1.19 \times 10^3}{300} = 3.97 \text{ A}$$

(iii) Reluctance of the air gap $= \dfrac{l}{\mu_0 A} = \dfrac{0.001}{4\pi \times 10^{-7} \times 0.001}$

$$= 7.96 \times 10^5 \text{ AT/wb}$$

Mmf required for the air gap = $7.96 \times 10^5 \times 0.001 = 796$ AT.

Hence, total mmf = mmf required for steel ring + mmf required for air gap

$$= 1.19 \times 10^3 + 796 = 1986 \text{ AT}$$

If $I'$ be the new value of current then

$$I' N = 1986$$

or, $$I' = \frac{1986}{300} = 6.62 \text{ A}$$

3. Write short notes on Hysteresis and eddy current loss. [2004]

   *Answer:* Refer Article 2.29 and 2.30

4. (a) Express the inductance in terms of its physical dimensions. [2005]

   *Answer:*

   Energy stored in inductance $E = \frac{1}{2} LI^2$, where $L$ is inductance and $I$ is current in ampere (A).

$$\therefore \quad L = \frac{2E}{I^2}$$

$$\text{Now Energy } E = \text{Work} = \text{mass} \times \text{acceleration} \times \text{displacement}$$

$$= \frac{\text{mass} \times (\text{displacement})^2}{\text{time}}$$

$$\therefore \quad \text{Inductance } L = 2\,\frac{\text{mass} \times (\text{displacement})^2}{\text{time} \times (\text{ampere})^2}$$

   $\therefore$ Physical dimension of inductance is $ML^2\,T^{-2}A^{-2}$

   (b) An air-cored toroidal coil has 450 turns and a mean diameter of 300 mm and cross sectional area of 300 mm$^2$. Determine the self-inductance of the coil and the average voltage induced in it when a current of 2 A is reversed in 40 mS.

   *Answer*: No. of turns $N = 450$

   Diameter $d = 300$ mm $= 0.3$ m

   Area $= 300 \text{ mm}^2 = 0.3 \times 10^{-3} \text{ m}^2$

$$\text{Self-inductance } L = \frac{\mu_0\,\mu_r\,AN^2}{l}$$

$$= \frac{4\pi \times 10^{-7} \times 1 \times 0.3 \times 10^{-3} \times (450)^2}{\pi \times 0.3}$$

$$= 810 \times 10^{-7} \text{ H} = 81\ \mu\text{H}$$

   Average emf induced in the coil

$$(v_L) = L\frac{di}{dt} = 81 \times 10^{-6} \times \frac{2}{40 \times 10^{-3}} = 4 \text{ mV}$$

5. Write short notes on following: (a) Ampere circuital law [2005]

   *Answer*: Refer Article 2.7.

   (b) Eddy current losses

   *Answer*: Refer Article 2.30.

6. In ac circuit laminated iron is invariably used in order to reduce eddy current losses. [2005]

   *Answer*: True. (Refer Article 2.30)

7. Flux in a magnetic circuit can be compared in an electric circuit to [2006]

   (i) voltage (ii) current (iii) resistance (iv) inductance

*Answer*: (ii) current

8. For a coil with *N*-turns the self inductance will be proportional to [2006]

   (i) $N$ (ii) $\frac{1}{N}$ (iii) $N^2$ (iv) $\frac{1}{N^2}$

   *Answer*: (iii) $N^2$

9. (a) Compare magnetic circuit with electric circuit. [2006, 2011, 2013]

   *Answer*: Refer Article 2.25.

10. (a) State and explain Farday's law of electromagnetic induction. [2006]

    *Answer*: Refer Article 2.8.1.

    (b) Calculate the mmf required to produce a flux of 0.01 Wb across an air gap of 2 mm of length having an effective area of 200 cm$^2$ of a wrought iron ring of mean iron path of 0.5 m and cross sectional area of 125 cm$^2$. Assume leakage coefficient of 1.25. The magnetization curve of wrought iron is given below:

| | | | | | |
|---|---|---|---|---|---|
| B(Wb/m$^2$): | 0.60 | 0.8 | 1 | 1.2 | 1.4 |
| H(AT/m): | 75 | 125 | 250 | 500 | 1000 |

*Answer*: Flux ($\phi$) = 0.01 Wb.

Cross sectional area of air gap (Ag) = 200 cm$^2$ = 0.02 m$^2$

Length of air gap (lg) = 2 mm = 0.002 m

$$\text{Reluctance of the air gap} = \frac{lg}{\mu_0 Ag} = \frac{0.002}{4\pi \times 10^{-7} \times 0.02} = 79617.8 \text{ AT/wb}$$

$\therefore$ mmf required for the air gap = flux × leakage coefficient × reluctance

= (0.01 × 1.25 × 79617.8) = 995 *AT*

Cross sectional area of iron ring (*Ai*) = 125 cm$^2$ = 0.0125 m$^2$

Length of the iron ring (*li*) = 0.5 m

$$\text{Flux density in iron ring } (B) = \frac{\phi \times 1.25}{Ai} = \frac{0.01 \times 1.25}{0.125} = 1 \text{ wb/m}^2$$

From the given magnetization curve the value of $H$ corresponding to $B = 1$ is 250 AT/m

Hence total mmf required for the iron part of ring is ($Hl_i$) i.e. 250 × 0.5 = 125 AT.

$\therefore$ Total mmf required is (995 + 125) = 1120 AT

11. Inductive reactance of a coil of inductance 0.2 H at 50 Hz is [2007, 2014]

    (a) 62.8 ohms (b) 628 ohms (c) 0.2 ohms (d) 20 ohms

    *Answer*: (a) 62.8 ohms.

12. The reluctance of a magnetic circuit depends upon its [2007]

    (a) length (b) cross-section and length (c) resistivity

    *Answer*: (b) cross-section and length.

13. State and prove Ampere's circuital law. [2007]

    *Answer*: Refer Article 2.7.

14. Define self and mutual inductance. What do you mean by co-efficient of coupling? [2007]

    *Answer*: Refer Article 2.9 and Article 2.10.

15. (i) For an inductive circuit, current [June 2008]

    (a) lags the voltage
    (b) leads the voltage
    (c) is in phase with the voltage
    (d) is independent of the voltage phase

    *Answer*: (a) lags the voltage.

    (ii) The reluctance of a magnetic circuit is given by [June 2008, 2014]

    (a) $\frac{l}{\mu_r \mu_0 A}$ (b) $\frac{\phi}{NI}$ (c) $\frac{1}{\mu_0 A}$ (d) $\frac{1}{\mu_r A}$

    *Answer*: (a) $\frac{l}{\mu_r \mu_0 A}$.

    (iii) The time constant of $L - R$ circuit is given by [June 2008, 2014]

    (a) $L/R$ (b) $R/L$ (c) $\frac{1}{LR}$ (d) $LR$

    *Answer*: (a) $L/R$.

    (iv) The unit of magnetic flux density is [June 2009]

    (a) weber (b) tesla
    (c) coulomb (d) none of these

    *Answer*: (b) tesla

16. Define self and mutual inductance. What is understood by co-efficient of coupling?

    *Answer*: Refer Article 2.9 and Article 2.10 [June 2008]

17. State Biot–Savart law applicable to electromagnetism.

    *Answer*: Refer Article 2.5. [June 2008]

18. (a) Derive an expression for energy stored in a magnetic field.

    *Answer*: Refer Article 2.12. [June 2008, 2013]

    (b) A horse-shoe electromagnet is required to lift a load of 200 kg. The electromagnet is wound with 500 turns and the length of the magnetic path is 50 cm and the cross section of each arm is 25 sq cm. Find the current in the coil. State the assumptions. [$\mu r = 400$]

    *Answer*: Force of attraction of each pole

    $$F = \frac{200}{2} \times 9.81 = 981 \text{ N}$$

    Area $A = 25$ sq cm $= 0.0025$ m$^2$

    Now $$F = \frac{B^2 A}{2\mu_0}$$

    $$\therefore \quad B = \sqrt{\frac{2F\mu_0}{A}} = \sqrt{\frac{2 \times 981 \times 4\pi \times 10^{-7}}{0.0025}}$$

$= 0.993 \text{ wb/m}^2$

$$\therefore \quad H = \frac{B}{\mu_0 \mu_r} = \frac{0.993}{4\pi \times 10^{-7} \times 400} = 1976.5 \text{ AT/m}$$

Length of the magnetic path is 50 cm i.e. 0.5 m

Required mmf = 1976.5 × 0.5 = 988.25 AT

Number of turns $N = 500$

$$\therefore \quad \text{Current in the coil } I = \frac{988.25}{500} = 1.9765 \text{ A}$$

The assumptions are

(i) No fringing of flux and no leakage co-efficient

(ii) Magnet is made of uniform material

(iii) The flux density remains constant.

19. Derive an expression for the hysteresis loss in a magnetic material.

*Answer*: Refer Article 2.29 [December 2008]

20. State and prove Amper's circuital law [December 2008, 2013]

*Answer*: Refer Article 2.7.

21. (a) Derive an expression for the lifting power of an electromagnet.

*Answer*: Refer Article 2.14. [December 2008, 2013, 2014]

(b) A horseshoe electromagnet is required to lift a load of 200 kg. The iron length of the horseshoe magnet and the load is 60 cm and of relative permeability 600. Find the current drawn by the electromagnet if it is wound with 1000 turns. The contact gapt of the load and magnet is 0.001 cm at each pole. The cross section of the pole is 20 sq cm. Assume, no leakage flux.

*Answer*:

$$F = \frac{200}{2} \times 9.81 = 981 \text{ N}$$

$$A = 20 \text{ sq cm} = 0.002 \text{ sq m}$$

Now

$$F = \frac{B^2 A}{2\mu_o}$$

$$B = \sqrt{\frac{2\mu_0 F}{A}}$$

$$= \sqrt{\frac{2 \times \pi \times 10^{-7} \times 981}{0.002}} = 1.11 \text{ wb/m}^2$$

∴ Total MMF = MMF for iron part + MMF for air gap

$$\frac{Bli}{\mu_o \mu_r} + \frac{Bl_g}{\mu_0} = (B/\mu_0)\left[\frac{li}{\mu_r} + l_g\right]$$

$$= \frac{1.11}{4\pi \times 10^{-7}}\left[\frac{0.6}{600} + 0.001 \times 10^{-2}\right] = 892.6 \text{ AT}$$

with

$$N = 1000, I = \frac{\text{Total MMF}}{N} = 0.893 \text{ A}$$

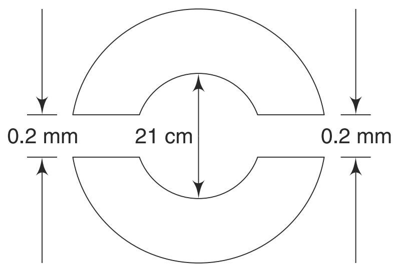
22. What is mutual inductance? Derive an expression of coefficient of coupling involving self inductance $L_1$ and $L_2$ and mutual inductance $M$. [WBUT 2014]
    *Answer*: Refer Article 2.9 and 2.10.
23. Draw an analogy between electric circuit and magnetic circuit.
    *Answer*: Refer Article 2.25.
24. The force experienced by a small conductor of length $L$, carrying a current $I$. placed in a magnetic field $\vec{B}$ at an angle $\theta$ with respect to $\vec{B}$ is given by [WBUT 2011, 2014]

    (a) $BIL$ (b) $BIL \sin \theta$
    (c) $BIL \cos \theta$ (d) zero

    *Answer:* (b) $BIL \sin \theta$
25. The mutual inductance between two coupled coils is 10 mH. If turns of one coil are doubled and that in the other are halved, the mutual inductance will be [WBUT 2011]

    (a) 5 mH (b) 10 mH
    (c) 14 mH (d) 20 mH

    *Answer:* (b) 10 mH
26. (a) State and explain Biot-Savart law. [WBUT 2011]
    *Answer*: Refer Article 2.5
    (b) A ring having a mean diameter of 21 cm and a cross-section of 10 $cm^2$ is made of two semicircular sections of cast iron and cast steel respectively with each joint having reluctance equal to air gap of 0.2 mm as shown in figure. Determine the ampere turns required to produce a flux of 0.8 mWb. The relative permeabilities of cast iron and cast steel are 166 and 800 respectively. Neglect fringing and leakage effects.




**Cast steel**

*Answer*:

$$d = 21 \text{ cm } A = 10 \text{ cm}^2$$

Total length of the ring including air gap is $\pi \times 21 = 65.97$ cm

Length of air gap = 0.04 cm

$\therefore$ Length of iron part = length of steel part

$$= \frac{65.97 - 0.04}{2} = 32.965 \text{ cm} = 0.32965 \text{ m.}$$

$$\text{Reluctance of the iron path} = \frac{0.32965}{4\pi \times 10^{-7} \times 166 \times 10 \times 10^{-4}}$$

$$= 1.58 \times 10^6 \text{ AT/Wb}$$

$$\text{Reluctance of steel path} = 1.58 \times 10^6 \times \frac{166}{200} = 1.3114 \times 10^6 \text{ AT/Wb}$$

$$\text{Reluctance of the air gap} = \frac{0.04 \times 10^{-2}}{4\pi \times 10^{-7} \times 10 \times 10^{-4}} = 1.27 \times 10^6 \text{ AT/Wb}$$

Flux = $0.8 \times 10^{-3}$ Wb.

$\therefore$ Mmf required for iron path = $1.58 \times 10^6 \times 0.8 \times 10^{-3}$

$= 1.264 \times 10^{-3}$ AT

Mmf required for steel path = $1.3114 \times 10^6 \times 0.8 \times 10^{-3}$

$= 1.049 \times 10^3$ AT

Mmf required for air gap = $1.27 \times 10^6 \times 0.8 \times 10^{-3}$

$= 1.016 \times 10^3$ AT

Hence, total ampere turns required is

$= (1.264 + 1.049 + 1.016)\ 10^3$ AT $= 3.329 \times 10^3$ AT

27. Conductance is analogous to
    (a) permeance (b) flux
    (c) reluctance (d) inductance

    *Answer:* (a) permeance

28. Energy stored by a capacitor is given by
    (a) $\frac{1}{2} CV^2$ (b) $\frac{1}{2} QV$
    (c) $\frac{Q^2}{2C}$

    *Answer:* (a) $\frac{1}{2} CV^2$

29. Define self and mutual inductance. Derive an expression for coefficient of coupling (k) involving self inductances $L_1$ and $L_2$ and mutual inductance $M$.

    *Answer:* Refer Article 2.9 and 2.10.

30. What is meant by hysteresis in a magnetic circuit? What is the significance of B-H curve?

    *Answer:* Refer Article 2.28.

31. Find an expression for the energy stored in a magnetic field.

    *Answer:* Refer Article 2.12.

32. An iron ring of mean length 50 cm has an air gap of 1 mm and winding of 200 turns. The relative permeability of iron is 300. When 1 A current flows through the coil, determine the flux density. [WBUT 2013]

    *Answer:* Refer Example 2.36

33 A coil of 250 turns carrying a current of 2 A produces a flux of 0.3 mWb. When the current is reduced to zero in 2 ms, the voltage induced in a nearly coil is 60 V. Calculate the self inductance of each coil and the mutual inductance between the two coils Assume co-efficient of coupling to be 0.7

[WBUT 2014]

*Answer:* Self inductance $L_1 = \dfrac{N\phi}{I}$

$$= \frac{250 \times 0.3 \times 10^{-3}}{2} \text{ H} = 37.5 \text{ mt.}$$

$$e_M = M\frac{dI_1}{dt}$$

$$60 = M\frac{2-0}{2 \times 10^{-3}} = M\frac{2 \times 10^3}{2} = 1000 \text{ M}$$

$$\therefore \quad M = \frac{60}{1000} \text{ H} = 60 \text{ mH}$$

If $L_2$ be the self inductance of the other coils then

$$M = K\sqrt{L_1 L_2}$$

or, $$60 = 0.7\sqrt{37.5\, L_2}$$

$$\therefore \quad L_2 = 196 \text{ mH}$$


# 3 AC FUNDAMENTALS

## 3.1 GENERATION OF ALTERNATING EMF

Let us consider a rectangular coil (Fig. 3.1), having $N$ number of turns and $A$ m$^2$ cross-sectional area, which is rotating in a uniform magnetic field with an angular velocity $\omega$ radian/s. If in $t$ seconds the coil rotates through an angle $\theta = \omega t$ from the $X$-axis, the component of the flux perpendicular to the plane of the coil is $\phi = \phi_m \cos \omega t$ (where $\phi_m$ = maximum flux density perpendicular to the axis of rotation, when the plane of the coil coincides with the $X$-axis).

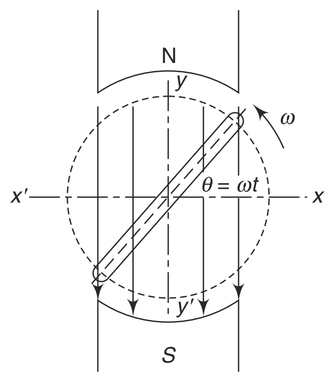



**Fig 3.1** ***Generation of alternating emf***

We know from Faraday's laws of electromagnetic induction that, "the induced emf in the coil is equal to the rate of change of flux linkages of the coil". Again, Lenz's law states that, "when a circuit and a magnetic field move relatively to each other the electric current induced in the circuit will have a magnetic field opposing the motion". Combining these two laws, the instantaneous induced emf at time $t$ is given by

$$e = -N\frac{d\phi}{dt} = -N\frac{d}{dt}(\phi_m \cos \omega t) \qquad [\because \quad \phi = \phi_m \text{ cost } \omega t]$$

$$= \omega N \phi_m \sin \omega t = (\omega N \phi_m \sin \theta) \text{ V}$$

when $\theta = 90°$, $e = \omega N \phi_m = E_m$ (say) where ($E_m$) is the maximum value of the instantaneous induced emf.

Now, if $f$ be the frequency of rotation of coil in Hertz and $B_m$ the maximum flux density in $\omega b$/m$^2$,

$$e = E_m \sin \theta = (2\pi f N B_m A) \sin \theta \text{ V} \qquad [\because \quad B_m \cdot A = \phi_m]$$

Let $i$ be the instantaneous value of the current in the coil. Therefore, $i = I_m \sin \omega t$, where $I_m$ is the maximum value of the current.

As both the induced emf and induced current varies sinusoidally hence the emf or current can be plotted against (time). A sinusoidal curve is obtained as shown in Fig. 3.2

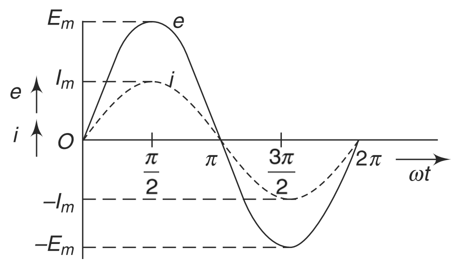



**Fig. 3.2** *AC Sinusoidal wave form*

## 3.2 DEFINITIONS RELATING TO ALTERNATING QUANTITY

**1. Amplitude (Peak Value)** It is the maximum value, positive or negative of an alternating quantity.

**2. Instantaneous Value** It is the value of the alternating quantity at any instant.

**3. Cycle** One complete set of positive and negative values of an alternating quantity is known as cycle.

**4. Time Period** It is the time required by an alternating quantity to complete 1 cycle; so for a 50 Hz a.c the time period is 1/50 second.

**5. Frequency** The number of cycles per second is called the frequency of the alternating quantity. Its unit is Hertz (Hz).

**6. Phase** Phase of an alternating quantity is fraction of the time period that has elapsed since the quantity last passed through the chosen zero position of reference.

**7. Phase Angle** It is the equivalent of phase in radians or degrees. So phase angle is $\left(2\pi \frac{t}{T}\right)$, where $t$ is the instantaneous time and $T$ is the time period.

**8. Phase Difference** Phase difference between two alternating quantities is the fractional part of a period by which one has advanced over or lags behind the other. To measure phase difference the frequency of the alternating quantities should be same.

(a) The alternating quantities are in phase when each pass through their zero value, maximum and minimum values at the same instant of time.

(b) A leading alternating quantity is one which reaches its maximum, minimum or zero value earlier than the other quantity. A lagging quantity is one which reaches the maximum, minimum and zero values later than the other quantity.

In Fig. 3.3 alternating quantity $e_B$ is leading with respect to $e_C$ and is lagging with respect to $e_A$. If we consider $e_B$ as reference then

$$e_B = E_m \sin \omega t,$$

where $E_m$ is the amplitude and $\omega$ is the angular frequency of ($e_B$).

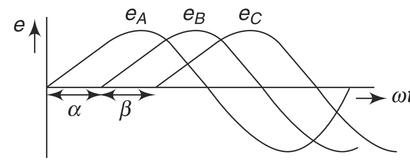



**Fig. 3.3** ***Lagging and leading alternating quantities***

Therefore $$e_A = E_m \sin(\omega t + \alpha) \text{ and } e_C = E_m \sin(\omega t - \beta)$$

where $\alpha$ is the phase difference between $e_A$ and $e_B$ and ($\beta$) is the phase difference between ($e_B$) and ($e_C$).

**9. Roots Mean Square (RMS Value)** The rms value of the alternating current is that steady current, i.e., d.c current which if passed through a circuit produces the same amount of heat as produced by the alternating current flowing through the same circuit for the same period of time. The heat produced by direct current $I$ or its equivalent rms value of the alternating quantity $i$ is proportional to $i^2$. So the area under the curve $i^2$ vs. $2\pi$ is the total heat produced by an alternating current [Fig. 3.4(a) and Fig. 3.4(b)].

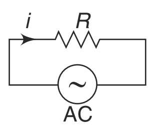



**Fig. 3.4(a)** ***AC through pure resistance***

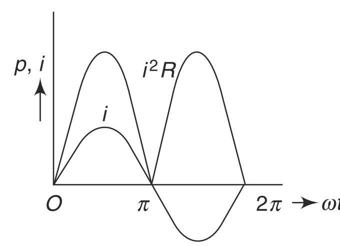



**Fig. 3.4(b)** ***RMS value of alternating quantity***

Rms value is given by

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\, dt}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2\theta\, d\theta} \qquad [\text{substituting } i = I_m \sin\theta]$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{2\pi} 2\sin^2\theta\, d\theta}$$

$$= \frac{I_m}{2}\sqrt{\frac{1}{\pi}\int_0^{2\pi} (1 - \cos 2\theta)\, d\theta}$$

$$= \frac{I_m}{2}\sqrt{\frac{1}{\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \frac{I_m}{2}\sqrt{\frac{1}{\pi}(2\pi)}$$

$$= \frac{I_m}{\sqrt{2}} = 0.707\ I_m.$$

Hence, the rms value of an alternating quantity = 0.707 × maximum value of that alternating quantity.

**10. Average (or Mean) Value** The average value of an alternating current is that steady or d.c. current which transfers across any circuit the same amount of charge as transferred by that alternating current during the same period of time.

The average value of an alternating current is given by

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i\, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi}[-\cos\theta]_0^{\pi} = \frac{2I_m}{\pi} = 0.637\ I_m.$$

Thus, the average value of an alternating quantity = 0.637 × maximum value of that alternating quantity.

**11. Crest or Amplitude or Peak Factor $K_a$** It is the ratio of the peak or maximum value to the rms value of an alternating quantity. For a sinusoidal wave,

$$K_a = \frac{I_m}{I_{rms}} = \frac{I_m}{0.707\ I_m} = 1.414$$

The knowledge of crest factor is important for measuring iron losses, as iron loss depends on the value of maximum flux. Also in dielectric insulation testing the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage.

**12. Form factor $K_f$** It is the ratio of the rms value to the average value of an alternating quantity. For a sinusoidal wave

$$K_f = \frac{I_{rms}}{I_{av}} = \frac{0.707\ I_m}{0.637\ I_m} = 1.11.$$

**3.1** An alternating emf of frequency 50 Hz, has an amplitude of 100 V. Write down the equation for the instantaneous value. Also find the instantaneous value of the emf after 1/600 second.

***Solution***

The instantaneous equation for the emf is

$$e = 100 \sin 2\pi ft = 100 \sin 2\pi \times 50t = 100 \sin 100\pi t$$

At $t = \frac{1}{600}$ sec,

$$e = 100 \sin 100\pi \times \frac{1}{600} = 100 \sin \frac{100 \times 180°}{600}$$

$$= 100 \sin 30° = 50\ \text{A}.$$

**3.2** An alternating current has rms value of 50 A and frequency 60 Hz. Find the time taken to reach 50 A for the first time.

***Solution***

Rms value = 50 A i.e $I_{rms} = 50$ A

So $I_m = 50\sqrt{2} = 70.71$ A.

The instantaneous equation of the current is

$$i = I_m \sin 2\pi ft = 70.71 \sin 2\pi \times 60t = 70.71 \sin 120\,\pi t$$

when $i = 50$ A

$$50 = 70.71 \sin 120\,\pi t$$

$$\therefore \quad \sin 120\,\pi t = \frac{50}{70.71} = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$$

Hence $t = \dfrac{1}{120 \times 4} = 2.08$ m.sec.

**3.3** An alternating sinusoidally varying voltage with angular frequency of 314 radian/second has an average value of 127.4 V. Find the instantaneous value of the emf (a) $\frac{1}{300}$ and (b) $\frac{1}{75}$ after passing through a positive maximum value.

***Solution***

$$E_{av} = 127.4 \text{ V}$$

$$\therefore \quad E_m = \frac{E_{av}}{0.637} = 200 \text{ V.}$$

Reckoning the time from the instant when the voltage waveform has maximum value, the equation of the sinusoidal voltage wave is $e = E_m \cos \omega t = 200 \cos 314\, t$.

(a) When $t = \dfrac{1}{300}$ sec

$$e = 200 \cos 314\, t = 200 \cos 100\,\pi \times \frac{1}{300}$$

$$= 200 \cos\frac{\pi}{3} = 100 \text{ V}$$

(b) When $t = \dfrac{1}{75}$

$$e = 200 \cos 314\, t = 200 \cos 100\,\pi t$$

$$= 200 \cos 100\pi \times \frac{1}{75} = 200 \cos\frac{4\pi}{3} = -100 \text{ V}$$

**3.4** An alternating voltage is given by the equation $v = 282.84 \sin\left(377t + \frac{\pi}{6}\right)$. Find the (a) rms value, (b) frequency, and (c) the time period.

***Solution***

(a) $V_m = 282.84$ V

$$V_{rms} = \frac{282.84}{\sqrt{2}} = 200 \text{ V}$$

(b) $\omega = 377$ rad/s

$$f = \frac{377}{2\pi} = \frac{377}{3.14 \times 2} = 60 \text{ Hz}$$

(c) $T = \dfrac{1}{f} = \dfrac{1}{60} = 0.0167$ sec.

**3.5** If the form factor of a current wave form is 2 and the amplitude factor is 2.5, find the average value of the current if the maximum value of the current is 500 A.

***Solution***

$$K_f = 2 \text{ and } K_a = 2.5, I_m = 500 \text{ (Given)}$$

Therefore $$K_f = 2 = \frac{I_{rms}}{I_{av}} \text{ and } K_a = 2.5 = \frac{I_m}{I_{rms}} = \frac{500}{I_{rms}}$$

So $$I_{rms} = \frac{500}{2.5} = 200 \text{ A and } I_{av} = \frac{I_{rms}}{2} = \frac{200}{2} = 100 \text{ A.}$$

**3.6** Find the average and rms value of the wave form shown in Fig. 3.5.

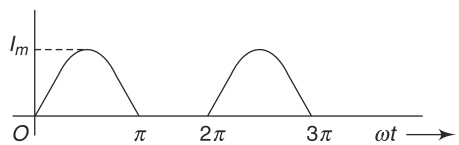



**Fig. 3.5** ***Waveform of Ex. 3.6***

***Solution***

$$I_{av} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin \omega t \, d(\omega t)$$

$$= \frac{I_m}{2\pi}[-\cos \omega t]_0^{\pi} = \frac{I_m}{\pi} = 0.318 \, I_m$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t \, d(\omega t)} = \sqrt{\frac{I_m^2}{4\pi}\int_0^{\pi}(1 - \cos 2\,\omega t)\, d(\omega t)}$$

$$= \frac{I_m}{2}\sqrt{\frac{1}{\pi}\left[\omega t - \frac{\sin^2 \omega t}{2}\right]_0^{\pi}} = \frac{I_m}{2}\sqrt{\frac{1}{\pi}(\pi)} = 0.5 \, I_m.$$

**3.7** Find the rms and average value of the waveform shown in Fig. 3.6

***Solution***

$$E_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} E_m^2 \sin^2 \omega t \, d(\omega t)}$$

$$= \sqrt{\frac{E_m^2}{2\pi}\int_0^{\pi}(1 - \cos 2\,\omega t)\, d(\omega t)}$$

$$= \frac{E_m}{\sqrt{2\pi}}\sqrt{\left[\omega t - \frac{\sin 2\,\omega t}{2}\right]_0^{\pi}}$$

$$= \frac{E_m}{\sqrt{2\pi}}\sqrt{\pi} = \frac{E_m}{\sqrt{2}}$$

$$= 0.707 \, E_m$$

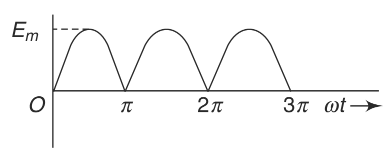



**Fig. 3.6** ***Waveform of Ex. 3.7***

$$E_{av} = \frac{1}{\pi}\int_0^{\pi} E_m \sin \omega t \, d(\omega t) = \frac{E_m}{\pi}[-\cos \omega t]_0^{\pi} = \frac{E_m}{\pi} \times 2$$

$$= 0.637 \, E_m.$$

**3.8** Find the average and rms of the wave form shown in Fig. 3.7.

***Solution***

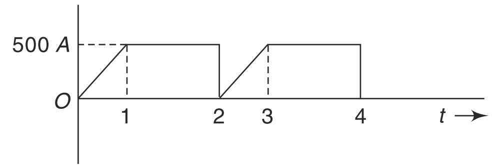



**Fig. 3.7** ***Waveform of Ex. 3.8***

$$I_{av} = \frac{1}{2}\int_0^2 i\,dt = \frac{1}{2}\left[\int_0^1 i_1\,dt + \int_1^2 i_2\,dt\right]$$

$$i_1 = \frac{500}{1}\,t + 0 = 500t$$

$$i_2 = 500$$

$$\text{Therefore } I_{av} = \frac{1}{2}\left[\int_0^1 500t\,dt + \int_1^2 500\,dt\right] = \frac{1}{2}\left[500\left[\frac{t^2}{2}\right]_0^1 + 500\,[t]_1^2\right]$$

$$= \frac{1}{2}\left[500 \times \frac{1}{2} + 500\right] = \frac{1}{2} \times 750 = 375 \text{ A}$$

$$I_{rms} = \sqrt{\frac{1}{2}\left\{\int_0^1 i_1^2\,dt + \int_1^2 i_2^2\,dt\right\}}$$

$$= \sqrt{\frac{1}{2}\left\{\int_0^1 (500t)^2\,dt + \int_1^2 (500)^2\,dt\right\}}$$

$$= \frac{500}{\sqrt{2}}\sqrt{\left[\frac{t^3}{3}\right]_0^1 + [t]_1^2}$$

$$= \frac{500}{\sqrt{2}}\sqrt{\frac{1}{3} + 1} = \frac{500}{\sqrt{2}}\,\frac{2}{\sqrt{3}} = 408.24 \text{ A.}$$

## 3.3 PHASOR REPRESENTATION OF AN ALTERNATING QUANTITY

Alternating quantities have varying magnitude and direction. So they are represented by a rotating vector. A phasor is a vector rotating at a constant angular velocity.

Let us consider that an *alternating* or *sinusoidal quantity* be represented by a phasor $Oa$. It rotates in the counter clockwise direction with a velocity of ($\omega$) radian/s as shown in Fig. 3.8. The projection of this vector on the vertical axis gives the instantaneous value $e$ of the induced emf (i.e sin $\omega t$). When $\omega t = 0$, then instantaneous

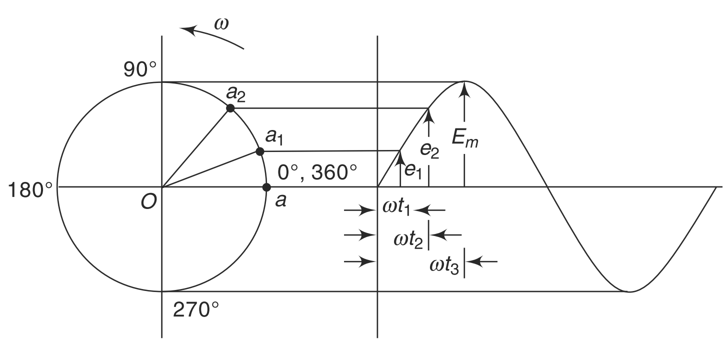



**Fig. 3.8** ***Phasor representation of alternating quantity***

value = $Oa$ sin $\omega t$ = 0. When $\omega t = \pi/2$, the instantaneous value = $Oa$ sin $\pi/2$; $Oa$ = $E_m$ (peak value).

The instantaneous value of emf at various intervals of time are:

at $t_1$, $e_1 = E_m \sin \omega t_1$,

at $t_2$, $e_2 = E_m \sin \omega t_2$

at $t_3$, $e_3 = E_m \sin \omega t_3$; and so on.

Phasor diagram is one in which different alternating or sinusoidal quantities of the same frequency are represented by phasors with their phase relationship.

Now consider two similar single turn coils $A$ and $B$ displaced from each other by an angle ($\theta$) rotating in a uniform magnetic field with the same angular velocity [Fig. 3.9(a)]. Suppose the emf wave of coil $A$ passes through zero in the positive direction at instant $t$ = 0 and at the same instant emf of coil $B$ attains a fixed positive value due to its advancement through an angle ($\theta$) from its zero value [Fig. 3.9(b)]. This can be represented as a still picture with the help of phasors in the phase diagram. [Fig. 3.9(c)] Obviously the angle between the two phasors is the phase difference between the two emfs.

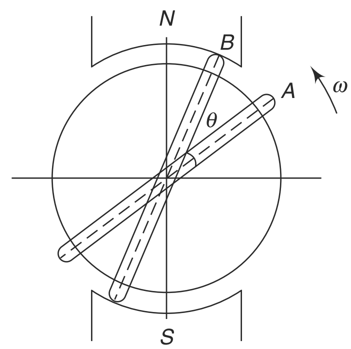



**Fig. 3.9(a)** ***Coil rotating in magnetic field***

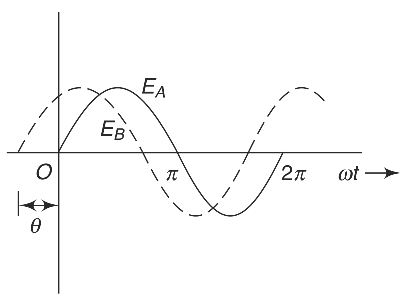



**Fig. 3.9(b)** ***Phasor diagram of ac emf***

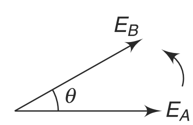



**Fig. 3.9(c)** ***Phase difference between $E_A$ and $E_B$***

It should be noted that normally phasors are drawn to represent the *rms* values and the reference phasor is drawn horizontally, e.g, the phasor ($E_A$). Also the phasors are assumed to rotate in the anticlockwise direction. So the phasor ahead in an anticlockwise direction from a given reference phasor is said to be *leading*, e.g., ($E_B$) leads phasor ($E_A$) by angle ($\theta$). The phasor which is behind the reference phasor is said to be *lagging*.

### 3.3.1 Addition and Subtraction of Sinusoidal Alternating Quantities

Draw the phasor diagram of the alternating quantity and then resolve each phasor into its horizontal and vertical components. Then add or substract the horizontal components and the vertical components separately. Suppose $I_x$ represents the addition or subtraction of the phasors in the horizontal axis and $I_y$ represents the addition and subtraction in the vertical axis. The diagonal of the rectangle formed by $I_x$ and $I_y$ denotes the resultant phasor $I$ as shown in Fig. 3.10. The magnitude of $I$ is given by

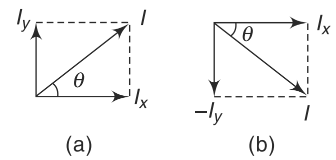



**Fig. 3.10** ***(a) Addition of alternating quantities (b) Subtraction of alternating quantities***

$$|I| = \sqrt{I_x^2 + I_y^2}$$

If $\theta$ represents the angle between the resultant phasor and the reference phasor (or horizontal line) then

$$\theta = \tan^{-1} \frac{I_y}{I_x}$$

### 3.3.2 Graphical Method

Let us take an example of adding voltages: $v_1 = 8 \sin (\omega t - 30°)$ and $v_2 = 6 \sin (\omega t + 45°)$.

Magnitude of $v_1$ is 8 and that of $v_2$ is 6, i.e., $V_1 = 8$ V and $V_2 = 6$ V. Choose the scale 1 cm = 2 V. Draw one horizontal line $OP$ as the reference line. Draw $OA = 8/2 = 4$ cm at –30° and $OB = 6/2 = 3$ cm at an angle of 45° with respect to the reference $OP$ to represent ($V_1$) and ($V_2$) respectively. Complete the parallelogram $OACB$. The diagonal of the parallelogram, i.e., $OC$ represents the resultant voltage $V_r$ (Fig. 3.11). By measurement $OC = 5.58$ cm. So $OC = (5.58 \times 2) = 11.16$ V. The angle $\theta$ between $OC$ and $OP = 1.2°$ (by measurement). So, $v_r = 11.16 \sin (\omega t + 1.2°)$ volts or $v_r = 11.16 \angle 1.2°$ V.

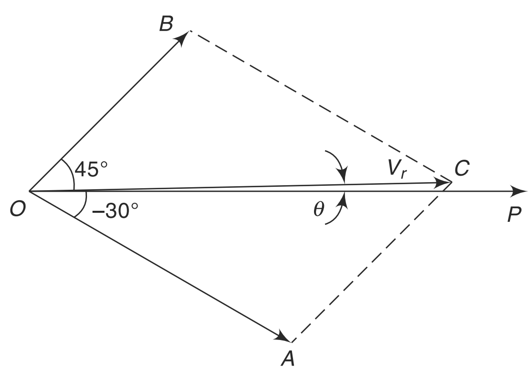



**Fig. 3.11** ***Addition of two vectors (graphical method)***

### 3.3.3 Analytical Method

At first draw the phasor diagram. The horizontal component of the resultant voltage

$$V_x = 8 \cos (-30°) + 6 \cos 45° = 8 \times \frac{\sqrt{3}}{2} + 6 \times \frac{1}{\sqrt{2}} = 11.17$$

The vertical component of the resultant voltage

$$V_y = 8 \sin(-30°) + 6 \sin(45°)$$
$$= -8 \times \frac{1}{2} + 6 \times \frac{1}{\sqrt{2}} = 0.24.$$

So the resultant voltage as shown in Fig. 3.12 is

$$V_r = \sqrt{V_x^2 + V_y^2} = \sqrt{11.17^2 + 0.24^2}$$
$$= 11.1726 \text{ V}.$$
$$\theta = \tan^{-1}\frac{V_y}{V_x} = \tan^{-1}\frac{0.24}{11.17} = 1.23°$$

i.e. $V_r = 11.1726 \sin(\omega t + 1.23°)$ V.

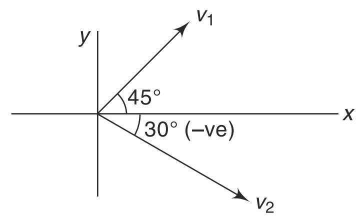



**Fig. 3.12** ***Addition of two phasors (analytical method)***

## 3.4 AC VOLTAGE AS APPLIED TO PURE RESISTANCE, PURE INDUCTANCE AND PURE CAPACITANCE

### 3.4.1 AC through Pure Resistance Alone

When a pure resistance is placed across a sinusoidal emf [Fig. 3.13(a)], the current will be in phase with the emf [Fig. 3.13(b)]. The corresponding phasor diagram is shown in Fig. 3.13(c):

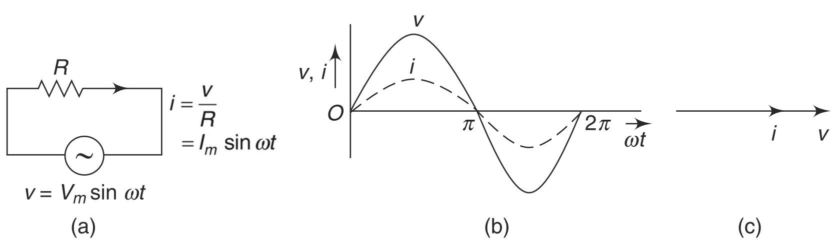



**Fig. 3.13** ***(a) AC through pure resistance (b) Phasor diagram of voltage and current through R (c) phasor diagram of voltage and current through R.***

The current is given by, $i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t \, I_m \sin \omega t$

where $I_m = \frac{V_m}{R}$.

Also, $I = \frac{V}{R}$

where $V$ = rms value of the appplied voltage

$I$ = rms value of current

and $R$ = resistance in ohms.

### 3.4.2 AC through Pure Inductance Alone

Whenever an alternating sinusoidal voltage is applied to a purely inductive coil [Fig. 3.14(a)] a back emf is produced due to the self-inductance of the coil. The applied voltage has to overcome this self-induced emf and therefore, $v = L\dfrac{di}{dt}$, where $L$ is the self-inductance of the coil, $v$ the back emf and $(di/dt)$ the rate of change of current.

i.e. $$\frac{di}{dt} = \frac{v}{L} = \frac{V_m}{L}\sin\omega t \qquad [\because \quad v = V_m \sin \omega t]$$

or $$i = \frac{V_m}{L}\int \sin\omega t\, dt = -\frac{V_m}{\omega L}\cos\omega t = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

or $$i = \frac{V_m}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

where $X_L = \omega L$ (= inductive reactance)

$\therefore$ $$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right), \text{ where } I_m = \frac{V_m}{X_L}.$$

So the current *lags* behind the voltage by $\left(\dfrac{\pi}{2}\right)$ and the phasor diagram is shown in Fig. 3.14(b) and (c).

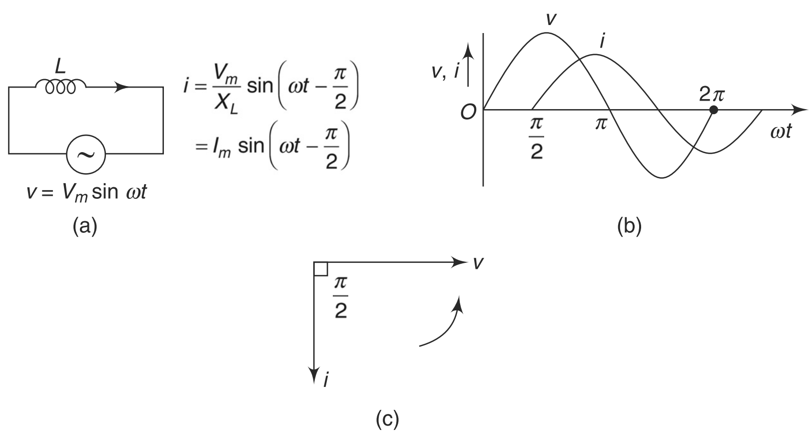



**Fig. 3.14** ***(a) AC through pure inductance (b) Phasor diagram of voltage and current through L (c) Current lags voltage by 90° in pure inductive circuit***

Also, $I = \dfrac{V}{X_L}$; where $I$ = rms value of the current

$V$ = rms value of the voltage and

$X_L = \omega L$ = inductive reactance in ohms.

### 3.4.3 AC through Pure Capacitance Alone

If a sinusoidal voltage is applied to the plates of a capacitor [Fig. 3.15(a)] then the instantaneous charge in the capacitors $q = Cv$, where $v$ is the instantaneous value of the applied voltage and $C$ is the capacitance.

If current $i$ is the rate of flow of charge, then

$$i = \frac{dq}{dt} = C\frac{d}{dt}v = C\frac{d}{dt}(V_m \sin \omega t)$$

$$= V_m \omega C \cos \omega t$$

$$= \frac{V_m}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right) = \frac{V_m}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

where $X_C = \dfrac{1}{\omega C}$ (= capacitive reactance).

Also, $i = I_m \sin\left(\omega t + \dfrac{\pi}{2}\right)$, where $I_m = \dfrac{V_m}{X_C}$.

So the current *leads* the applied voltage by $\left(\dfrac{\pi}{2}\right)$ and the phasor diagram is shown in Fig. 3.15(b) and (c).

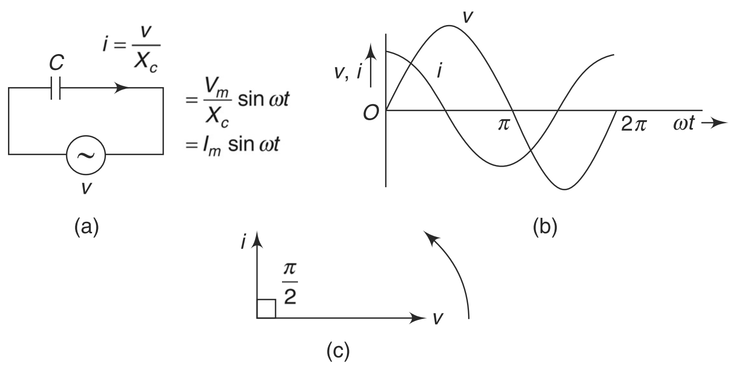



**Fig. 3.15** ***(a) AC through pure capacitance (b) Phasor diagram of voltage and current through pure capacitance (c) Current leads voltage by 90° in pure capacitance circuit.***

Also $I = \dfrac{V}{X_C}$, where $I$ = rms value of the current,

$V$ = rms value of the voltage

and $X_C$ = capacitive reactance in ohms.

## 3.5 SERIES RL CIRCUIT

Consider a coil of resistance $R$ ohms and inductance $L$ henries. The coil is represented by $R$ in series with $L$ [Fig. 3.16(a)]. Let $V$ = rms value of applied voltage, $I$ = rms value of resultant current, $V_R$ = voltage drop across $R$ and $V_L$ = voltage drop across $L$.

In the phasor diagram of Fig. 3.16(b) the current $I$ flowing in the circuit is drawn in the horizontal axis as reference. $V_R$ is drawn in the same direction as that of $I$ and $V_R = IR$. $V_L$ is drawn leading with respect to $I$ by 90° and $V_L = IX_L$. The resultant of the phasors $V_R$ and $V_L$ gives the supply voltage $V$. The magnitude of the supply voltage is $|V| = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = |I|\sqrt{R^2 + X_L^2} = |Z|$, where $Z$ is the *impedance* of the circuit and is expressed in ohms;

also, $$|I| = \frac{|V|}{|Z|} \text{ and } |Z| = \sqrt{R^2 + X_L^2}$$

or $$(\text{Impedance})^2 = (\text{Resistance})^2 + (\text{Inductive reactance})^2.$$

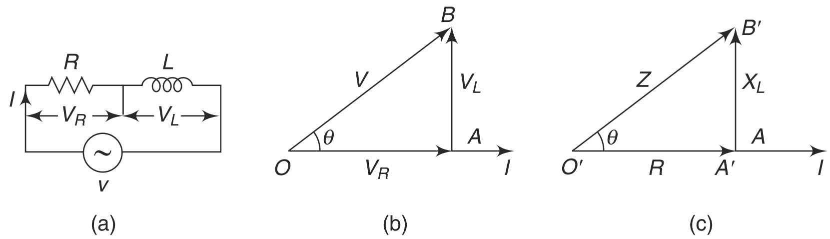



**Fig. 3.16** ***(a) AC through inductive coil (b) Voltage triangle (c) Impedance triangle.***

Triangle $OAB$ [Fig. 3.16(b)] is called the *voltage triangle* and triangle $O'A'B'$ [Fig. 3.16(c)] is called the *impedance triangle*. It is noticed that current $I$ lags the applied voltage $V$ by an angle ($\theta$) where $\theta = \tan^{-1}\frac{V_L}{V_R} = \tan^{-1}\frac{IX_L}{IR} = \tan^{-1}\frac{X_L}{R}$. So, if $v = V_m \sin \omega t$ than, $I = I_m \sin(\omega t - \theta) = \frac{V_m}{Z} \sin(\omega t - \theta)$. The phasor diagrams of the applied voltage and current are shown in Fig. 3.17.

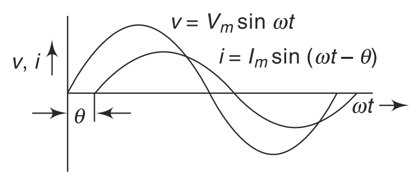



**Fig. 3.17** ***Phasor diagram of voltage and current through inductive coil.***

## 3.6 SERIES *RC* CIRCUIT

Consider a simple ac circuit in which a resistor of $R$ ohms and capacitance of $C$ farad are connected in series [Fig. 3.18(a)]. Let $V$ = rms value of applied voltage, $I$ = rms value of resultant current, $V_R$ = voltage drop across $R$ and $V_C$ = voltage drop across $C$.

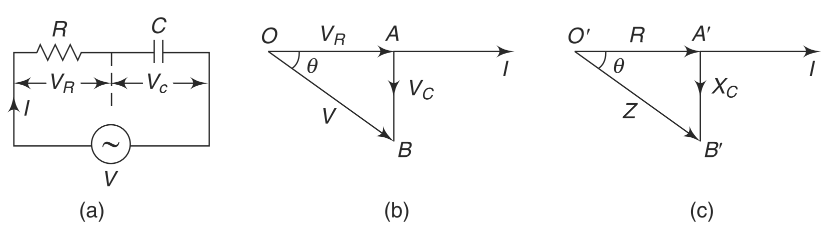



**Fig. 3.18** ***(a) AC through series R circuit (b) Voltage triangle (c) Impedance triangle.***

In the phasor diagram of Fig. 3.18(b) the current $I$ flowing in the circuit is drawn in the horizontal axis as reference $V_R$ is drawn in the same direction as that of $I$ and

$V_R = IR$. $V_C$ is drawn lagging with respect to $I$ by 90° and $V_C = IX_C$. The resultant of the vectors $V_R$ and $V_C$ gives the supply voltage $V$. The magnitude of the supply voltage is

$$|V| = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2} = |I|\sqrt{R^2 + X_C^2} = |IZ|.$$

where $Z$ is the impedance of the circuit and is expressed in ohms.

So, $$|I| = \frac{|V|}{|Z|} \quad \text{and} \quad |Z| = \sqrt{R^2 + X_C^2}$$

or (Impedance)$^2$ = (Resistance)$^2$ + (Capacitive reactance)$^2$.

Triangle $OAB$ [Fig. 3.18(b)] is called the *voltage triangle* and the triangle $O'A'B'$ [Fig. 3.15(c)] is called the *impedance triangle*. It is seen that the current $I$ leads the applied voltage $V$ by an angle $\theta$ where $\theta = \tan^{-1}\frac{V_C}{V_R} = \tan^{-1}\frac{IX_C}{IR} = \tan^{-1}\frac{X_C}{R}$. So, if $v = V_m \sin \omega t$; then $i = I_m \sin(\omega t + \theta) = \frac{V_m}{Z}\sin(\omega t + \theta)$. The phasor diagram of the applied voltage and currents are shown in Fig. 3.19.

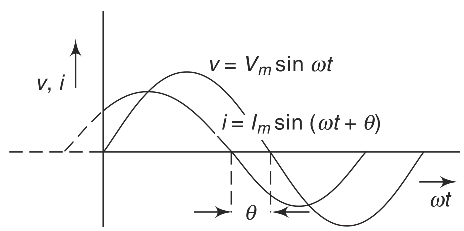



**Fig. 3.19** ***Phasor diagram of voltage and current in RC circuit.***

## 3.7 SERIES *RLC* CIRCUIT

Consider a simple series ac circuit containing a resistor of resistance $R$ ohms, an inductor of inductance $L$ henries and a capacitor of capacitance $C$ farad across an ac supply of rms voltage $V$ volts [Fig. 3.20(a)].

$I$ = rms value of the current flow in the circuit

$V_R$ = rms value of voltage across $R = IR$

$V_L$ = rms value of voltage across $L = IX_L$

and $V_C$ = rms value of the voltage across the capacitor = $IX_C$.

In the voltage triangle $OAB$ [Fig. 3.20(b)] $OA$, $AC$ and $AD$ represents $V_R$, $V_L$ and $V_C$ repectively. If $|V_L| > |V_C|$ then $AB$ represents the resultant of $(V_L - V_C)$. The vector sum of $(V_R)$ and $(V_L - V_C)$ gives the resultant voltage $(V)$.

Hence, $$|V| = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2} = I\sqrt{R^2 + X^2} = IZ$$

where $X$ = Net reactance in ohm = $(X_L - X_C)$

The phase angle of ($V$) is given by, $\theta = \tan^{-1}\dfrac{(V_L - V_C)}{(V_R)}$

$$= \tan^{-1}\frac{(IX_L - IX_C)}{IR}$$

$$= \tan^{-1}\frac{(X_L - X_C)}{R}$$

$$= \tan^{-1}\frac{X}{R};\ X\ (\text{say}) = (X_L - X_C),$$

If, $v = V_m \sin \omega t,$

then $i = \dfrac{V_m}{Z}\sin(\omega t - \theta) = I_m \sin(\omega t - \theta)$

Hence, when $|V_L| > |V_C|$, we have

$X_L > X_C$ and current $I$ is lagging with respect to $V$ by an angle less than 90°.

In the voltage triangle $O'A'B'$ in Fig. 3.20(c) $|V_C| > |V_L|$. $OA'$ represents $V_R$, $A'C'$ represents $V_L$ and $A'D'$ represents $V_C$. The phasor $(V_C - V_L)$ is represented by $A'B'$ and $O'B'$ denotes resultant voltage $V$.

Here, $|V| = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$

$$= I\sqrt{R^2 + (X_L - X_C)^2} = I\sqrt{R^2 + X^2} = IZ$$

where $X = X_L - X_C$ = Net reactance in ohms.

The phase angle of $V$ is given by, $\theta = \tan^{-1}\dfrac{(V_L - V_C)}{V_R}$

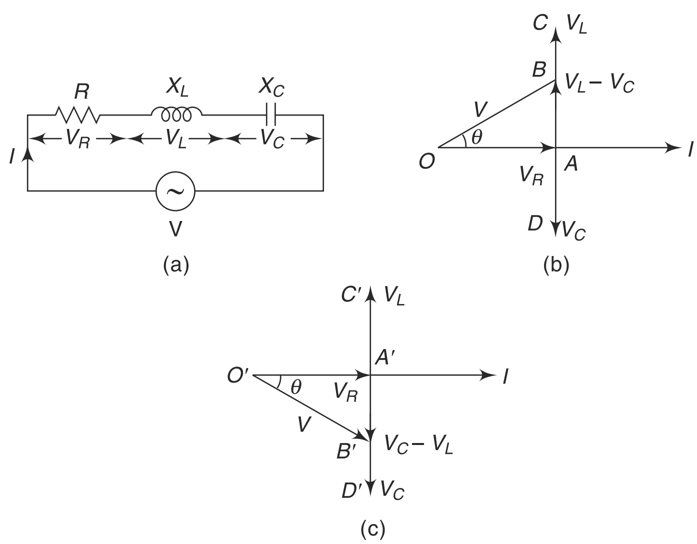



**Fig. 3.20** ***(a) AC through RLC series circuit. (b) Voltage triangle for lagging p.f. (c) Voltage triangle for leading p.f.***

$$= \tan^{-1} \frac{IX_L - IX_C}{IR}$$

$$= \tan^{-1} \frac{X_L - X_C}{R} = \tan^{-1} \frac{X}{R}.$$

If $v = V_m \sin \omega t$ then

$$i = \frac{V_m}{Z} \sin (\omega t + \theta) = I_m \sin (\omega t + \theta).$$

When $V_C > V_L$ or $IX_C > IX_L$ or $X_C > X_L$ then current $I$ is leading with respect to the resultant voltage $V$ by an angle less than 90°.

The impedance triangle when $V_L > V_C$ is shown in Fig. 3.21(a) and the impedance triangle when $V_C > V_L$ is shown in Fig. 3.21(b).

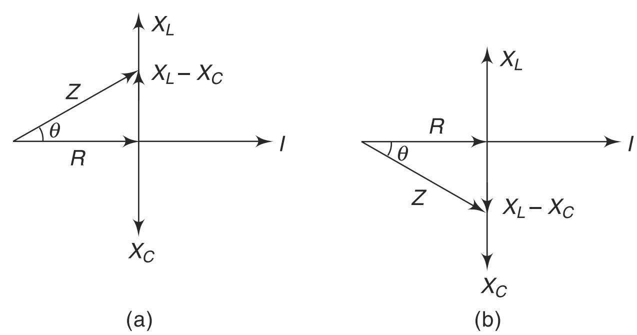



**Fig. 3.21** ***(a) Impedance triangle for lagging p.f. (b) Impedance triangle for leading p.f***

## 3.8 IMPEDANCES IN SERIES

When several impedances are connected in series the net impedance can be found out by using the following steps:

(a) Add all the resistances in the circuit to get total $R$.
(b) Add all the inductive reactances to get total $X_L$.
(c) Add all the capacitive reactances to get total $X_C$.
(d) Total impedance is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$

[*Note:* all additions in step (b) and (c) are phasor additions.]

**3.9** A coil has an inductance of 50 mH and negligible resistance. Find its reactance at 100 Hz.

***Solution***

$$L = 50 \text{ mH} \quad \text{and} \quad f = 100 \text{ Hz}$$

Inductive reactance $X_L = \omega L = 2\pi f L$, where $\omega$ is the angular frequency.

So, $X_L = 2 \times 3.14 \times 100 \times 50 \times 10^{-3}$

$= 31.4159\ \Omega.$

**3.10** If the frequency of applied voltage is 5 kHz, calculate the reactance of a 10 μF capacitor.

***Solution***

$$f = 5 \text{ kHz} = 5000 \text{ Hz} \quad \text{and} \quad C = 10\ \mu\text{F} = 10 \times 10^{-6} \text{ F}$$

$$\therefore \quad \text{Capacitive reactance } (X_C) = \frac{1}{\omega C} = \frac{1}{2\pi fc} = \frac{1}{2 \times 3.14 \times 5000 \times 10 \times 10^{-6}}$$

$$= \frac{10^6}{2 \times 3.14 \times 5000 \times 10} = 3.18\ \Omega$$

**3.11** A circuit containing a (a) resistance of 20 Ω alone (b) inductance of 10 mH alone and (c) capacitance of 300 μF alone is connected across an alternating voltage source; write the expressions for the current when $v = 100 \sin 100\ \pi t$.

***Solution***

$$v = 100 \sin 100\ \pi t, \quad \therefore \quad V_m = 100 \text{ V and } \omega = 100\ \pi \text{ rad/s.}$$

(a) $R = 20\ \Omega \quad \therefore i_R = \dfrac{V_m}{R} \sin 100\ \pi t = \dfrac{100 \sin 100\ \pi t}{20} = 5 \sin 100\ \pi t$

(b) $L = 10 \text{ mH} = 0.01 \text{ H}$

Therefore $X_L = \omega L$ (= Inductive reactance) = $100\ \pi \times 0.01 = 3.14\ \Omega$.

$$i_L = \frac{V_m}{X_L} \sin\left(100\pi t - \frac{\pi}{2}\right) = \frac{100}{3.14} \sin\left(100\pi t - \frac{\pi}{2}\right)$$

$$= 31.85 \sin\left(100\ \pi t - \frac{\pi}{2}\right)$$

(c) $C = 300\ \mu\text{F} = 300 \times 10^{-6} \text{ F}$

$$\therefore \text{ capacitive reactance } (X_C) = \frac{1}{\omega C} = \frac{10^6}{100\pi \times 300}\ \Omega$$

$$= 10.61\ \Omega$$

$$i_C = \frac{V_m}{X_C} \sin\left(100\pi t + \frac{\pi}{2}\right) = \frac{100}{10.61} \sin\left(100\pi t + \frac{\pi}{2}\right)$$

$$= 9.425 \left(\sin \pi t + \frac{\pi}{2}\right).$$

**3.12** A coil of resistance 100 Ω and inductive reactance 200 Ω is connected across a supply voltage of 230 V. Find the supply current.

***Solution***

$R = 100\ \Omega, X_L = 200\ \Omega$

$$\text{Impedance } |Z| = \sqrt{R^2 + X_L^2} = \sqrt{(100)^2 + (200)^2} = 223.61\ \Omega$$

$$\therefore \text{ supply current } I = \frac{V}{Z} = \frac{230}{223.61} = 1.028 \text{ A.}$$

**3.13** A circuit takes a current $i = 50 \sin\left(314t - \dfrac{\pi}{3}\right)$ when the supply voltage is $v = 400 \sin 314\ t$. Find the impedance, resistance, and the inductance of the circuit.

***Solution***

$$v = 400 \sin 314\ t$$

$$i = 50 \sin\left(314 - \frac{\pi}{3}\right)$$

$$I_m = 50 \text{ A}$$

and $\theta = \frac{\pi}{3}$

Hence $V_m = 400$ V and $\omega = 314$ rad/s

$\therefore$ Impedance $|Z| = \frac{|V|}{|I|} = \frac{V_m}{I_m} = \frac{400}{50} = 8\ \Omega$

$$\theta = \frac{\pi}{3} = \tan^{-1}\frac{X_L}{R} \quad \text{or,} \quad \frac{X_L}{R} = \tan\frac{\pi}{3} = 1.732 = \sqrt{3}$$

$\therefore$ $X_L = 1.732\ R$ or, $x_L^2 = (1.732)^2 R^2$ or, $Z^2 - R^2 = 3R^2$

or $4R^2 = Z^2 = (8)^2 = 64.$ So, $R = \sqrt{\frac{64}{4}} = 4\ \Omega$

Thus, $X_L = 1.732 \times 4 = 6.93\ \Omega$

Therefore $L = \frac{X_L}{\omega} = \frac{6.93}{314} = 0.022$ H $= 0.022$ mH.

**3.14** When a resistor and coil in series are connected to a 240 V supply, a current of 5 A is flowing lagging 60° behind the supply voltage, and the voltage across the coil is 220 V. Find the resistance of the resistor and the resistance and reactance of coil.

***Solution***

Let $R_L$ be the resistance of the coil and $X_L$ be the reactance of the coil. If $\theta$ be the angle of the current then, $\cos\theta = \cos 60° = 0.5 = R/Z$ where $R$ and $Z$ are the resistance and reactance of the whole circuit respectively.

Therefore, $R = Z \times 0.5$

But, $|Z| = \frac{|V|}{|I|} = \frac{240}{5} = 48\ \Omega$

$R = 48 \times 0.5 = 24\ \Omega$

Also, $\frac{X_L}{Z} = \sin 60° = 0.866.$

Hence $X_L = 48 \times 0.8666 = 41.57\ \Omega.$

Now, impedance of the coil $= \sqrt{R_L^2 + X_L^2} = \frac{220}{5} = 44\ \Omega$

$\therefore$ $R_L = \sqrt{(44)^2 - (41.57)^2} = 14.42\ \Omega.$

Thus resistance of the resistor is (24 – 14.42) = 9.58 Ω, resistance of coil is 14.42 Ω and reactance of coil is 41.57 Ω.

**3.15** When a certain inductive coil is connected to a dc supply at 200 V, the current in the coil is 10 A. When the same coil is connected to an ac supply at 200 V, 50 Hz the current is 8 A. Calculate the resistance and reactance of the coil.

***Solution***

For dc the reactance of the coil is zero ($\because f = 0$).

Hence, resistance of the coil $= \frac{200}{10} = 20\ \Omega$

For ac supply, impedance $= \frac{200}{8} = 25\ \Omega$

Hence reactance of the coil $= \sqrt{(25)^2 - (20)^2} = \sqrt{625 - 400} = 15\ \Omega$

**3.16** A 200 V, 120 W lamp is to be operated on 240 V, 50 Hz. supply. Calculate the value of the capacitor that would be placed in series with the lamp in order that it may be used at its rated voltage.

***Solution***

Let $R$ be the resistance of the lamp as shown in Fig. 3.22. The current flowing through the

$$\text{circuit} = \frac{P}{V} = \frac{120}{200} = 0.6 \text{ A.}$$

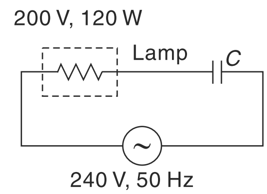



**Fig. 3.22** ***Circuit for Ex. 3.16***

It $Z$ be the impedance of the whole circuit,

$$|Z| = \frac{|V|}{|I|} = \frac{240}{0.6} = 400 \ \Omega.$$

Now, $$\frac{V^2}{R} = P \text{ or, } R = \frac{(200)^2}{120} = 333.33 \ \Omega.$$

Hence the capacitive reactance is

$$X_C = \sqrt{Z^2 - R^2}$$

$$= \sqrt{(400)^2 - (333.33)^2} = 221.11 \ \Omega.$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2 \times 3.14 \times 50 \times 221.11} \text{ F}$$

$$= 0.0000144 \text{ F} = 14.4 \ \mu\text{F.}$$

Hence the value of the capacitor is 14.4 μF.

**3.17** A capacitor and a 50 Ω resistor are connected in series to an alternating current supply. The voltage across the capacitor is 200 V rms and across the resistor is 150 V rms. Determine (a) rms value of supply voltage, (b) peak value of the voltage across the capacitor assuming sinusoidal wave form, (c) power used in the resistor.

***Solution***

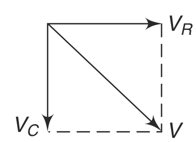



**Fig. 3.23** ***Determination of*** $|V|$

Resistance $R = 50 \ \Omega$

Voltage acrosss resistor, $V_R = 150$ V

Voltage across capacitor $V_C = 200$ V

$$\therefore \quad \text{current } |I| = \frac{V_R}{R} = \frac{150}{50} = 3 \text{ A.}$$

$$\text{Supply voltage} = \sqrt{V_R^2 + V_C^2} = \sqrt{(150)^2 + (200)^2} = 250 \text{ V.}$$

$$\text{Peak value of the voltage across capacitor} = \sqrt{2} \ V_{C \text{ rms}}$$

$$= \sqrt{2} \times 200 = 282.8 \text{ V.}$$

$$\text{Power used in the resistor} = I^2 R = (3)^2 \times 50$$

$$= 450 \text{ W.}$$

**3.18** A resistance of 10 Ω is connected in series with an inductance of 0.05 H and a capacitance of 300 μF to a 100 V ac supply. Calculate the value and phase angle of the current when the frequency is (a) 25 Hz (b) 50 Hz.

***Solution***

(a) $f = 25$ Hz, $R = 10 \ \Omega$, $L = 0.05$ H, $C = 300 \times 10^{-6}$ F;
$V = 100$ V; Hence $X_L = 2\pi f L = 2\pi \times 25 \times 0.05 = 7.85 \ \Omega$,

and $$X_C = \frac{1}{2\pi f C} = \frac{10^6}{2 \times 3.14 \times 25 \times 300} = 21.23\ \Omega.$$

∴ impedance $$|Z| = \sqrt{R^2 + (X_C - X_L)^2}$$

$$= \sqrt{(10)^2 + (21.23 - 7.85)^2} = 16.7\ \Omega;$$

and net reactance $|X| = |X_C - X_L| = 13.38\ \Omega.$

As $X_C > X_L$ so the current is leading.

If $\theta$ be the angle of lead then,

$$\theta = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{13.38}{10} = 53.23°.$$

Current $$|I| = \frac{|V|}{|Z|} = \frac{100}{16.7} = 5.988 \text{ A}.$$

(b) $f = 50$ Hz

So $$X_L = 2\pi \times 50 \times 0.05 = 15.7\ \Omega$$

and $$X_C = \frac{10^6}{2\pi \times 50 \times 300} = 10.61\ \Omega.$$

∴ $$|X_L - X_C| = 5.09\ \Omega$$

and $$Z = \sqrt{R^2 + (X_L - X_C)^2} = 11.22\ \Omega$$

As $X_L > X_C$ so the current is lagging.

It $\theta$ be the angle of lag then,

$$\theta = \tan^{-1}\frac{5.09}{10} = 26.97°$$

and current $$|I| = \frac{100}{11.22} = 8.91 \text{ A}.$$

**3.19** A 230 V, 50 Hz voltage is applied to a coil $L = 5$ H and $R = 2\ \Omega$ in series with a capacitance $C$. What value must $C$ have in order that the voltage across the coil be 400 V?

***Solution***

Impedance of the coil $= \sqrt{R^2 + X_L^2} = \sqrt{2^2 + (2\pi \times 50 \times 5)^2} = 1570\ \Omega$

Voltage across the coil = 400 V

∴ current $I = \frac{400}{1570} = 0.2547$ A

Impedance of the circuit $= \frac{230}{0.2547} = 903.023\ \Omega.$

If $X_C$ be the capacitive reactance, we have

$$\sqrt{R^2 + (X_L \sim X_C)^2} = 903.023$$

∴ $$2^2 + (100\ \pi \times 5 \sim X_C)^2 = (903.023)^2$$

$$500\ \pi \sim X_C = 903.023$$

So $$X_C = 666.78\ \Omega \text{ and } X_L > X_C$$

∴ $$C = \frac{1}{2\pi \times 50 \times 666.78} F = 4.776\ \mu\text{F}.$$

**3.20** A voltage of 400 V is applied to a series circuit containing a resistor, an inductor and a capacitor. The respective voltages across the components are 250 V, 200 V and 180 V and the current is 5 A. Determine the phase angle of the current.

***Solution***

Resistance $R = \frac{250}{5} = 50\ \Omega$

Inductive reactance $|X_L| = \frac{250}{5} = 40\ \Omega$.

Capacitive reactance $|X_C| = \frac{180}{5} = 36\ \Omega$

Impedance $|Z| = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50)^2 + (40 - 36)^2}$

$= \sqrt{2500 + 16} = 50.16\ \Omega$

$\therefore$ Phase angle of the current $= \tan^{-1} \frac{X}{R}$ $(X = X_L - X_C) = \tan^{-1} \frac{4}{50} = 4.57°$ lagging.

## 3.9 PARALLEL AC CIRCUIT

Two circuits are said to be connected in parallel if the voltage across them is the same. Consider a parallel ac circuit where an inductive coil is in parallel with a resistor and capacitor in series [Fig. 3.24(a)]. The inductive coil in branch 1 consists of a resistance of $R_L$ Ω and inductance $L$ henry. The resistance in the other branch, i.e branch 2 is $R_C$ and the capacitance is $C$ Farad. So for branch 1 $Z_1 = \sqrt{R_L^2 + X_L^2}$ and $I_1 = V/Z_1$, where $V$ is the supply voltage and $X_L = \omega L$ (the inductive reactance). The phase angle of the current $\theta_1 = \tan^{-1} \frac{X_L}{R_L}$.

Similarly, for branch 2

$Z_2 = \sqrt{R_C^2 + X_C^2}$, where $X_C = \frac{1}{\omega C}$ is the capacitive reactance and $I_2 = \frac{V}{Z_Z}$.

The phase angle of the current is $\theta_2 = \tan^{-1} \frac{X_C}{R_C}$.

The current $I_1$ lags behind the applied voltage by $\theta_1$ and current $I_2$ leads the applied voltage by $\theta_2$ as shown in Fig. 3.24(b).

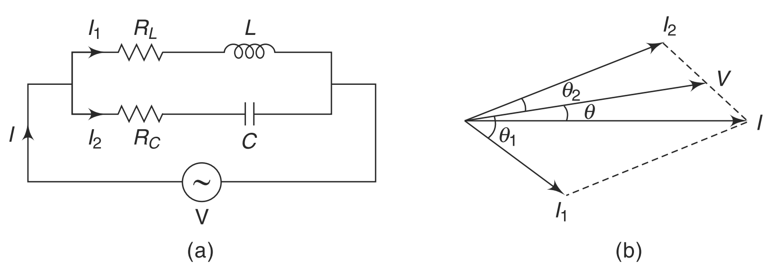



**Fig. 3.24** ***(a) RL circuit in parallel with RC circuit (b) Phasor diagram of voltage and current of Fig. 3.24(a)***

The resultant current $I$ is the vector sum of $I_1$ and $I_2$. Resolving $I_1$ and $I_2$ into the $X$ and $Y$ components and then by adding or subtracting [as in Fig. 3.25 (a)] we get

Sum of $X$ axis components of $I_1$ and $I_2 = I_1 \cos \theta_1 + I_2 \cos \theta_2$.

Sum of $Y$ axis components of $I_1$ and $I_2 = -I_1 \sin \theta_1 + I_2 \sin \theta_2$.

If $\theta$ be the phase angle of the resultant current $I$ then

$$I\cos\theta = I_1\cos\theta_1 + I_2\cos\theta_2$$

$$I\sin\theta = -I_1\sin\theta_1 + I_2\sin\theta_2.$$

Squaring the above two equations on both sides and then by adding, we get

$$I^2\cos^2\theta + I^2\sin^2\theta = (I_1\cos\theta_1 + I_2\cos\theta_2)^2 + (-I_1\sin\theta_1 + I_2\sin\theta_2)^2$$

$$I = \sqrt{(I_1\cos\theta_1 + I_2\cos\theta_2)^2 + (-I_1\sin\theta_1 + I_2\sin\theta_2)^2}$$

and the phase angle $\theta = \tan^{-1}\dfrac{-I_1\sin\theta_1 + I_2\sin\theta_2}{I_1\cos\theta_1 + I_2\cos\theta_2}$

The resultant current is shown in Fig. 3.25(b). If $\theta$ is positive the current $I$ leads the applied voltage $V$ and if $\theta$ is negtative the current $I$ lags the applied voltage $V$.

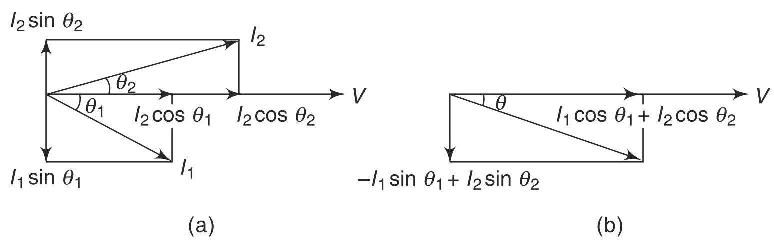



**Fig. 3.25** ***Branch currents in ac parallel circuit***

## 3.10 ADMITTANCE, CONDUCTANCE AND SUSCEPTANCE OF AN AC CIRCUIT

Admittance $Y$ is the reciprocal of impedance $Z$ of an ac circuit.

$$Y = \frac{1}{Z} = \frac{I}{V}$$

Just as impedance has two components viz., resistance $R$ and reactance $X$, the admittance also has two components, viz. *conductance G* along the horizontal axis and *susceptance B* along the vertical axis [Fig. 3.26].

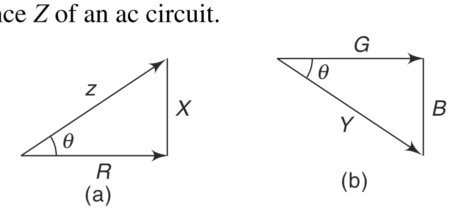



**Fig. 3.26** ***(a) Impedance triangle (b) Admittance triangle***

Hence, conductance $(G) = Y\cos\theta = \dfrac{1}{Z}\cdot\dfrac{R}{Z} = \dfrac{R}{Z^2}$

and susceptance $(B) = Y\sin\theta = \dfrac{1}{Z}\cdot\dfrac{X}{Z} = \dfrac{X}{Z^2}$

Admittance $(Y) = \sqrt{G^2 + B^2}$

The units of $Y$, $G$ and $B$ are mho or ohm$^{-1}$ or Siemens (S). It is to be noted that *inductive susceptance is considered negative and capacitive susceptance is considered positive*.

***Use of Admitance in Solving Parallel Circuits***

Consider a three branch parallel circuit, as shown in Fig. 3.27.

The total conductance, $G = g_1 + g_2 + g_3$

and total susceptance, $B = -b_1 + b_2$

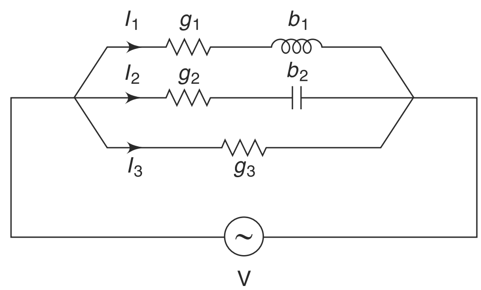



**Fig. 3.27** ***Parallel ac circuit***

Current in branch 1, $I_1 = Vy_1$

Current in branch 2, $I_2 = Vy_2$

Current in branch 3, $I_3 = Vy_3$

Net admittance $Y = \sqrt{G^2 + B^2}$

$= \sqrt{(g_1 + g_2 + g_3)^2 + (-b_1 + b_2)^2}$

$= y_1 + y_2 + y_3$.

The current, $I = (V \times y)$ and the phase angle of the current is, $\theta = \tan^{-1} \dfrac{B}{G}$.

## 3.11 AVERAGE POWER IN AC CIRCUITS

Power in a dc circuit is given by $P_{dc} = VI = I^2R = \dfrac{V^2}{R}$. In an ac circuit the instantaneous power is the power at any instant of time. It is equal to the product of voltage and current at that instant.

$$p = v.i.$$

Like voltage and current, power is also continuously changing with time. So the average power is given by

$$P = \frac{1}{T}\int_0^T p\,dt$$

By convention, $P$ always means average power and no subscript is used.

Also, $$P = \frac{1}{2\pi}\int_0^{2\pi} p\,d\theta = \frac{1}{2\pi};\int_0^{2\pi} vi\,d\theta$$

Average power is also called *active power* or *real power* or *true power*. Its unit is watts.

### 3.11.1 Power in a Purely Resistive Circuit

In a purely resistive circuit voltage and current are in phase. Hence, $v = V_m \sin\theta$ and $i = I_m \sin\theta$.

Instantaneous power $p = vi = V_m I_m \sin^2\theta = \dfrac{1}{2} V_m I_m (1 - \cos 2\theta)$. The voltage, current and power waveform are shown in Fig. 3.28.

The power waveform in Fig. 3.28 is obtained by multiplying together at every instant the corresponding (instantaneous) values of voltage and current. It is seen that $p$ remains positive throughout the cycle irrespective of the direction of voltage and

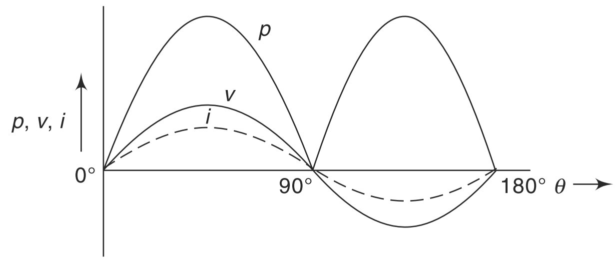



**Fig. 3.28** ***Instantaneous power in a pure resistive circuit***

current in the circuit. This is due to the fact that as voltage and current are in phase so either both voltage and current are positive or both are negative at any instant of time. So their product ($p$) is always positive. This shows that power flow is only in the direction from the source to the load resistance ($R$) and this power is called active or real or true power ($P_R$).

$$\text{Active power } (P_R) = \frac{1}{2\pi}\int_0^{2\pi} p\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} V_m I_m (1 - \cos 2\theta)\, d\theta$$

$$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} d\theta - \frac{V_m I_m}{4\pi}\int_0^{2\pi} \cos 2\theta\, d\theta$$

$$= \frac{V_m I_m}{4\pi} \times 2\pi - \frac{V_m I_m}{4\pi}\left[\frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{4\pi} \times 0 = \frac{V_m I_m}{2}$$

$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = VI$$

In a purely resistive circuit $P_R = VI = (IR)I = I^2R = \left(\frac{V}{R}\right)^2 R = \frac{V^2}{R}$.

Also, the active or real power in ac circuit is $VI \cos \theta$. A simple reasoning leads to the conclusion that p.f. of a pure resistive circuit is 1(one) ans $\cos \theta$ is 1 ($\because$ $VI \cos \theta = VI$, in pure resistive circuit). $VI = (V^2/R) = I^2R$ is the energy dissipated in resistive circuit.

### 3.11.2 Power in a Purely Inductive Circuit

In a purely inductive circuit current lags the applied voltage by 90°.

So, $v = V_m \sin \theta$ and $i = I_m \sin\left(\theta - \frac{\pi}{2}\right)$

$$\text{Instantantaeous power } (p) = vi = V_m \sin \theta\, I_m \sin\left(\theta - \frac{\pi}{2}\right)$$

$$= -\frac{1}{2} \times 2V_m I_m \sin \theta \cos \theta = -\frac{1}{2} V_m I_m \sin 2\theta.$$

The voltage, current and power waveform are shown in Fig. 3.29.

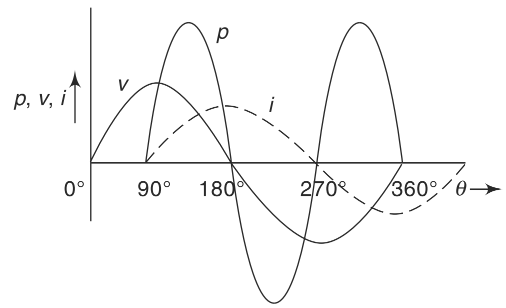



**Fig. 3.29** ***Instantaneous power in pure inductive circuit***

The power waveform in Fig. 3.29 is obtained by multiplying at every instant together the corresponding (or instantaneous) values of voltage and current.

The power curve is a sine wave of twice the frequency of the current and voltage wave. During the first quarter cycle, the power curve is above the horizontal axis and is positive. The circuit draws energy from the source. This energy is stored in the magnetic field of the inductance. During the second quarter cycle the power curve is below the horizontal axis and is negative. The previously stored energy is now returned to the source. Thus energy stored in the circuit during the first quarter cycle is equal to the energy returned to the source during the second quarter cycle in ideal inductive circuits. So the total energy dissipated, called the *active energy*, during every cycle of the current is zero. The rate of energy dissipated, called the active or average power over the complete cycle of the current in a purely inductive circuit, is also zero.

$$\text{Active power } P_L = \frac{1}{2\pi}\int_0^{2\pi} p\,d\theta = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2}\sin 2\theta d\theta$$

$$= \frac{V_m I_m}{4\pi} \times \frac{1}{2}[\cos 2\theta]_0^{2\pi}$$

$$= -\frac{V_m I_m}{8\pi}[\cos 4\pi - \cos 0]$$

$$= -\frac{V_m I_m}{8\pi}[1 - 1]$$

$$= 0.$$

*Thus in a purely inductive circuit the active power over a complete cycle is zero.*

The peak value of ($p$) is $\frac{V_m I_m}{2} = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}} = VI$

*Reactive power $Q_L$*

In a purely inductive circuit $V = V_L = X_L I$, where $X_L$ is the inductive reactance.

$$\text{Reactive power } Q_L = V_L I = I^2 X_L$$

or

$$Q_L = \frac{V_L^2}{X_L}\ VAR \text{ (volt-ampere reactive).}$$

$Q_L$ is called the *reactive volt amperes* for an inductive circuit. It is measured in VAR. The energy which is continually exchanged between the source and the reactive load is called the *reactive energy*. By convention $Q_L$ is considered positive for inductive circuits.

Also, $Q_L = VI \sin \theta$, ($\theta$) being 90° for pure inductive circuits. Obviously (cos $\theta$) for inductance is zero.

### 3.11.3 Power in a Purely Capacitive Circuit

In a purely capacitive circuit the current leads the applied voltage by 90°.

So, $$v = V_m \sin \theta \text{ and } i = I_m \sin\left(\theta + \frac{\pi}{2}\right)$$

Instantaneous power $$p = vi = V_m I_m \sin \theta \sin\left(\theta + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin \theta \cos \theta = \frac{1}{2} V_m I_m \cdot 2 \sin \theta \cos \theta$$

$$= \frac{V_m I_m}{2} \sin 2\theta.$$

The voltage, current and power waveform are shown in Fig. 3.30.

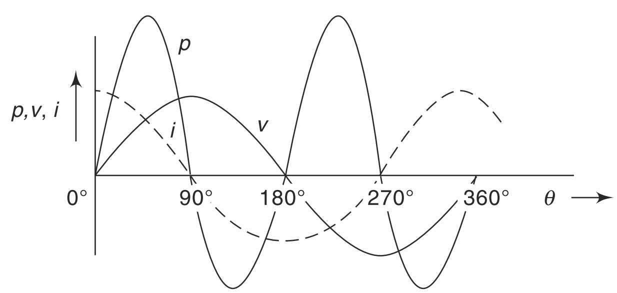



**Fig. 3.30** ***Instantaneous power impure capacitive circuit***

The power curve is a sine wave of twice the frequency of the current or voltage curve. During the first quarter cycle the power curve is above the horizontal axis and is positive. The circuit draws energy from the source and the capacitor is charged. The energy is stored in the electric field of the capacitor. During the second quarter cycle the power curve is below the horizontal and is negative. The capacitor is discharged and the energy from the dielectric field is returned to the source. The energy stored in the electric field during the first quarter cycle is equal to the energy returned to the source during the second quarter cycle in a purely capacitive circuit. Therefore the total active energy during each cycle of the current is zero. The active power over a complete cycle of current in a purely capacitive circuit is zero.

Active power$$(P_C) = \frac{1}{2\pi}\int_0^{2\pi} p \, d\theta = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} d\theta \sin 2\theta \, d\theta$$

$$= \frac{V_m I_m}{4\pi}\left[\frac{-\cos 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{-V_m I_m}{8\pi}[\cos 4\theta - \cos 0°]$$

$$= \frac{-V_m I_m}{8\pi}[1 - 1] = 0$$

*Reactive power ($Q_C$)*

The peak value of ($p$) is $\frac{I_m V_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = VI.$

In a purely capacitive circuit

$$V = V_C = IX_C,$$

$$\therefore \qquad Q_C = V_C I = I^2 X_C = \frac{V_C^2}{X_C}\text{ VAR.}$$

$Q_C$ is called the reactive volt amperes for a capacitive circuit. It is measured in VAR. It is the rate of interchange of reactive energy between a capacitive load and the source. By convention $Q_C$ is considered negative. Obviously, $Q_C = VI \sin\theta$, $\theta$ being 90°. Power factor of such a circuit is also zero.

### 3.11.4 Power in a General Series Circuit

Consider a general case where $v = V_m \sin\theta$ and $i = I_m \sin(\theta - \phi)$, where $\phi$ is the phase angle of the current with respect to the voltage.

Instantaneous power $p = vi$

$$= V_m I_m \sin\theta \sin(\theta - \phi)$$

$$= \frac{V_m I_m}{2}[\cos\phi - \cos(2\theta - \phi)]$$

Active power

$$P = \frac{1}{2\pi}\int_0^{2\pi} p\, d\theta = \frac{V_m I_m}{2\pi \times 2}\int_0^{2\pi}[\cos\phi - \cos(2\theta - \phi)]\, d\theta$$

$$= \frac{V_m I_m}{4\pi}\int_0^{2\pi}\cos\phi\, d\theta - \frac{V_m I_m}{4\pi}\int_0^{2\pi}\cos(2\theta - \phi)\, d\theta$$

$$= \frac{V_m I_m}{4\pi}\cos\phi\,[\theta]_0^{2\pi} - \frac{V_m I_m}{4\pi}\left[\frac{1}{2}\sin(2\theta - \phi)\right]_0^{2\pi}$$

$$= \frac{(\sqrt{2}V)(\sqrt{2}I)}{4\pi}(\cos\phi)(2\pi) - \frac{V_m I_m}{8\pi}[\sin(4\pi - \phi) - \sin(-\phi)]$$

$$= VI\cos\phi - \frac{V_m I_m}{8\pi}[-\sin\phi + \sin\phi]$$

$$= VI\cos\phi.$$

The voltage, current and power waveform are shown in Fig. 3.31.

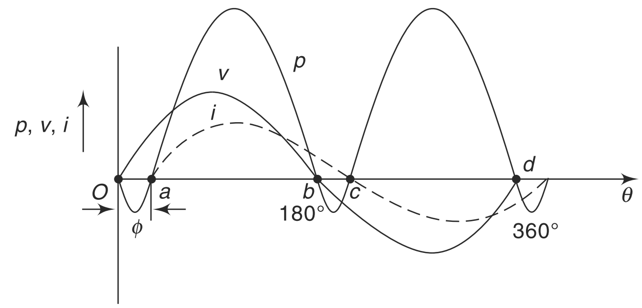



**Fig. 3.31** ***Instantaneous power in RLC series circuit***

From the power curve it is observed that during the interval *Oa* the power is negative. During the interval *ab* the power is positive. The interval *bc* is the repetition of interval *Oa* and interval *cd* is the repetition of interval *ab*. The negative area under curve *p* between interval *Oa* represents the energy returned from the circuit to the source. The positive area under curve *p* in the interval *ab* represents the energy supplied from the source to the load. So, during each current or voltage cycle a part of the energy called active energy is consumed, while the other part called the reactive energy is interchanged between the source and the load. The rate of energy consumption is the active power. The difference between the total positive and total negative areas during a cycle of current or voltage gives the net active energy of the circuit.

### 3.11.5 Voltamperes Power (Complex Power)

The product of rms values of voltage and current in a circuit is called the *circuit voltamperes*. It is also called *apparent power* or *complex power*. It is denoted by *S* and is measured in voltamperes VA.

$$S = VI = (IZ)I = I^2Z;\ \text{Also, } S = P + jQ = \sqrt{P^2 + Q^2}\left(\tan^{-1}\frac{Q}{P}\right)$$

### 3.11.6 Power Triangle

From the previous sections we know that

$P_R = V_R I$ and $Q_R = 0$, for purely resistive circuit

$P_L = 0$ and $Q_L = (V_L I)$ for purely inductive circuit

$P_C = 0$ and $Q_C = (V_C I)$ for purely capacitive circuit.

The net reactive power in the RLC series circuit is $Q = Q_L - Q_C$

[$\because$ $Q_L$ is considered positive and $Q_C$ negative]

$$= I^2X_L - I^2X_C = I^2(X_L - X_C) = I^2X$$

The active power $P = P_R = I^2R.$

The impedance triangle is represented in Fig. 3.32(a)

Multiplying each side of the impedance triangle by $I^2$ we get the power triangle as shown in Fig. 3.32(b).

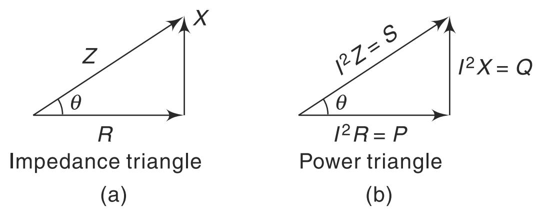



**Fig. 3.32** ***(a) Impedance triangle(a) (b) Power triangle***

Here, $P = S\cos\theta$ and $Q = S\sin\theta$

As $S = VI$,

Hence, $P = VI\cos\theta$ and $Q = VI\sin\theta$

Also $|S| = \sqrt{P^2 + Q^2}$ and $\theta = \tan^{-1} Q/P$.

A power triangle can be obtained from a voltage triangle by multiplying each of its sides by the current as shown in Fig. 3.33.

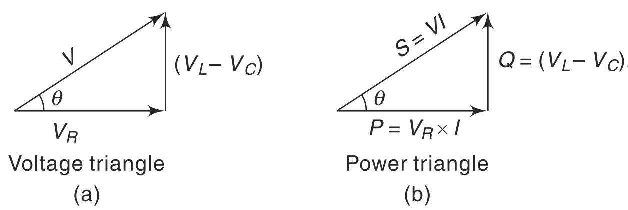



**Fig. 3.33** ***(a) Voltage triangle and (b) Power triangle***

### 3.11.7 Power Factor in Resistive, Inductive and Capacitive Circuit

The ratio of the active power to the apparent power in an ac circuit is defined as the *power factor* (p.f.) of the circuit.

$$\text{Power factor} = \frac{P}{S} = \frac{VI\cos\theta}{VI} = \cos\theta$$

*So power factor in an ac circuit is also equal to the cosine of the phase angle between the applied voltage and the circuit current.*

The power factor is lagging in a circuit in which the current lags the applied voltage. An inductive circuit has lagging power factor.

The power factor is leading in a circuit where the current leads the applied voltage. A capacitive circuit has the leading power factor.

From the impedance triangle, power factor is also given by

$$\cos\theta = \frac{R}{Z}.$$

From voltage triangle power factor can be obtained as

$$\cos\theta = \frac{V_R}{V}.$$

Combining all results,

$$\text{p.f.} = \cos\,\theta = \frac{P}{S} = \frac{R}{Z} = \frac{V_R}{V}.$$

For purely resistive circuit p.f. = cos 0° = 1.

For purely inductive and capacitive circuits p.f. = cos 90° = 0.

We know that power consumed in a circuit is $VI\cos\theta$

Power consumed in a purely resistive circuit = $VI$ cos 0° = $VI$

Power consumed in a purely inductive or purely capacitive circuit = $VI$ cos 90° = 0.

Hence, we can conclude that the power is consumed only in the resistor and there is no power consumption in either the pure inductor or the pure capacitor.

### 3.11.8 Active and Reactive Components of Current

When the current is not in phase with the voltage it lags or leads the applied voltage by an angle $\theta$. The component of the current which is in phase with the voltage namely ($I\cos\theta$) is called the *active component* of current. The other component which is in quadrature with the voltage namely ($I\sin\theta$) is called the *reactive component* of current.

**3.21** Two inductive coils $A$ and $B$ are connected in parallel across a 200 V, 50 Hz. supply. Coil $A$ takes 15 A and 0.85 p.f. and the supply current is 30 A and 0.8 p.f. Determine the (a) equivalent resistance and equivalent reactance and (b) resistance and reactance of each coil.

***Solution***

Considering supply voltage $V$ as the reference phasor,

$$V = 200\angle 0°$$

$$I_1 = 15\angle{-\cos^{-1} 0.85} = 15\angle{-31.79°}\text{ A}$$

The power factor is lagging since the coil is inductive.

Total current $I = 30\angle{-\cos^{-1} 0.8} = 30\angle{-36.86°}$ A.

(a) Equivalent impedance of the circuit

$$Z_{eq} = \frac{V}{I} = \frac{200\angle 0°}{30\angle{-36.86°}} = 6.67\angle 36.86°\Omega.$$

Hence, equivalent resistance ($R_{eq}$) = 6.67 cos 36.86° = 5.336 Ω and equivalent reactance ($X_{eq}$) = 6.67 sin 36.86° = 4 Ω.

(b) If $I_2$ be the current in coil $B$ lagging by $\theta$ angle with respect to the supply voltage, then the horizontal and vertical components of $I_2$ are

$$I_{2x} = I_2\cos(-\theta) \text{ and } I_{2y} = I_2\sin(-\theta)$$

Similarly, $I_{1x} = I_1\cos(-31.79)° = 15\cos(-31.79)° = 12.75$ A

and $I_{1y} = I_1\sin(-31.79°) = -15\sin 31.79° = -7.9$ A.

The two components of $I$ are

$$I_x = 30\cos(-36.86°) = 24\text{ A}$$

and $I_y = 30\sin(-36.86°) = -18$ A.

Since, $I_x = I_{1x} + I_{2x}$

and $I_y = I_{1y} + I_{2y}$

So $24 = 12.75 + I_{2x}$

or $I_{2x} = 24 - 12.75 = 11.25$ A

and $-18 = -7.9 + I_{2y}$

or $I_{2y} = +7.9 - 18 = -10.1$ A.

Thus the current in coil 2 is $\sqrt{(11.25)^2 + (10.1)^2} \angle \tan^{-1} \dfrac{-10.1}{11.25}$

$= 15.1186\angle -41.917°$ A.

The impedance of coil 2 is $(Z_2) = \dfrac{V}{I_2} = \dfrac{200\angle 0°}{15.1186\angle -41.917°}$

$= 13.229\angle 41.917°\ \Omega$.

The resistance of coil 2 is $(R_2) = 13.229 \cos 41.917° = 9.844\ \Omega$ and the reactance of coil 2 is $X_2 = 13.229 \sin 41.917° = 8.837\ \Omega$.

**3.22** Find the branch currents, total current, $Z$ and $Y$, apparent, active and reactive power and power factor in the parallel circuit shown in Fig. 3.34.

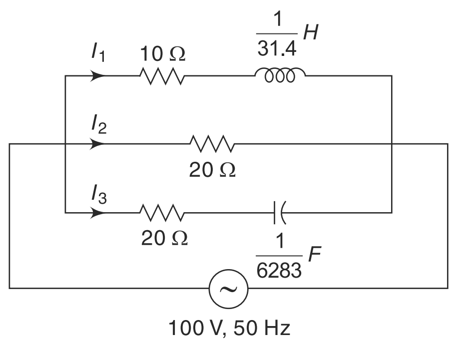



**Fig. 3.34** ***Circuit diagram for Ex. 3.22***

### *Solution*

Let us consider the supply voltage as reference.

The current in second branch 2,

$$I_2 = \frac{100\angle 0°}{20\ \Omega} = 5\angle 0° \text{ A.}$$

The current in branch 1, $I_1 = \dfrac{V}{z_1}$.

$$Z_1 = \sqrt{(10)^2 + \left(100\pi \times \frac{1}{31.4}\right)^2} \angle \tan^{-1} \frac{100\pi \times \frac{1}{31.4}}{10}$$

$$= 14.14\angle 45°\ \Omega$$

$$\therefore \quad I_1 = \frac{100\angle 0°}{14.14} \angle -45° = 7.07\angle -45° \text{ A.}$$

The impedance of branch 3, $Z_3 = \sqrt{(20)^2 + \left(\dfrac{6283}{100\pi}\right)^2} \angle -\tan^{-1} \dfrac{\frac{6283}{100\pi}}{20}$

$= 28.29\angle -45°\ \Omega$.

The current in branch 3, $I_3 = \dfrac{100\angle 0°}{28.29\angle -45°} = 3.53\angle 45°$ A.

Total admittance $y = y_1 + y_2 + y_3$

$$= \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}$$

$$= \frac{1}{14.14\angle 45°} + \frac{1}{20} + \frac{1}{28.29\angle -45°}$$

$$= 0.0707\angle -45° + 0.05 + 0.035\angle 45°.$$

Now, horizontal component of $y$ is

$$y_x = 0.0707 \cos(-45°) + 0.05 \cos 0° + 0.035 \cos 45°$$
$$= 0.1247 \text{ Siemens}$$

and vertical component of $y$ is

$$y_y = 0.0707 \sin(-45°) + 0.05 \sin 0° + 0.035 \sin 45°$$
$$= -0.025 \text{ Siemens.}$$

Therefore $$y = \sqrt{y_x^2 + y_y^2}\ \angle\tan^{-1}\frac{y_y}{y_x}$$

$$= \sqrt{(0.1247)^2 + (-0.025)^2}\ \angle\tan^{-1}\frac{-0.025}{0.1247}$$

$$= 0.127\angle{-11.336°} \text{ Siemens}$$

$\therefore$ $$Z = \frac{1}{y} = \frac{1}{0.127\angle{-11.336°}} = 7.87\angle 11.336°\ \Omega.$$

Total current $I = Vy = 100\angle 0° \times 0.127\angle{-11.336°} = 12.7\angle{-11.336°}$ A.

Apparent power $(VI) = 100 \times 12.7 = 1270$ VA.

Active power $(VI \cos\theta) = 100 \times 12.7 \cos(11.336°) = 1245.22$ W.

Reactive power $(VI \sin\theta) = 100 \times 12.7 \sin(11.336°) = 249.63$ VAR (inductive).

Power factor $(\cos\theta) = \cos 11.336° = 0.98$ lagging.

**3.23** A lamp rated 400 W takes a current of 4 A when in series with an inductance. (a) Find the value of the inductance connected in series to operate the combination from 240 V, 50 Hz mains (b) Also find the value of the capacitance which should be connected in parallel with the above combination to raise the overall power factor to unity.

***Solution***

The resistance of the lamp $R = \dfrac{P}{I^2} = \dfrac{400}{(4)^2} = 25\ \Omega.$

Voltage across the lamp $= IR = 4 \times 25 = 100$ V.

(a) The impedance of the circuit $\dfrac{V}{I} = \dfrac{240}{4} = 60\ \Omega.$

If $V_L$ is the voltage across inductance

$$V^2 = V_R^2 + V_L^2$$

or $$V_L = \sqrt{V^2 - V_R^2} = \sqrt{(240)^2 - (100)^2} = 218.17 \text{ V.}$$

So the inductive reactance $= \dfrac{V_L}{I} = \dfrac{218.17}{4} = 54.54\ \Omega$

and the inductance $= \dfrac{54.54}{\omega} = \dfrac{54.54}{2\pi \times 50} = 0.173$ H.

(b) Let $I_C$ be the current through the capacitance.

The current through the inductive coil is $I_L$. If the overall power factor is unity the vertical component of total current $(I)$ is 0.

$\therefore$ $$I_C + 4 \sin(-65.37°) = 0$$

or $$I_C = 4 \sin 65.37° = 3.636 \text{ A.}$$

Hence, capacitive reactance $(X_C) = \dfrac{V}{I_C} = \dfrac{100}{3.636} = 27.5\ \Omega$

and capacitance $= \dfrac{1}{100\pi \times 27.5} F = 115.79\ \mu$F.

**3.24** A fluorescent lamp taking 100 W at 0.75 p.f. lagging from a 240 V, 50 Hz. supply is to be corrected to unity p.f. Determine the value of the correcting apparatus required.

***Solution***

Power = 100 W

p.f. = 0.75 lagging

Voltage = 240 V

$VI \cos \theta = P,$

$$\therefore \quad I = \frac{100}{240 \times 0.75} = 0.555\angle -\cos^{-1} 0.75 = 0.555\angle -41.41° \text{ A}$$

and impedance $(Z) = \frac{V}{I} = \frac{240}{0.555}\ \Omega = 432.43\ \Omega.$

If the power factor becomes unity the net reactive components of current is zero and to improve p.f. from 0.75 lag to 1, a capacitance should be connected in parallel. If $I_C$ be the capacitive current,

net reactive component of current = $I_C + 0.555 \sin(-41.41°) = 0.$

So $I_C = 0.555 \sin 41.41° = 0.367$ A.

$\therefore$ capacitive reactance $(X_C) = \frac{V}{I_C} = \frac{240}{0.367} = 653.95\ \Omega$

and capacitance $= \frac{1}{100\pi \times 653.95}$ F = 4.87 μF.

**3.25** A 4 kW load takes a current of 20 A from a 240 V ac supply. Calculate the kVA and KVAR of the load.

***Solution***

$V = 240$ V

$I = 20$ A

$P = 4$ kW.

If $\cos \theta$ be the power factor then

$VI \cos \theta = P$ or, $240 \times 20 \times \cos \theta = 4000$

or $$\cos \theta = \frac{4000}{240 \times 20} = 0.833.$$

Therefore, $\sin \theta = 0.553$

$$\text{kVA } (= VI) = \frac{240 \times 20}{10^3} = 4.8 \text{ kVA}$$

and $$\text{KVAR } (= VI \sin \theta) = \frac{240 \times 20 \times 0.553}{10^3} \text{ KVAR} = 2.65 \text{ KVAR}.$$

**3.26** A 240 V, single phase induction motor delivers 15 kW at full load. The efficiency of the motor at this load is 82% and the p.f. is 0.8 lagging. Calculate (a) the input current of the motor, (b) the kW input and (c) kVA input.

***Solution***

$$\left.\begin{aligned} V &= 240 \text{ V} \\ \eta &= 82\% \\ \cos \theta &= 0.8 \text{ lag} \end{aligned}\right\} \text{(given)}$$

Output power = 15 kW = 15,000 W

So, input power $(P) = \dfrac{\text{Output power}}{\text{Efficiency}} = \dfrac{15{,}000}{0.82} = 18292.68$ W.

(a) If $I$ be the input current then $VI \cos\theta = P$

or $I = \dfrac{P}{V\cos\theta} = \dfrac{18292.68}{240 \times 0.8}$ A = 95.27 A.

(b) kW input $= \dfrac{18292.68}{10^3} = 18.29$.

(c) kVA input $= VI = \dfrac{240 \times 95.27}{10^3} = 22.86$.

**3.27** A single phase 50 Hz motor takes 100 A at 0.85 p.f. lagging from a 240 V supply. Calculate the (a) active and reactive components of the current and (b) the power taken from the supply.

***Solution***

$$I = 100 \text{ A}$$
$$\cos\theta = 0.85$$
$$V = 240 \text{ V.}$$

(a) Active component of current $(I\cos\theta) = 100 \times 0.85 = 85$ A.

Reactive component of current $(I\sin\theta) = 100\sqrt{1 - (0.85)^2}$
$= 52.67$ A.

(b) Real power taken from the supply $(VI\cos\theta) = 240 \times 100 \times 0.85$
$= 20400$ W = 20.4 kW.

**3.28** A series $RL$ circuit having $R = 15\ \Omega$ amd $L = 0.03$ H is connected across a 240 V, 50 Hz. supply. Find the (a) rms current in the circuit; (b) average power absorbed by the inductance and (c) the power factor of the circuit.

***Solution***

$$R = 15\ \Omega$$
$$X_L = 2\pi fL = 2\pi \times 50 \times 0.03 = 9.42\ \Omega$$
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(15)^2 + (9.42)^2} = 17.71\ \Omega.$$

(a) rms current $|I| = \dfrac{V}{Z} = \dfrac{240}{17.71} = 13.55$ A.

(b) Average power absorbed by the inductance is 0.

(c) Power factor of the circuit $\dfrac{R}{Z} = \dfrac{15}{17.71} = 0.847$ lag.

**3.29** A 200 V, 50 Hz. inductive circuit takes a current of 15 A, lagging the voltage by 45°. Calculate the resistance and inductance of the circuit.

***Solution***

$$V = 200 \text{ V}$$
$$I = 15 \text{ A}$$
$$\cos\theta = \cos 45° = 0.707$$

Impedance $Z = \frac{V}{I} = \frac{200}{15} = 13.33\ \Omega$.

Resistance $R = Z \cos \theta = 13.33 \cos 45° = 9.42\ \Omega$.

Inductive reactance $X_L = Z \sin \theta = 13.33 \sin 45° = 9.42\ \Omega$.

Hence the inductance $L = \frac{X_L}{\omega} = \frac{9.42}{2\pi \times 50} = 0.03$ H.

**3.30** A 2-element series circuit consumes 700 W and has a p.f. of 0.707 leading. If the applied voltage is $v = 141 \sin (314t + 30°)$ find the circuit constants.

***Solution***

As p.f is leading so the circuit contains a capacitor along with a resistor.

Power ($P$) = 700 W

p.f. ($\cos \theta$) = 0.707, hence ($\sin \theta$) = $\sin (\cos^{-1} 0.707) = 0.707$

Instantaneous voltage ($v$) = $141 \sin (314\,t + 30°)$

rms value of voltage ($V$) = $\frac{141}{\sqrt{2}} = 100$ V.

Angular frequency ($\omega$) = 314 rad/s.

If $I$ be the rms value of current then,

$$P = VI \cos \theta$$

or
$$I = \frac{P}{V \cos \theta} = \frac{700}{100 \times 0.707}\ \text{A} = 9.9\ \text{A}.$$

Now, Impedance $Z = \frac{V}{I} = \frac{100}{9.9}\ \Omega = 10.1\ \Omega$.

$\therefore$ Resistance ($R$) = $Z \cos \theta = 10.1 \times 0.707 = 7.14\ \Omega$ and capacitive reactance ($X_C$) = $Z \sin \theta$ = $10.1 \times 0.707 = 7.14\ \Omega$.

Therefore, capacitance $C = \frac{1}{\omega X_C} = \frac{1}{314 \times 7.14}$ F = 446 μF.

Hence the circuit constants are 7.14 Ω and 446 μF.

**3.31** A circuit takes a current of 3 A at a p.f of 0.6 lagging when connected to a 115 V, 50 Hz supply. Another circuit takes a current of 5 A at a p.f. of 0.707 leading when connected to the same supply. If the two circuits are connected in series across a 230 V, 50 Hz. supply, calculate (a) the current (b) the power consumed and (c) the p.f. of the circuit.

***Solution***

As the p.f. is lagging in the first circuit so it contains a resistor along with an inductor. The second circuit contains a resistor along with a capacitor as the p.f. is leading in that circuit. The circuit is shown in Fig. 3.35.

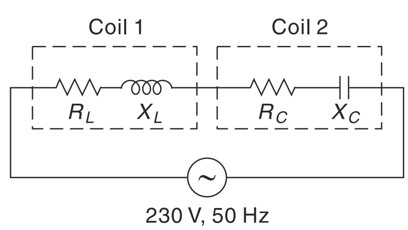



**Fig. 3.35** ***Circuit diagram for Ex. 3.31***

Supply voltage $V = 230$ V,

Frequency $f = 50$ Hz.

For circuit 1,

$$I = 3\ \text{A}, \cos \theta = 0.6\ \text{lag, and}\ V = 115\ \text{V}$$

$$\therefore \quad |Z| = \frac{V}{I} = \frac{115}{3} = 38.33\ \Omega$$

$$R_L = Z \cos\theta = 38.33 \times 0.6 = 22.998\ \Omega$$

$$X_L = Z \sin\theta = 38.33 \times \sin(\cos^{-1} 0.6) = 30.664\ \Omega.$$

For circuit 2,

$$I = 5\text{ A}, \cos\theta = 0.707 \text{ lead and } V = 115\text{ V}.$$

$$\therefore \quad |Z| = \frac{V}{I} = \frac{115}{3} = 23\ \Omega$$

$$R_C = Z\cos\theta = 23 \times 0.707 = 16.261\ \Omega$$

$$X_C = Z\sin\theta = 23 \times \sin(\cos^{-1} 0.707)$$

$$= 16.261\ \Omega$$

when the two circuits are connected in series,

$$\text{Impedance} = \sqrt{(R_L + R_C)^2 + (X_L - X_C)^2}$$

$$= \sqrt{(39.259)^2 + (14.403)^2} = 41.81\ \Omega$$

(a) The current $(I) = \dfrac{V}{Z} = \dfrac{230}{41.81}$ A = 5.5 A.

(b) The power is consumed in the resistors only.

$\therefore$ the power consumed $= I^2(R_L + R_C)$

$= (5.5)^2\,(39.259)$

$= 1187.58$ W

$= 1.187$ kW.

(c) Power factor of the circuit $= \dfrac{\text{Net resistance}}{\text{Net impedence}}$

$= \dfrac{39.259}{41.81} = 0.94.$

As $X_L > X_C$ so the p.f. is lagging. Therefore p.f. of the circuit is 0.94 lagging.

**3.32** The impedances $Z_1$ and $Z_2$ are connected in parallel across a 200 V, 50 Hz single phase ac supply. $Z_1$ carries 2 A at 0.8 lag p.f. If the total current is 5 A at 0.985 lagging p.f., determine (a) value of $Z_1$ and $Z_2$ (b) total power and power consumed by $Z_2$.

***Solution***

For the Ist circuit,

$$V = 200\text{ V}$$

$$I_1 = 2\text{ A}$$

$$\cos\theta_1 = 0.8 \text{ lag} \quad \text{so} \quad \theta_1 = \cos^{-1} 0.8 = 36.86° \text{ (lag)}.$$

$$|Z_1| = \frac{V}{I} = \frac{200}{2} = 100\ \Omega. \text{ Also, } \sin\theta_1 = 0.6$$

Resistance $R_1 = Z_1 \cos\theta_1 = 100 \times 0.8 = 80\ \Omega$.

Inductive reactance $X_L = Z\sin\theta = 100 \times 0.6 = 60\ \Omega$.

Total current $(I) = 5$ A and net p.f. $(\cos\theta) = 0.985$ lag.

So $\quad \theta = \cos^{-1}(0.985) = 9.93°$

and $\quad \sin\theta = 0.172$

Total power $= VI\cos\theta$

$= 200 \times 5 \times 0.985$

$= 985$ W

Total $|Z| = \frac{V}{I} = \frac{200}{2} = 40\ \Omega.$

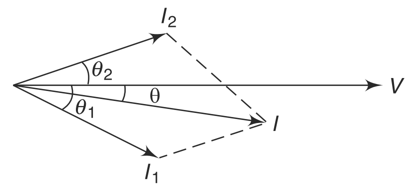



**Fig. 3.36** ***Phasor diagram***

The phasor diagram is shown in Fig. 3.36

Horizontal component of $I_1$ is $I_{1x} = I_1 \cos \theta_1$

$= 2 \times 0.8 = 1.6$ A

Horizontal component of $I$ is $I_x = I \cos \theta$

$= 5 \times 0.985 = 4.925$ A

Vertical component of $I_1$ is $I_{1y} = -I_1 \sin \theta_1 = 2 \times 0.6 = -1.2$ A

Vertical component of $I$ is $I_y = -I \sin \theta$

$= -5 \times 0.172 = -0.86$ A.

If $I_{2x}$ and $I_{2y}$ be the horizontal and vertical component of the current in the second circuit then

$$I_x = I_{1x} + I_{2x}$$

and $$I_y = I_{1y} + I_{2y}.$$

So, $$I_{2x} = I_x - I_{1x} = 4.925 - 1.6 = 3.325$$

and $$I_{2y} = I_y - I_{2y} = -0.86 + 1.2 = +0.34.$$

The current in circuit 2 is $(I_2) = \sqrt{I_x^2 + I_y^2} = \sqrt{(3.325)^2 + (0.34)^2}$

$= 3.34$ A.

Impedance $Z_2 = \frac{V}{I_2} = \frac{200}{3.34}\ \Omega = 59.88\ \Omega.$

Power factor of circuit 2 $= \frac{I_{2x}}{I_2} = 0.9955.$

Hence power consumed by $Z_2$ is $P_2 = VI_2 \cos \theta_2$

$= 200 \times 3.34\ (0.9955)$

$= 665$ W.

**3.33** An iron cored electromagnet has a dc resistance of 7.5 Ω and when connected to a 400 V, 50 Hz supply takes 10 A and consumes 2 kW. Calculate for this value of current (a) power loss in iron core, (b) the inductance of coil, (c) the p.f., and (d) the value of series resistance which is equivalent to the effect of iron loss.

***Solution***

When the electromagnet is connected to a dc source it is required to consider the resistance of the coil only.

Given, resistance of coil ($R_C$) = 7.5 Ω.

When connected to ac source both the resistance of the coil and the equivalent resistance of iron part should be considered.

However, $V = 400$ V; $I = 10$ A and $P$ (= 2 kW) = 2000 W.

Equivalent impedance $Z = \frac{V}{I} = \frac{400}{10} = 40\ \Omega.$

Power factor $(\cos \theta) = \frac{P}{VI} = \frac{2000}{400 \times 10} = 0.5.$

Total resistance = $Z \cos \theta = 40 \times 0.5 = 20\ \Omega$

Total reactance = $Z \sin \theta = 40 \sin (\cos^{-1} 0.5) = 34.64\ \Omega$.

$\therefore$ resistance of iron core = 20 – 7.5 = 12.5 Ω.

(a) Power loss in iron core is $I^2 \times 12.5 = (10)^2 \times 12.5 = 1250$ W = (1.25 kW).

(b) Inductance of coil = $\dfrac{34.64}{\omega} = \dfrac{34.64}{100\pi} = 0.1103$ H

(c) Power factor ($\cos \theta$) = 0.5

(d) The value of series resistance is 12.5 ohm which is equivalent to iron loss.

**3.34** An iron cored choking coil takes 4 A at p.f. of 0.5 when connected to a 200 V, 50 Hz. supply. When the core is removed and the applied voltage is reduced to 50 V, 50 Hz, the current is 8 A and the p.f. 0.8 lag. Calculate the (a) core loss and (b) inductance of the choke with and without the core.

***Solution***

With core,

$V = 200$ V; $\cos \theta = 0.5$ and $I = 4$ A.

Hence $|Z| = \dfrac{V}{I} = \dfrac{200}{4} = 50\ \Omega$.

Resistance of core along with coil = $Z \cos \theta = 50 \times 0.5 = 25\ \Omega$.

Reactance of the core and coil = $Z \sin \theta = 50 \sin (\cos^{-1} 0.5) = 43.3\ \Omega$.

Without core,

$V = 50$ V; $I = 8$ A and $\cos \theta = 0.8$.

Hence $|Z| = \dfrac{V}{I} = \dfrac{50}{8} = 6.25\ \Omega$.

Resistance of coil ($Z \cos \theta$) = $6.25 \times 0.8 = 5\ \Omega$

Reactance of coil ($Z \sin \theta$) = $6.25 \times \sin (\cos^{-1} 0.8) = 3.75\ \Omega$.

$\therefore$ Resistance of core = 25 – 5 = 20 Ω.

Core loss = $I^2 \times$ (Resistance of core) = $(4)^2 \times 20 = 320$ W.

$$\text{Inductance of choke with core} = \frac{43.3}{\omega} = \frac{43.3}{2\pi \times 50} = 0.13794$$
$$= 137.9 \text{ mH}$$

$$\text{Inductance of choke without core} = \frac{3.75}{\omega} = \frac{3.75}{2\pi \times 50} = 0.01194 \text{ H}$$
$$= 11.94 \text{ mH.}$$

**3.35** The following loads are connected in parallel:

(a) 100 kVA at 0.8 p.f. lagging,

(b) 250 kVA at 0.8 p.f. leading,

(c) 200 kVA at 0.6 p.f. lagging

(d) 50 kW at unity p.f.

Determine (a) the total kVA, (b) total kW, (c) total KVAR and (d) the overall p.f.

***Solution***

$$\text{Total kW} = 100 \times 0.8 + 250 \times 0.8 + 200 \times 0.6 + 50$$
$$= 80 + 200 + 120 + 50 = 450 \text{ kW.}$$

$$\text{Total KVAR} = 100 \sin(\cos^{-1} 0.8) - 250 \sin(\cos^{-1} 0.8) + 200 \sin(\cos^{-1} 0.6) + 0$$
$$= 100 \times 0.6 - 250 \times 0.6 + 200 \times 0.8$$
$$= 60 - 150 + 160 = 70.$$

or Total KVAR is 70 (lagging)

$$\text{Total kVA} = \sqrt{(450)^2 + (70)^2}$$
$$= 455.4.$$
$$\text{Overall p.f.} = \frac{\text{Total kW}}{\text{Total KVA}} = \frac{450}{455.4} = 0.988.$$

As KVAR is lagging, p.f. is also 0.988 (lagging).

## 3.12 COMPLEX NOTATION APPLIED TO AC CIRCUITS

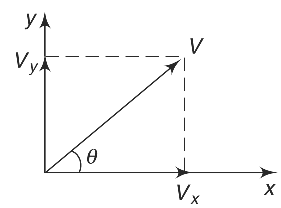



**Fig. 3.37** ***Horizontal and vertical components of phasor V***

For solving complicated ac circuit problems complex algebra is used. In this method a phasor is resolved into two components at right angles to each other. If a phasor $V$ is resolved into two components $V_x$ (horizontal component) and $V_y$ (vertical component) [Fig. 3.37] then $V^2 = (V_x^2 + V_y^2)$ and ($V$) can be represented in cartesian form as,

$$V = V_x + j\, V_y = V(\cos\theta + j \sin\theta).$$

The symbol $j$ is an operator indicating the anticlockwise rotation of the phasor by 90°. It is assigned by a value $\sqrt{-1}$.

In polar form, the phasor $V$ is represented by

$$V = V\angle\theta, \text{ where } V = \sqrt{V_x^2 + V_y^2} \text{ and } \theta = \tan^{-1}\frac{V_y}{V_x}.$$

### *Addition and Subtraction of Complex Quantities*

Let us consider two phasors $v_1$ and $v_2$ which are represented in cartesian form as

$$v_1 = a_1 + jb_1 \text{ and } v_2 = a_2 + j\, b_2.$$

Then, $v_1 + v_2 = (a_1 + a_2) + j(b_1 + b_2)$

and $v_1 - v_2 = (a_1 - a_2) + j(b_1 - b_2).$

### *Multiplication and Division of Complex Quantities*

$$v_1\, v_2 = (a_1 + jb_1)\,(a_2 + jb_2)$$
$$= (a_1a_2 - b_1b_2) + j(b_1a_2 + a_1b_2).$$

Also,
$$\frac{v_1}{v_2} = \frac{a_1 + jb_1}{a_2 + jb_2} = \frac{(a_1 + jb_1)\,(a_2 - jb_2)}{a_2 + jb_2)\,(a_2 - jb_2)}$$
$$= \frac{(a_1a_2 + b_1b_2) + j(b_1a_2 - a_1b_2)}{a_2^2 + b_2^2}$$

$$= \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} + j\,\frac{b_1a_2 - a_1b_2}{a_2^2 + b_2^2}$$

During calculations the horizontal and vertical components of phasors are summed up separately in cartesian form. This form is convenient for addition and subtraction while the polar form is convenient for multiplication and division.

Let $v_1 = a_1 + jb_1 = |V_1| \angle\theta_1$

and $v_2 = a_2 + jb_2 = |V_2| \angle\theta_2$

where $|V_1| = \sqrt{a_1^2\, b_1^2}$ and $\theta_1 = \tan^{-1}\frac{b_1}{a_1}$

and $|V_2| = \sqrt{a_2^2 + b_2^2}$ and $\theta_2 = \tan^{-1}\frac{b_2}{a_2}$

$$v_1\, v_2 = |V_1| \angle\theta_1 \times |V_2| \angle\theta_2 = |V_1||V_2| \angle(\theta_1 + \theta_2)$$

and $$\frac{v_1}{v_2} = \frac{|V_1| \angle\theta_1}{|V_2| \angle\theta_2} = \frac{|V_1|}{|V_2|} \angle(\theta_1 + \theta_2)$$

## 3.13 SERIES PARALLEL AC CIRCUITS

Consider a series parallel ac circuit as shown in Fig. 3.38.

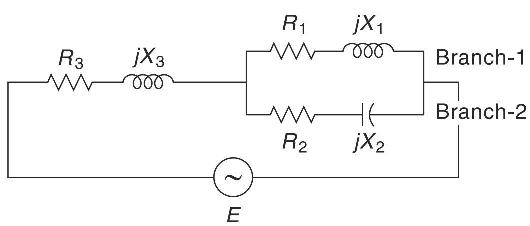



**Fig. 3.38** ***Series parallel ac circuit***

First, the impedance of the parallel branches 1 and 2 are considered.

For branch 1:

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_1}$$

For branch 2: $Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_2}$

The admittance for parallel circuits 1 and 2 is obtained as

$$Y_{12} = Y_1 + Y_2 = \frac{1}{R_1 + jX_1} + \frac{1}{R_2 - jX_2}.$$

and impedance $(Z_{12}) = \frac{1}{Y_{12}}.$

Total impedance of series parallel ac circuit

$$Z = Z_{12} + Z_3 = Z_{12} + (R_3 + jX_3)$$

Thus, current $I = \dfrac{E}{Z}$.

**3.36** The voltage across a circuit is given by (300 + *j*60) V and the current through it by (10 – *j*5) A. Determine the (a) active power, (b) reactive power and (c) apparent power.

***Solution***

$$V = 300 + j\,60 = \sqrt{(300)^2 + (60)^2}\ \angle\tan^{-1}\frac{60}{300}$$
$$= 305.94\angle 11.31°\text{ V}$$
$$I = 10 - j5 = \sqrt{(10)^2 + (5)^2}\ \angle -\tan^{-1}\frac{5}{10} = 11.18\angle -26.56°\text{ A.}$$

Angle between voltage and current is $\theta = 11.31° - (-26.56°) = 37.87°$, and current is lagging with respect to the voltage.

(a) Active power ($VI\cos\theta$) $= 305.94 \times 11.18\cos 37.87°$
$= 2700\text{ W} = 2.7\text{ kW}$.

(b) Reactive power ($VI\sin\theta$) $= 305.94 \times 11.18\sin 37.87°$
$= 2099.68$ VAR
$= 2.099$ KVAR (lagging).

(c) Apparent power ($VI$) $= 305.94 \times 11.18 = 3420.4$ VA $= 3.42$ kVA.

**3.37** Three impedances (4 – *j*6) Ω, (6 + *j*8) Ω and (5 – *j*3) Ω are connected in parallel. Calculate the current in each branch when the total supply current is 20 A.

***Solution***

$$Z_1 = (4 - j6)\ \Omega;\ y_1 = \frac{1}{Z_1} = \frac{1}{(4 - j6)} = \frac{(4 + j6)}{(4)^2 + (6)^2} = 0.077 + j0.115$$
$$Z_2 = (6 + j8)\ \Omega;\ y_2 = \frac{1}{Z_2} = \frac{1}{6 + j8} = \frac{6 - j8}{(6)^2 + (8)^2} = 0.06 - j0.08$$
$$Z_3 = (5 - j3)\ \Omega\ \ y_3 = \frac{1}{Z_3} = \frac{1}{5 - j3} = \frac{5 + j3}{(5)^2 + (3)^2} = 0.147 + j0.088$$

Total admittance $y = y_1 + y_2 + y_3 = (0.077 + 0.06 + 0.147) + j\,(0.115 - 0.08 + 0.088)$
$= 0.284 + j\,0.123 = 0.31\angle 23.4°$.

Supply voltage $V = \dfrac{I}{y} = \dfrac{20}{0.31\angle 23.4°} = 64.5\angle -23.4°$ V.

$$I_1 = Vy_1 = 64.5\angle -23.4°(0.077 + j0.115)$$
$$= 64.5\angle -23.4° \times 0.138\angle 56.19°$$
$$I_2 = Vy_2 = 64.5\angle -23.4°\ (0.06 - j0.08)$$
$$= 64.5\angle -23.4° \times 0.1\angle -53.13°$$
$$I_3 = Vy_3 = 64.5\angle -23.4°(0.147 + j0.088)$$
$$= 64.5\angle -23.4° \times 0.171\angle 30.9°$$

i.e.,
$$I_1 = 8.9\angle 32.79°\text{ A}$$
$$I_2 = 6.45\angle -76.5°\text{ A}$$
$$I_3 = 11.03\angle 7.5°\text{ A.}$$

**3.38** Find the value of unknown reactance '$X$' so that p.f. of the circuit will be unity in Fig. 3.39. Also calculate the current drawn from the supply.

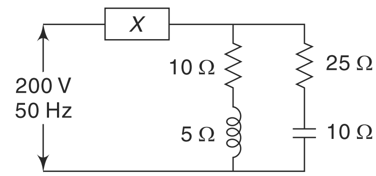



**Fig. 3.39** *Circuit diagram for Ex. 3.38*

***Solution***

The combined impedance of the two parallel branches

$$Z = \frac{(10 + j5)\,(25 - j10)}{(10 + j5) + (25 - j10)} = \frac{(250 + 50) + j(125 - 100)}{35 - j5}$$

$$= \frac{300 + j25}{35 - j5} = \frac{(300 + j25)\,(35 + j5)}{(35)^2 + (5)^2} = \frac{10375 + j2375}{1250} = (8.3 + j\,1.9)\ \Omega$$

If the p.f. becomes unity then the net reactance of the circuit should be zero i.e., $X = -j1.9$ or $X = 1.9\ \Omega$ (capacitive).

So, total impedance is 8.3 Ω. Therefore current is 200/8.3 = 24.1 A at u.p.f

**3.39** Determine the current drawn by the series parallel circuit shown in Fig. 3.40 and find the overall p.f.

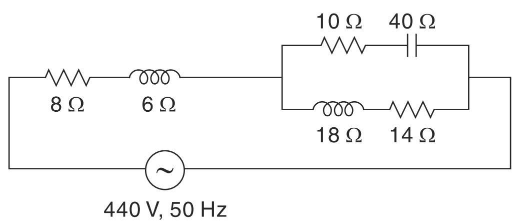



**Fig. 3.40** *Circuit diagram for Ex. 3.39*

***Solution***

The equivalent impedance of the two parallel branches is

$$= \frac{(10 - j40)\,(14 + j18)}{(10 - j40\,) + (14 + j18)} = \frac{860 - j380}{24 - j22}$$

$$= 27.358 + j9.24\ \Omega.$$

$\therefore$ The impedance of the whole circuit $= 8 + j6 + 27.358 + j9.24$

$= 35.358 + j15.24$

$= 38.5\angle 23.32°$.

Hence the current drawn by the circuit $= \dfrac{440}{38.5\angle 23.3°} = 11.43\angle{-23.3°}$.

Overall power factor (= cos 23.3° lag) = 0.918 lag.

**3.40** In the circuit shown in Fig. 3.41 determine the impedance of circuit $AB$; what voltage of 50 Hz frequency is to be applied across $AB$ that will cause a current of 10 A to flow in the capacitor?

***Solution***

The combined impedance of the two parallel branches is

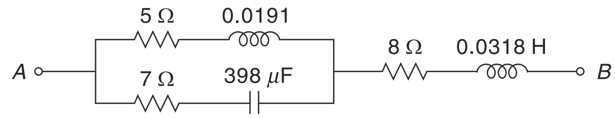



**Fig. 3.41** ***Circuit diagram for Ex. 3.40***

$$\frac{(5 + j2\pi \times 50 \times 0.0191) \times \left(7 + \dfrac{10^6}{j2\pi \times 50 \times 398}\right)}{(5 + j2\pi \times 50 \times 0.0191) \times \left(7 + \dfrac{10^6}{j2\pi \times 50 \times 398}\right)}$$

$$= \frac{(5 + j6)\,(7 - j8)}{5 + j6 + 7 - j8} = \frac{35 + 48 + j42 - j40}{12 - j2} = \frac{83 + j2}{12 - j2}$$

$$= \frac{83.024\angle 1.38°}{12.16\angle 9.46°} = 6.83\angle 10.84° = 6.7 + j1.28\ \Omega.$$

The impedance ($Z$) of the whole circuit ($AB$) is given by

$$\begin{aligned}(Z) &= 6.7 + j\ 1.28 + 8 + j\ 2\pi \times 50 \times 0.0318\\ &= 14.7 + j(1.28 + 10)\\ &= 14.7 + j11.28\\ &= 18.52\angle 37.5°\ \Omega.\end{aligned}$$

The impedance ($Z$) of the capacitor branch = $7 - j8\ \Omega$

The voltage across this branch = $10 \times (7 - j8) = (70 - j80)$ V.

∴ the current in the other parallel branch

$$= \frac{70 - j80}{5 + j6} = \frac{106.3\angle{-48.8°}}{7.81\angle 50.19°}$$

$$= 13.61\angle{-99°} = -2.129 - j13.44.$$

Thus total current = $10 - 2.129 - j13.44$

$$= 7.87 - j13.44 = 15.57\angle{-59.65°}$$

The voltage across the third branch is = $15.57\angle{-59.65°} \times (8 + j10)$

$$= 15.57\angle{-59.65°} \times 12.8\angle 51.34°$$

$$= 200\ \angle{-8.31°} = (198 - j28)\ \text{V}.$$

Hence the supply voltage is $[(70 - j\ 80) + (198 - j28)]$ V,

i.e., $(268 - j\ 108)$ V or, $288.9\angle{-21.95°}$ V.

[Note that in this problem we have assumed 10 A current to be the reference phasor having angle $10\angle 0°$. Other currents and voltages are expressed accordingly.]

**3.41** Two circuits having the same numerical value of impedance are connected in parallel. The p.f. of one circuit is 0.8 (lead) and the other is 0.6 (lead). What is the p.f. of the combination?

***Solution***

Let the numerical value of impedance be $Z$. So Impedance of one circuit is $Z_1 = Z(\cos\theta_1 + j\sin\theta_1)$ and that of the second circuit is $Z_2 = Z(\cos\theta_2 + j\sin\theta_2)$.

However, $\cos\theta_1 = 0.8$ and $\cos\theta_2 = 0.6$

∴ $\sin\theta_1 = 0.6$ and $\sin\theta_2 = 0.8$.

Hence, $Z_1 = Z(0.8 + j\,0.6)$ and $Z_2 = Z(0.6 + j\,0.8)$.

Now net impedance $= \dfrac{Z_1Z_2}{Z_1 + Z_2} = \dfrac{(0.8 + j0.6)\,(0.6 + j0.8)}{(0.8 + j0.6)\,(0.6 + j0.8)}$

$$\frac{1\angle 36.86^\circ \times 1\angle 53.13^\circ}{1.4 + j1.4} = \frac{1\angle 90^\circ}{1.98\angle 45^\circ} = 0.505\angle 45^\circ$$

So the p.f. of the circuit is cos 45° = 0.707 (lead).

**3.42** A circuit with two branches having admittances $y_1 = 0.16 + j0.12$ and $y_2 = -j0.15$ are in parallel and connected to a 100 V supply. Find the total loss and phase relationship between the branch currents and the supply current.

***Solution***

$$y_1 = (0.16 + j\,0.12)\text{ S}$$

$$y_2 = (-j\,0.15)\text{ S}$$

$$I_1 = Vy_1 = 16 + j12 = 20\angle 36.87^\circ\text{ A}$$

and $I_2 = Vy_2 = -j15 = 15\angle -90^\circ$ A.

Total current $I = I_1 + I_2 = 16 + j12 - j15 = 16 - j3$

$= 16.28\angle -10.62^\circ$ A.

∴ total loss $= VI\cos\theta$

$= 100 \times 16.28 \cos 10.62^\circ$

$= 1600$ W.

$I_1$ leads $I$ by $(36.87^\circ + 10.62^\circ) = 47.5^\circ$

and $I_2$ lags $I$ by $(90^\circ - 10.62^\circ) = 79.38^\circ$.

**3.43** A small single phase 240 V induction motor is tested in parallel with a 160 Ω resistor, the motor takes 2 A and the total current is 3 A. Find the power and p.f. of (a) the whole circuit and (b) the motor.

***Solution***

(a) The current in the resistor is $(I_1) = \dfrac{240}{160} = 1.5$ A.

The impedance of the motor is $\dfrac{240}{2} = 120\ \Omega$.

Let the motor current is $(a + jb)$ A.

∴ $a^2 + b^2 = 2^2 = 4$.

The total current is $(a + jb + 1.5)$

or $(a + 1.5)^2 + b^2 = 3^2 = 9$

or $4 + 2.25 + 3a = 9$ $[\therefore a^2 + b^2 = 4]$

or $3a = 2.75$ or, $a = 0.917$

and $b = \pm\sqrt{4 - (0.917)^2} = \pm 1.78$.

In induction motor current is lagging. So $b = -1.78$.

The motor current is thus $(0.917 - j1.78)$ A.

The total current $= 1.5 + 0.917 - j1.78$

$= 2.417 - j\,1.78$

$= 3\angle -36.37^\circ$ A.

p.f. of the whole circuit is cos 36.37 = 0.8 lagging.

Power of the whole circuit is $VI\cos\theta = 240 \times 3 \times 0.8 = 576$ W.

(b) p.f. of the motor is $\cos\left(\tan^{-1}\dfrac{1.78}{0.917}\right)$ lagging = 0.458 lagging.

Power of the motor is $240 \times 2 \times 0.458 = 220$ W.

**3.44** Find the phase angle of the input impedance of a series circuit consisting of a 500 Ω resistor, a 60 mH inductor and a 0.053 μF capacitor at frequencies of (a) 2000 Hz. and (b) 4000 Hz.

***Solution***

(a) Phase angle of impedance $\theta = \tan^{-1}\dfrac{\text{Reactance}}{\text{Resistance}} = \tan^{-1}\dfrac{X_L - X_C}{R}$

When $f = 2000$ Hz,

$$(X_L - X_C) = 2\pi \times 2000 \times 60 \times 10^{-3} - \frac{10^6}{2\pi \times 2000 \times 0.053}$$

$$= 753.6 - 1502.22 = 748.6 \text{ (–ve).}$$

so $\theta = \tan^{-1}\dfrac{-748.6}{500} = 56.26°$ (lead).

(b) When $f = 4000$ Hz

$$X_L - X_C = 2\pi \times 4000 \times 60 \times 10^{-3} - \frac{10^6}{2\pi \times 4000 \times 0.053}$$

$$= 1507.2 - 751$$

$$= 756.2.$$

Therefore phase angle $\theta = \tan^{-1}\dfrac{756.2}{500} = 56.53°$ (lag).

**3.45** A resistor $R$ in series with a capacitor $C$ is connected to a 50 Hz. 240 V supply. Find the value of $C$ so that $R$ absorbs 300 W at 100 V. Find also the maximum charge and the maximum stored energy in $C$.

***Solution***

Supply voltage = 240 V

Voltage across $R$ is 100 V.

Power across $R$ is 300 W

∴ Current through ($R$) is $\dfrac{300}{100} = 3$ A

Voltage across capacitor = $\sqrt{(240)^2 - (100)^2} = 218.17$ V.

Hence, maximum voltage ($V_m$) = $\sqrt{2} \times 218.17 = 308.54$ V.

Thus maximum charge is ($C\,V_m$)

Now, capacitive reactance $X_C = \dfrac{218.17}{3} = 72.72$ Ω.

Hence, $C = \dfrac{1}{72.72 \times 2\pi \times 50}$ F = 43.79 μF

Maximum charge (= $CV_m$) = $43.79 \times 10^{-6} \times 308.54 = 0.0135$ C.

Maximum energy stored = $\dfrac{1}{2}C\,V_m^2 = \dfrac{1}{2} \times 43.79 \times 10^{-6} \times (308.54)^2 = 2.08$ J.

## 3.14 SERIES RESONANCE

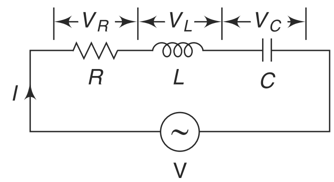



**Fig. 3.42** ***RLC series circuit***

An ac circuit is said to be in resonance when the circuit current is in phase with the applied voltage. So the power factor of the circuit becomes unity at resonance and the impedance of the circuit consists of only resistance.

In the series RLC circuit in Fig. 3.42 the impedance is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The current in the circuit is

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

The power factor is: $\cos\theta = \dfrac{R}{Z} = \dfrac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

At resonance $Z = R$

$$\therefore \quad \omega L - \frac{1}{\omega C} = 0$$

or $$\omega = \frac{1}{\sqrt{LC}}$$

If $f_o$ be the resonant frequency then

$$2\pi f_o = \frac{1}{\sqrt{LC}}$$

or $$f_o = \frac{1}{2\pi\sqrt{LC}}$$

The p.f. at resonance: $\cos\theta = \dfrac{R}{Z} = \dfrac{R}{R} = 1$ [ at resonance, $Z = R$]

and the current $I = \dfrac{V}{Z} = \dfrac{V}{R}$.

***Properties of Series Resonant Circuits***

(a) The circuit impedance $Z$ is minimum and equal to the circuit resistance $R$.

(b) The power factor is unity.

(c) The circuit current $I = \dfrac{V}{R}$, and the current is maximum.

(d) The power dissipated is maximum, i.e. $P_o = \dfrac{V^2}{R}$,

(e) The resonant frequency is $f_o = \frac{1}{2\pi\sqrt{LC}}$,

(f) The voltage across inductor is equal and opposite to the voltage across capacitor.

Since the circuit current is maximum at resonance it produces large voltage drops across $L$ and $C$. But as these voltages are equal and opposite to each other so the net voltage across $L$ and $C$ is zero however large the current is flowing. If $R$ is not present then the circuit would act like a short circuit at resonant frequency. Hence a series circuit is sometimes called an *acceptor circuit* and the series resonance is often referred to as the *voltage resonance*. Figure 3.43 represents variation of $R$, $X_L$, $X_C$, $Z$ and $I$ with frequency. As $R$ is the independent of frequency so it is a straight line. Inductive reactance $X_L$ is directly proportional to the frequency so it is a straight line passing through origin. Capacitive reactance $X_C$ is inversely proportional to frequency and its

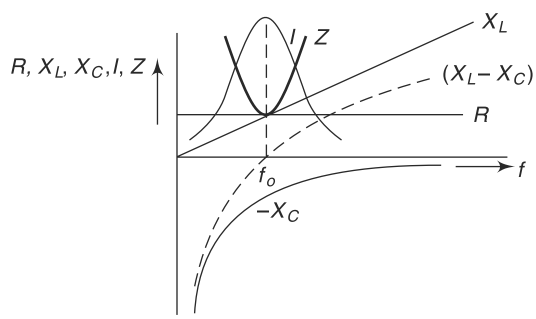



**Fig. 3.43** ***Variation of R, $X_L$, $X_C$, Z, I with frequency***

characteristic is a rectangular hyperbola. At resonant frequency $Z$ is minimum and $I$ is maximum.

## 3.15 *Q* FACTOR IN SERIES RESONANCE

*Q factor* of a series *RLC* circuit is defined as the voltage magnification produced in the circuit at resonance.

Voltage magnification is the ratio of voltage drop across the inductor or capacitor to the voltage drop across the resistor.

Hence, $$Q \text{ factor} = \frac{IX_L}{IR} = \frac{\omega_0 L}{R} = \frac{2\pi f_o L}{R} = \frac{2\pi \frac{1}{2\pi\sqrt{LC}} L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

*Q* factor is also referred as the *magnification factor* of the circuit.

## 3.16 DIFFERENT ASPECTS OF RESONANCE

### 3.16.1 Variation of Current and Voltage Across *L* and *C* with Frequency

The voltage across the capacitor is $V_C = I \cdot \frac{1}{\omega C}$

In an $RLC$ series circuit $I = \dfrac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$

$$\therefore \qquad V_C = \frac{V}{\omega C \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

or
$$V_C^2 = \frac{V^2}{\omega^2 C^2 \left\{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2\right\}}$$

$$= \frac{V^2}{\omega^2 C^2 \left\{R^2 + \dfrac{(\omega^2 LC - 1)^2}{\omega^2 C^2}\right\}} = \frac{V^2}{R^2\omega^2C^2 + (\omega^2 LC - 1)^2}.$$

To find the frequency at which $V_C$ is maximum $\left(\dfrac{dV_C}{d\omega}\right)$ should be zero i.e., $\left(\dfrac{dV_C^2}{d\omega}\right)$ should be zero.

As $\dfrac{dV_C^2}{d\omega} = 0$, we have

$$\frac{dV_C^2}{d\omega} = V^2 \left[\frac{-\{2\omega C^2R^2 + 2(\omega^2 LC - 1)\, 2\omega LC\}}{\{R^2\omega^2C^2 + (\omega^2 LC - 1)^2\}^2}\right] = 0$$

or $\quad 2\omega C^2R^2 + (2\omega^2 LC - 2)\, 2\omega LC = 0$

or $\quad \omega^2 = \dfrac{1}{2L^2C}(2L - CR^2) = \dfrac{1}{LC} - \dfrac{R^2}{2L^2}.$

Hence the frequency at which $V_C$ is maximum is $f_C = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{2L^2}}.$

To find the frequency at which voltage across $L$ is maximum, we have

$$\frac{dV_L}{d\omega} = 0 \quad \text{or,} \quad \frac{dV_L^2}{d\omega} = 0.$$

Now,
$$V_L = IX_L = \frac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}\,\omega L$$

$$V_L^2 = \frac{V^2\omega^2L^2}{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2} = \frac{V^2\omega^4L^2C^2}{\omega^2C^2R^2 + (\omega^2LC - 1)^2}$$

By differentiating $(V_L^2)$ with respect to $\omega$ and setting $\dfrac{dV_L^2}{d\omega} = 0$, we have on simplification,

$$2\omega^2LC - \omega^2C^2R^2 - 2 = 0$$

or
$$\omega^2[2LC - C^2R^2] = 2$$

or $$\omega^2 = \frac{2}{2LC - R^2C^2} = \frac{1}{LC - \frac{R^2C^2}{2}}$$

or $$\omega = \frac{1}{\sqrt{LC - \frac{R^2C^2}{2}}}$$

The variation of voltages across the capacitor and the inductor with frequency are shown in Fig. 3.44. It is seen that maximum value of $V_C$ occurs at $f_C$ below $f_o$ (resonant frequency) and maximum value of $V_L$ occurs at $f_L$ which is above $f_o$.

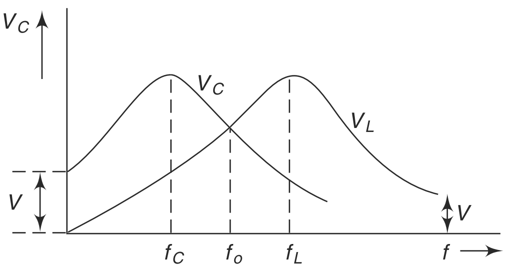



**Fig. 3.44** *Variation of voltage across L and C with frequency*

If $R$ is very small then $f_L$, $f_C$ and $f_o$ correspond to a single value of frequency $f_o = \frac{1}{2\pi\sqrt{LC}}$.

### 3.16.2 Effect of Resistance on the Frequency Response Curve

Figure 3.45 shows the nature of resonance curve for different values of $R$.

As seen from the Fig. 3.45 that when the resistance is small the curve rises steeply while with large resistance value the curve has a low peak.

A circuit with a flat frequency response curve will be more responsive and so less selective at frequencies in the neighbourhood of the resonant frequency. On the other hand, a circuit for which the curve has a tall narrow peak will be less responsive and so more selective at frequencies in the neighbourhood of resonant frequency.

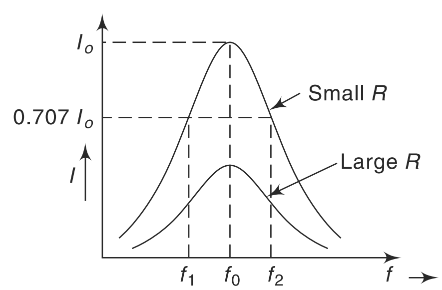



**Fig. 3.45** *Effect of resistance on frequency response curve*

### 3.16.3 Bandwidth and Selectivity in Series Resonance Circuit

The bandwidth of a given circuit is given by the band of frequencies which lies between two points on either side of resonant frequency where the current is $1/\sqrt{2}$ times of the current at resonance and hence the power is half of the power at resonance.

Figure 3.46 shows the variation of circuit current with frequency and this curve is known as the resonance curve. $f_1$ and $f_2$ are known as half power frequency where current is $I_o/\sqrt{2}$ ($I_o$ is the current at resonance). $f_1$ and $f_2$ are also called *corner* or *edge frequencies*. The power at the two points of frequencies $f_1$ and $f_2$ is

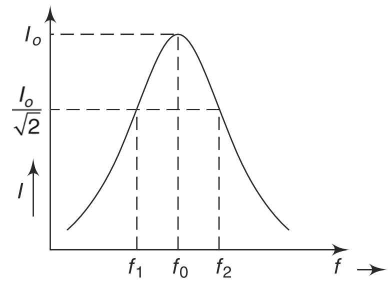



**Fig. 3.46** ***Variation of current with frequency***

$$P_1 = P_2 = I^2R = \left(\frac{I_o}{\sqrt{2}}\right)^2 R = \frac{I_o^2 R}{2}$$

$$= \frac{1}{2} \times \text{(power at resonance)}$$

Now $(I) = \dfrac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$ at any frequency ($\omega$)

At half power frequencies $I = \dfrac{I_o}{\sqrt{2}} = \dfrac{V}{R\sqrt{2}}$ $\left(\text{As } I_o = \dfrac{V}{R}\right)$

Squaring both sides and comparing above expressions we have

$$I^2 = \frac{V}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{V^2}{2R^2}$$

or $$\frac{V^2}{2R^2} = \frac{V^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad \text{or,} = \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2.$$

or $$\omega L - \frac{1}{\omega C} = \pm R \quad \text{or} \quad \omega = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2 + \frac{1}{LC}}}.$$

As $\left(\dfrac{R^2}{4L^2}\right)$ is much less than $\dfrac{1}{\sqrt{LC}}$ so,

$$(\omega) = \pm \frac{R}{2L} \pm \frac{1}{\sqrt{LC}} = \pm \frac{R}{2L} \pm \omega_o.$$

Since only positive values of $\omega_o$ is considered so

$$\omega = \pm \frac{R}{2L} + \omega_o$$

or $$\omega_1 = \omega_o - \frac{R}{2L} \quad \text{and} \quad \omega_2 = \omega_o + \frac{R}{2L}$$

or $$f_1 = f_o - \frac{R}{4\pi L} \quad \text{and} \quad f_2 = f_o + \frac{R}{4\pi L}.$$

Bandwidth $(\Delta\omega) = \omega_2 - \omega_1 = \dfrac{R}{L}$ rad/s, and $(\Delta f) = f_2 - f_1 = \dfrac{R}{2\pi L}$ Hz.

The ratio of the bandwidth to the resonant frequency is defined as the *selectivity* of the circuit. Thus selectivity $= \dfrac{(f_2 - f_1)}{f_0}$.

Thus, *narrower the bandwidth higher is the selectivity property of the circuit*.

**3.46** A circuit consists of a coil of resistance 100 Ω and inductance 1 H in series with a capacitor of capacitance 1 μF. Calculate (a) the resonant frequency, (b) current at resonant frequency and (c) voltage across each element when the supply voltage is 50 V.

***Solution***

Resistance $R = 100\ \Omega$

Inductance $L = 1$ H

Capacitance $C = 1 \times 10^{-6}$ F.

(a) Resonant frequency $f_o = \dfrac{1}{2\pi\sqrt{LC}}$

$= \dfrac{1}{2\pi\sqrt{1 \times 1 \times 10^{-6}}}$

$= 159$ Hz.

(b) Current at resonant frequency $I_o = \dfrac{V}{R} = \dfrac{50}{100} = 0.5$ A.

(c) Voltage across resistance $V_R = I_o \times R = 0.5 \times 100 = 50$ V.

∴ Voltage across inductance $V_L = I_o X_L = 0.5 \times 2\pi f_o L$

$= 0.5 \times 2\pi \times 159 \times 1$

$= 500$ V

and voltage across capacitance $V_C = I_o X_C = 0.5 \times \dfrac{10^6}{2\pi \times 159 \times 1} = 500$ V.

**3.47** An inductive coil is connected in series with a 8 μF capacitor. With a constant supply voltage of 400 V the circuit takes minimum current of 80 A when the supply frequency is 50 Hz. Calculate the (a) resistance and inductance of the coil and (b) voltage across the capacitor.

***Solution***

Supply voltage $V = 400$ V

The current is minimum at 50 Hz, so the resonant frequency is 50 Hz and the current at resonant frequency $(I_o) = 80$ A.

Capacitance $C = 8 \times 10^{-6}$ F.

Hence, resistance $R = \dfrac{V}{I_o} = \dfrac{400}{80} = 5\ \Omega$.

At resonance,

$$X_L = X_C$$

or $$\omega L = \frac{1}{\omega C}$$

or $$\omega = \frac{1}{\sqrt{LC}}$$

or $$2\pi f = \frac{1}{\sqrt{L \times 8 \times 10^{-6}}}$$

or $$2\pi \times 50 = \frac{10^3}{\sqrt{8L}}$$

or $$L = \left(\frac{10^3}{2\pi \times 50}\right)^2 \times \frac{1}{8} = \left(\frac{10}{\pi}\right)^2 \times \frac{1}{8} = 1.266 \text{ H}$$

**3.48** The resistor and a capacitor are connected in series with a variable inductor. When a circuit is connected to a 240 V, 50 Hz supply, the maximum current by varying the inductance is 0.5 A. At this current the voltage across the capacitor is 250 V. Calculate $R$, $C$ and $L$.

***Solution***

Supply voltage $V = 240$ V.

Resonant frequency ($f_o$) = 50 Hz.

Current at resonant frequency $I_o = 0.5$ A.

Voltage across capacitor $V_C = 250$ V

Now if $X_C$ and $X_L$ be the capacitive and inductive reactance then

$$I_o X_C = 250$$

and $$I_o X_L = 250.$$

At resonance, $$X_C = X_L = \frac{250}{0.5} = 500\ \Omega$$

Therefore, $$\frac{1}{2\pi f_o C} = 2\pi f_o L = 500$$

or $$C = \frac{1}{2\pi \times 50 \times 500} \text{ F} = 6.37\ \mu\text{F},$$

and $$L = \frac{500}{2\pi \times 50} \text{ H} = 1.59 \text{ H}.$$

Resistance $$R = \frac{V}{I_o} = \frac{240}{0.5} = 480\ \Omega.$$

**3.49** A constant voltage at a frequency of 1 MHz is applied to an inductor in series with a variable capacitor. When the capacitor is set at 500 pF, the current has its maximum value, and it is reduced to half of its maximum value when the capacitance is 600 pF. Find (a) the resistance, (b) the inductance and (c) $Q$ factor of the inductor.

***Solution***

The current is maximum at resonant frequency.

∴ $$f_o = 10^6 \text{ Hz}.$$

It is given that is $C = 500 \times 10^{-12}$ F.

Now, $$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{500 \times 10^{-12} L}} = 10^6$$

or $$\frac{10^6}{2\pi\sqrt{500L}} = 10^6$$

or $$2\pi\sqrt{500L} = 1$$

or $$L = \left(\frac{1}{2\pi}\right)^2 \times \frac{1}{500} \text{ H} = 0.05 \text{ mH}$$

when $C = 600 \times 10^{-12}$ F.

$\therefore$ Capacitive reactance $X_C = \dfrac{1}{2\pi \times 10^6 \times 600 \times 10^{-12}} = \dfrac{10^6}{2\pi \times 600} = 265\ \Omega.$

Inductive reactance $X_L = 2\pi f_o L = 2\pi \times 10^6 \times 0.05 \times 10^{-3} = 314\ \Omega.$

$\therefore$ Net reactance is $(X_L - X_C) = 314 - 265 = 49\ \Omega.$

Current $I = \dfrac{I_o}{2}\left(= \dfrac{V}{2R}\right)$, where $R$ is the resistance of the circuit and $I_o$ is the current at resonance.

So, $$I = \frac{V}{2R} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

or $$2R = \sqrt{R^2 + (X_L - X_C)^2}$$

or $$4R^2 = R^2 + (49)^2$$

or $$3R^2 = (49)^2$$

or $$R = 28.29\ \Omega$$

$\therefore$ $Q$ factor of the inductor is $\dfrac{\omega_o L}{R} = \dfrac{2\pi \times 10^6 \times 0.05 \times 10^{-3}}{28.29} = 11.1.$

**3.50** A series resonant circuit has an impedance of 500 Ω at resonant frequency and cut off frequencies are 10 kHz and 100 kHz. Determine (a) the resonant frequency, (b) value of $R$, $L$ and $C$, (c) quality factor at resonant frequency and (d) p.f. of the circuit at resonant frequency.

***Solution***

At resonance

Impedance = Resistance or $Z_o = R = 500\ \Omega.$

$$f_1 = 10 \times 10^3 \text{ Hz } \& f_2 = 100 \times 10^3 \text{ Hz.}$$

$$(f_2 - f_1) = 90 \times 10^3 = \frac{R}{2\pi L} = \frac{500}{2\pi L}.$$

Now $$L = \frac{500}{2\pi \times 90 \times 10^3} = 0.88 \text{ mH}$$

$\therefore$ Resonant frequency $f_o = f_1 + \dfrac{R}{4\pi L} = 10 \times 10^3 + \dfrac{500}{4\pi \times 0.88 \times 10^{-3}} = 55$ kHz.

As $$f_o = \frac{1}{2\pi\sqrt{LC}}$$

so $$C = \left(\frac{1}{2\pi \times 55 \times 10^3}\right)^2 \times \frac{1}{0.88 \times 10^{-3}} = 0.095 \times 10^{-7} \text{ F}$$

$$Q \text{ factor} = \frac{2\pi f_o L}{R} = \frac{2\pi \times 55 \times 10^3 \times 0.88 \times 10^{-3}}{500} = 0.61$$

and the p.f. of the circuit at resonant frequency is 1.

**3.51** A series RLC circuit has $R = 10\ \Omega$, $L = 0.1$ H and $C = 8\ \mu$F. Determine (a) the resonant frequency (b) $Q$ factor of the circuit at resonance (c) half power frequencies.

***Solution***

(a) Resonant frequency $f_o = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.1 \times 8 \times 10^{-6}}} = 178$ Hz.

(b) $Q_{\text{factor}} = \dfrac{2\pi f_o L}{2} = \dfrac{2\pi \times 178 \times 0.1}{10} = 11.18.$

(c) Lower half power frequency ($f_1$) $= f_o - \dfrac{R}{4\pi L}$

$$= 178 - \frac{10}{4\pi \times 0.1} = 178 - 7.96 = 170 \text{ Hz.}$$

Upper half power frequency ($f_2$) = 178 + 7.96 = 186 Hz.

**3.52** Calculate the half power frequencies of a series resonant circuit when the resonant frequency is $150 \times 10^3$ Hz and the bandwidth is 75 kHz.

***Solution***

Resonant frequency $f_o = 150 \times 10^3$ Hz.

Bandwidth $(\Delta f) = \dfrac{R}{2\pi L} = 75 \times 10^3.$

Lower half power frequency $\left(f_o - \dfrac{R}{4\pi L}\right) = \left(150 - \dfrac{75}{2}\right) = 10^3 = 112.5$ kHz.

Upper half power frequency $\left(f_o + \dfrac{R}{4\pi L}\right) = \left(150 + \dfrac{75}{2}\right) = 10^3$ Hz. = 187.5 kHz.

**3.53** A 25 μF condensor is connected in series with a coil having an inductance of 5 mH. Determine the frequency of resonance, resistance of the coil if a 25 V source operating at resonance frequency causes a circuit current of 4 mA. Determine the $Q$ factor of the coil.

***Solution***

$C = 25 \times 10^{-6}$ F $L = 5 \times 10^{-3}$ H

$I_o = 4 \times 10^{-3}$ A $V = 25$ V.

Frequency of resonance $(f_o) = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{5 \times 10^{-3} \times 25 \times 10^{-6}}}$$

$$= 450 \text{ Hz.}$$

∴ Resistance of the circuit $= \dfrac{V}{I_o} = \dfrac{25}{4 \times 10^{-3}} = 6250\ \Omega$

and $Q_{\text{factor}} = \dfrac{2\pi f_o L}{R} = \dfrac{2\pi \times 450 \times 5 \times 10^{-3}}{6250} = 2.26 \times 10^{-3}.$

## 3.17 RESONANCE IN A PARALLEL CIRCUIT

Let us consider a circuit where a capacitance $C$ is connected in parallel with an inductive coil of resistance $R$ and inductive reactance $X_L$ as shown in Fig. 3.47.

If $I_L$ be the current through the coil, $I_C$ be the current through the capacitor and the total current is $I$, then the vector diagram is shown in Fig. 3.48.

From Fig. 3.48 it is clear that under resonance as the p.f. is unity the reactive component of the total current is zero. The reactive component of the current $(I_C - I_L \sin \phi) = 0$, where $\phi$ is the power factor angle of the coil. Therefore

$$I_C = I_L \sin \phi$$

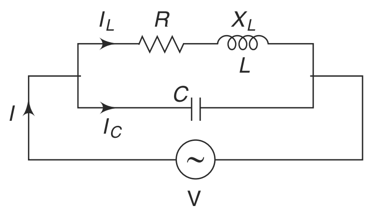



**Fig. 3.47** *AC parallel circuit*

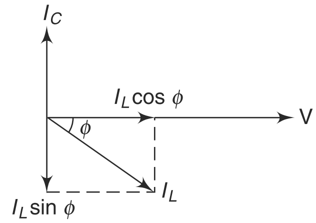



**Fig. 3.48** *Branch currents of Fig. 3.47*

or $$\frac{V}{X_C} = \frac{V}{Z_L} \times \frac{X_L}{Z_L}$$

where $Z_L$ is the impedance of the coil and $\left[Z_L = \sqrt{R^2 + X_L^2}\right]$

or $$X_C X_L = Z_L^2$$

or $$\frac{\omega L}{\omega C} = Z_L^2 = R^2 + \omega^2 L^2$$

or $$\omega^2 L^2 = \frac{L}{C} - R^2$$

or at resonance $$\omega_o = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

and resonant frequency $$f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}.$$

If resistance is neglected then

$$\omega_o = \frac{1}{\sqrt{LC}}$$

and $$f_o = \frac{1}{2\pi}\sqrt{LC}$$

Thus *if resistance is neglected the resonant frequency of the parallel circuit is equal to that of series circuit*. Also at resonance the net susceptance is zero.

$$\text{Net susceptance} = \left(\omega C - \frac{1}{\omega L}\right) = 0$$

$\therefore$ $$\omega_o = \frac{1}{\sqrt{LC}}$$

As the net reactive component of the current is zero at resonance so the supply current $I$ is equal to the active component of the current.

So $$I = I_L \cos\phi = \frac{V}{Z_L}\frac{R}{Z_L} = \frac{VR}{Z_L^2} = \frac{VR}{L/C} \quad (\because Z_L^2 = X_C \cdot X_L = L/C)$$

from previous equation

or $$I = \frac{V}{L/CR}$$

Thus at resonance the net impedance is given by $L/CR$ and is known as the *dynamic impedance* of the parallel circuit at resonance. This impedance is resistive only.

The current is minimum at resonance as its reactive part is zero and thus, ($L/CR$) represent the maximum impedance of the circuit. It is called a *rejector circuit*.

## 3.18 PROPERTIES OF PARALLEL RESONANT CIRCUITS

(a) At resonance the net reactive component of the line current is zero and the circuit current is equal to the active component of the total current, i.e. $I = I_L \cos \phi$.

(b) The line current is minimum at resonance or $I = \dfrac{V}{L/CR}$

(c) The power factor is unity at resonance.

(d) Net susceptance is zero at resonance i.e $\left(\omega C - \dfrac{1}{\omega L}\right) = 0$.

(e) The resonant frequency is $f_o = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$.

Since the current at resonance is minimum hence such a circuit is sometimes known as *rejector circuit* because it rejects (or takes minimum current) at resonant frequency. This resonance is often referred to as current resonance because the current, circulating between the two branches, is many times greater than the line current taken from the supply.

Figure 3.49 represents the variation of $R$, $B_L$ (inductive susceptance), $B_C$ (capacitive susceptance) $Z$ and $I$ with frequency. As $R$ is independent of frequency so it is a straight line. The capacitive susceptance ($B_C = \omega C$) is a straight line passing through the origin and the characteristic of inductive susceptance $\left(B = -\dfrac{1}{\omega L}\right)$ is a rectangular hyperbola. At resonance $I$ is minimum and so $Z$ is maximum.

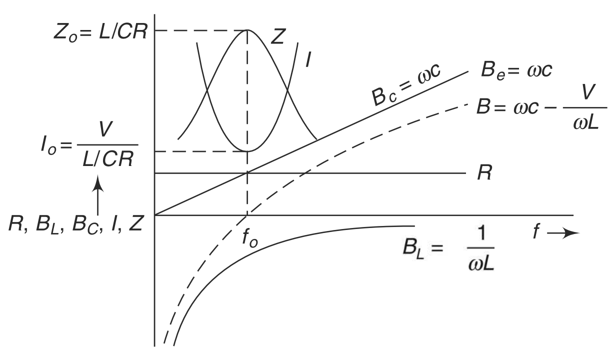



**Fig. 3.49** *Variation of R, $B_L$, $B_C$, Z, I with frequency*

## 3.19 *Q* FACTOR IN A PARALLEL CIRCUIT

It is defined as the ratio of the current, circulating between the two branches of the parallel circuit to the line current.

$$\therefore \qquad Q_{\text{factor}} = \frac{\frac{V}{XC}}{I_o} = \frac{I_C}{I_o} = \frac{V\omega C}{I_o}$$

$$I_o = \frac{V}{L/CR}.$$

Therefore $\qquad Q_{\text{factor}} = \dfrac{\omega CL}{CR} = \dfrac{\omega L}{R}.$

Now $Q_{\text{factor}}$ at resonance is $\dfrac{\omega_o L}{R} = \dfrac{1}{\sqrt{LC}}\dfrac{L}{R}$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

*In a series circuit $Q_{factor}$ gives the voltage magnification while in a parallel circuit it gives the current magnification.*

## 3.20 PARALLEL RESONANCE IN *RLC* CIRCUITS

In the circuit of Fig. 3.50 the resonance occurs when the net susceptance is zero.

Admittance $y = G + jB = \dfrac{1}{R} + j\,\omega\,C - j\,\dfrac{1}{\omega L}$

At resonance net susceptance $\left(\omega C - \dfrac{1}{\omega L}\right) = 0$

$$\therefore \qquad \omega_o = \frac{1}{\sqrt{LC}}$$

and $\qquad (f_o) = \dfrac{1}{2\pi\sqrt{LC}}$

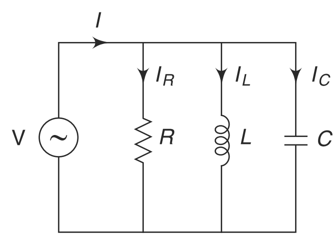



**Fig. 3.50** ***RLC parallel circuit***

*At resonant frequency ($f_o$) the admittance is minimum so the impedance is maximum and the current is minimum.*

## 3.21 PARALLEL RESONANCE IN *RC-RL* CIRCUITS

A parallel combination of *RL* and *RC* branches connected to a source of emf *E* is shown in Fig. 3.51.

The above circuit will produce parallel resonance when the resultant current is in phase with the applied voltage or the net susceptance of the above circuit is zero.

Total admittance $Y = y_1 + y_2$

$$= \frac{1}{R_L + j\omega L} + \frac{1}{R_C - j\cdot\frac{1}{\omega C}}$$

$$= \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C - j\cdot\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

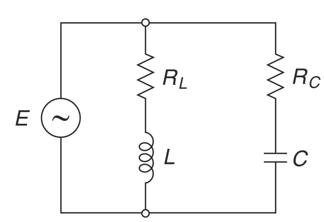



**Fig. 3.51** ***RL and RC parallel circuit***

So the net susceptance $\dfrac{-\omega L}{R_L^2 + \omega^2 L^2} + \dfrac{\dfrac{1}{\omega C}}{R_C^2 + \dfrac{1}{\omega^2 C^2}}$

$$\frac{\omega L}{R_L^2 + \omega^2 L^2} = \frac{\omega C}{\omega^2 C^2 R_C^2 + 1}$$

$$\omega^2 R_C^2 C^2 L + L = R_L^2 C + \omega^2 L^2 C$$

or $$\omega^2 (R_L^2 C^2 L + L^2 C) = R_L^2 C - L$$

or $$\omega^2 = \frac{R_L^2 C - L}{LC (R_C^2 C - L)} \quad \text{or,} \quad \omega = \sqrt{\frac{1}{LC} \frac{R_L^2 C - L}{R_C^2 C - L}}$$

So, Resonant frequency is $\left(\dfrac{1}{\sqrt{LC}} \sqrt{\dfrac{R_L^2 C - L}{R_C^2 C - L}}\right)$ rad/s.

or $$f_o = \frac{1}{2\pi \sqrt{LC}} \sqrt{\frac{R_L^2 C - L}{R_C^2 C - L}} \text{ Hz.}$$

**3.54** A coil of 10 Ω resistance has an inductance of 0.1 H and is connected in parallel with a 200 μF capacitor. Calculate the frequency at which the circuit will act as a non-inductive resistor of $R$ Ω. Find also the value of $R$.

***Solution***

Resistance of coil $R_L = 10\ \Omega$

Inductance of coil $L = 0.1$ H

Capacitance $C = 200 \times 10^{-6}$ F

Resonant frequency $f_o = \dfrac{1}{2\pi} \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}} = \dfrac{1}{2\pi} \sqrt{\dfrac{1}{0.1 \times 200 \times 10^{-6}} - \dfrac{(10)^2}{(0.1)^2}} = 31.8$ Hz.

The value of non-inductive resistor $R$ at resonance is the dynamic impedance of the circuit.

∴ $$R = \frac{L}{CR} = \frac{0.1}{200 \times 10^{-6} \times 10} = 50\ \Omega.$$

**3.55** In the circuit shown in Fig. 3.52 show that the circulating current at resonance is given by $(I) = V\sqrt{\dfrac{C}{L}}$ for a supply voltage of $V$ volts.

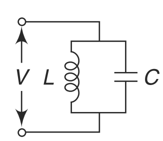



**Fig. 3.52** ***Circuit diagram for Ex. 3.55***

***Solution***

At resonance, Inductive reactance = Capacitive reactance

∴ $X_L = X_C$ or, $\omega L = \dfrac{1}{\omega C}$

Therefore $\omega_o = \dfrac{1}{\sqrt{LC}}$

Current through $L$ (or $C$) $= \dfrac{V}{X_L} = \dfrac{V}{\omega_o L} = \dfrac{V}{\dfrac{1}{\sqrt{LC}} L} = V\sqrt{\dfrac{C}{L}}$

**3.56** Calculate the value of $R_C$ in the circuit shown in Fig. 3.53 which yields resonance.

***Solution***

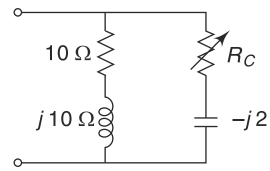



**Fig. 3.53** ***Circuit diagram for Ex. 3.56***

Admittance of the inductive branch

$$y_1 = \frac{1}{10 + j10} = \frac{10 - j10}{100 + 100}$$

Admittance of the capacitive branch

$$y_2 = \frac{1}{RC - 2j} = \frac{R_C + 2j}{R_C^2 + 4}.$$

Net admittance $y = y_1 + y_2 = \dfrac{10 - j10}{100 + 100} + \dfrac{R_C + 2j}{R_C^2 + 4}$

$$= \left(\frac{10}{200} + \frac{R_C}{R_C^2 + 4}\right) + j\left(\frac{-10}{200} + \frac{2}{R_C^2 + 4}\right)$$

As net susceptance is zero at resonance, so

$\therefore$ $$\frac{-10}{200} + \frac{2}{R_C^2 + 4} = 0$$

so, $$400 - 40 - 10R_C^2 = 0$$

or $$R_C = \sqrt{\frac{360}{10}} = 6\ \Omega.$$

**3.57** Show that no value of $R_L$ in the circuit shown in Fig. 3.54 will make it resonant.

***Solution***

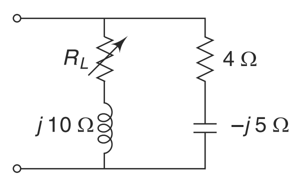



**Fig. 3.54** ***Circuit diagram for Ex. 3.57***

Net admittance $= \dfrac{1}{R_L + j10} + \dfrac{1}{4 - j5} = \dfrac{(R_L - j10)}{R_L^2 + 100} + \dfrac{(4 + j5)}{16 + 25}$

At resonance net suseptance is 0.

$\therefore$ $$\frac{-10}{R_L^2 + 100} + \frac{5}{16 + 25} = 0$$

or $$\frac{10}{R_L^2 + 100} = \frac{5}{41}$$

or, $$R_L^2 = \frac{41 \times 10}{5} - 100 = (\sqrt{-18})^2.$$

This value of $R_L$ is physically impossible and so no value of $R_L$ can make the circuit resonant.

**3.58** A 200 V, 50 Hz. source is connected across a 10∠30° Ω branch in parallel with a 10∠–60° Ω branch. Find the impedance of the circuit element which when connected in series with the supply produces resonance.

***Solution***

$$V = 200\ \text{V}$$

$$Z_1 = 10\angle 30°\ \Omega = 8.66 + j\,5$$

$$Z_2 = 10\angle -60° = 5 - j\,8.66$$

Combined impedance of the parallel branches

$$= \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{10\angle 30^\circ \times 10\angle -60^\circ}{8.66 + j5 + 5 - j8.66}$$

$$= \frac{100\angle -30^\circ}{13.66 - j3.66} = \frac{100\angle -30^\circ}{14.14\angle -15^\circ}$$

$$= 7.07\angle -15^\circ = 6.83 - j\,1.83.$$

At resonance the net reactance of the circuit should be zero. So the element which is connected in series to produce resonance must have reactance of $j1.83$. So inductive reactance $X_L = 1.83\ \Omega$ and inductance

$$L = \frac{X_L}{\omega} = \frac{1.83}{2\pi \times 50} = 5.83 \text{ mH}.$$

**3.59** For the circuit shown in Fig. 3.55 find the frequency at which the circuit will be at resonance. If the capacitor and inductor are interchanged what would be the value of new resonant frequency.

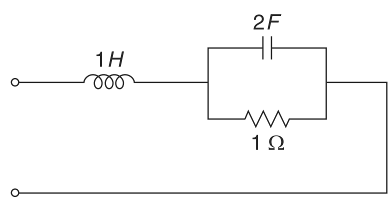



**Fig. 3.55** ***Circuit diagram for Ex. 3.59***

***Solution***

$$\text{Total impedance} = j\omega \times 1 + \frac{1 \times \frac{1}{2j\omega}}{1 + \frac{1}{2j\omega}}$$

$$= j\omega + \frac{1}{1 + 2j\omega} = j\omega + \frac{1 - j2\omega}{1 + 4\omega^2}$$

$$= \frac{1}{1 + 4\omega^2} + j\left(\omega - \frac{2\omega}{1 + 4\omega^2}\right).$$

∴ Net reactance is zero at resonance

Hence, $$\omega - \frac{2\omega}{1 + 4\omega^2} = 0$$

or $$1 + 4\omega^2 = 2$$

or, $\omega^2 = \frac{1}{4}$ or, $\omega = \frac{1}{2} = 0.5$ rad/s

when the capacitor and inductor are interchanged

Net impedance is $$\frac{1}{2j\omega} + \frac{1 \times j\omega}{1 + j\omega} = -\frac{1}{j2\omega} + \frac{j\omega}{1 + j\omega}$$

$$= -\frac{j}{2\omega} + \frac{j\omega\,(1 - j\omega)}{1 + \omega^2}$$

$$= -\frac{j}{2\omega} + \frac{j\omega + \omega^2}{1 + \omega^2}$$

$$= \frac{\omega^2}{1 + \omega^2} + j\left(\frac{\omega}{1 + \omega^2} - \frac{1}{2\omega}\right)$$

so $$\frac{\omega}{1+\omega^2} - \frac{1}{2\omega} = 0$$

or $$2\omega^2 = 1 + \omega^2$$

or $$\omega^2 = 1 \quad \text{or,} \quad \omega = 1 \text{ rad/s.}$$

## ADDITIONAL PROBLEMS

**A3.1** Two impedances $Z_1 = (10 + j15)\ \Omega$ and $Z_2 = (20 - j25)\ \Omega$ are connected in parallel and this parallel combination is connected in series with impedance $(Z_3) = (25 + jX)\ \Omega$. Find for what value of $X$ resonance occurs.

***Solution***

The net impedance of the whole circuit

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} + Z_3 = \frac{(10 + j15)\,(20 - j25)}{10 + j15 + 20 - j25} + 25 + jX$$

$$= \frac{18.03\angle 56.31° \times 32\angle -51.34°}{30 - j10} + 25 + jX$$

$$= \frac{576.96\angle 4.97°}{31.62\angle -18.43°} + 25 + jX$$

$$= 18.247\angle 23.4° + 25 + jX$$

$$= 16.74 + j7.25 + 25 + jX$$

$$= 41.74 + j(X + 7.25).$$

For resonance the reactive part of the impedance must be zero, i.e. net reactance is 0.

$\therefore$ $$X + 7.25 = 0$$

or $$X = -7.25\ \Omega;$$

**A3.2** In the arrangement shown in Fig. 3.56 calculate the impedance between $A$ and $B$ and the phase angle between voltage and current.

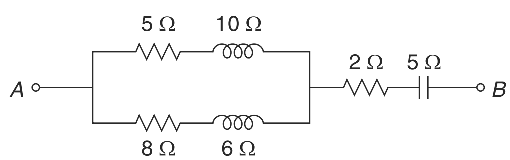



**Fig. 3.56** ***Circuit diagram for Ex. A3.2***

***Solution***

$$Z_{AB} = \frac{(5 + j10)\,(8 + j6)}{5 + j10 + 8 + j6} + (2 - j5)$$

$$= \frac{40 - 60 + j(80 + 30)}{13 + j16} + (2 - j5)$$

$$= \frac{-20 + j110}{13 + j16} + (2 - j5) = \frac{111.8\angle 100.3°}{20.61\angle 50.9°} + (2 - j5)$$

$$= 5.42\angle 49.4° + 2 - j5$$

$$= 5.53 - j\,0.88$$

$$= 5.6\angle -9.04°.$$

The impedance is 5.6 Ω and the angle between voltage and current is 9.04° (current is lagging w.r.t voltage).

**A3.3** A coil of resistance 50 Ω and inductance 0.5 H forms part of a series circuit for which the resonant frequency is 60 Hz. If the supply voltage is 230 V, 50 Hz. find (a) the line current, (b) power factor and (c) the voltage across the coil.

***Solution***

$$60 = \frac{1}{2\pi\sqrt{0.5C}}, \text{ where } C \text{ is the capacitance}$$

or
$$C = \frac{1}{0.5(2\pi \times 60)^2} = 14\ \mu\text{F}.$$

(a) Line current
$$= \left(\frac{V}{Z}\right) = \frac{V}{\sqrt{(50)^2 + \left(314 \times 0.5 - \frac{10^6}{314 \times 14}\right)^2}}$$

$$= \frac{230}{\sqrt{(50)^2 + (157 - 227.48)^2}}$$

$$= \frac{230}{86.4}\ \text{A} = 2.66\ \text{A}.$$

(b) Power factor $= \left(\frac{R}{Z}\right) = \frac{50}{86.4} = 0.578.$

As capacitive reactance is greater than the inductive reactance so p.f. is leading.

(c) Voltage across coil
$$= 2.66\sqrt{(50)^2 + (2\pi \times 50 \times 0.5)^2}$$
$$= 2.66 \times 164.77 = 438.28\ \text{V}.$$

**A3.4** A coil of resistance 5 Ω and inductance 0.1 H is connected in parallel with a circuit containing a coil of resistance 4 Ω and inductance 0.05 *H* in series with a capacitor *C* and a pure resistor *R*. Calculate the values of *C* and *R* so that currents in either branch are equal but differ in phase by 90°.

***Solution***

The impedance of coil 1 is
$$Z_1 = 5 + j\omega \times 0.1$$
$$= 5 \times j\,2\pi \times 50 \times 0.1$$
$$= 5 + j\,31.4.$$

The impedance of coil 2 is
$$(Z_2) = 4 + R + j\left(2\pi \times 50 \times 0.05 - \frac{1}{2\pi \times 50C}\right)$$
$$= 4 + R + j\left(15.7 - \frac{1}{314C}\right).$$

As the currents in either branch are equal but differ in phase by 90°

∴ $Z_2 = -jZ_1,$

i.e. $Z_2 = 31.4 - j\,5$

Thus, $4 + R = 31.4$ and $15.7 - \frac{1}{314C} = -5$

or $R = 31.4 - 4 = 27.4\ \Omega$

and
$$C = \frac{1}{314\,(15.7 + 5)}\ \text{F} = 153.85\ \mu\text{F}.$$

**A3.5** A 230 V 50 Hz supply is applied across a resistor of 10 Ω in parallel with a pure inductor. The total current is 25 A. What should be the value of the frequency if the total current is 36 A?

***Solution***

Let the inductance of the pure inductor be $L$.

When frequency is 50 Hz the admittance of the circuit is

$$y_1 = \frac{1}{10} + \frac{1}{j \times 2\pi \times 50L} = 0.1 - \frac{j}{314L}.$$

$$\therefore \qquad \sqrt{(0.1)^2 + \left(\frac{1}{314L}\right)^2} = \frac{25}{230}$$

or
$$0.01 + \frac{1}{98596\,L^2} = 0.0118$$

or
$$L = \frac{1}{\sqrt{98596\,(0.0118 - 0.01)}} \text{ H} = 0.075 \text{ H}$$

Let at frequency $f$ the total current be 36 A.

Then,
$$\sqrt{(0.1)^2 + \frac{1}{2 \times 3.14f \times 0.075}} = \frac{36}{230} = 0.156$$

or
$$\frac{1}{0.471f} = (0.156)^2 - 0.01 = 0.014$$

or
$$f = \frac{1}{0.471 \times 0.014} \text{ Hz} = 151 \text{ Hz.}$$

**A3.6** Two impedances $Z_1 = (47.92 + j76.73)\ \Omega$ and $(Z_2) = (10 - j5)\ \Omega$ are connected in parallel across a 200 V, 50 Hz supply. Find the current through each impedance and total current. What are the angles of phase difference of the branch currents with respect to the applied voltage?

***Solution***

Current through impedance $Z_1$ is

$$I_1 = \frac{200}{(47.92 + j76.73)} = \frac{200}{90.43\angle 58^\circ} = 21.21\angle{-58^\circ} \text{ A.}$$

Current through impedance $Z_2$ is

$$I_2 = \frac{200}{10 - j5} = \frac{200}{11.18\angle{-26.56^\circ}} = 17.89\angle 26.56^\circ \text{ A.}$$

Total current $(I) = I_1 + I_2 = 2.21\angle{-58^\circ} + 17.89\angle 26.56^\circ$

$= 17.17 + j\,6.12 = 18.23\angle 19.62^\circ$ A.

Current in branch 1 is lagging the applied voltage by 58° and

Current in branch 2 is leading the applied voltage by 26.56°.

**A3.7** A series connected $RLC$ circuit has $R = 15\ \Omega$, $L = 40$ mH and $C = 40\ \mu$F. Find the resonant frequency and under resonant condition calculate the current, power, voltage drops across various elements if the applied voltage is 75 V.

***Solution***

Resonant frequency $f_o = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2 \times 3.14\sqrt{0.04 \times 40 \times 10^{-6}}} = 125.88$ Hz.

Current at resonant condition $(I_o) = \dfrac{V}{R} = \dfrac{75}{15} = 5$ A.

Power $(= VI_o) = 75 \times 5 = 375$ W.

Voltage drop across $R = I_o R = 5 \times 15 = 75$ V = Applied voltage.

Voltage drop across $L = I_o X_L = 5 \times 2\pi \times 125.88 \times 0.04 = 158.1$ V.

Voltage drop across $C = I_o X_C = \dfrac{5 \times 10^6}{2\pi \times 125.88 \times 40} = 158.1$ V.

**A3.8** In a circuit two parallel branches $Z_1$ and $Z_2$ are in series with $Z_3$. The impedances are $Z_1 = 5 + j8$, $Z_2 = 3 - j4$ and $Z_3 = 8 + j10$. The voltage across $Z_3$ is 50 V. Find currents through the parallel branches and phase angle between them.

***Solution***

Current through $Z_3$ = Total current $= \dfrac{50\angle 10°}{8 + j10} = \dfrac{50}{12.8\angle 51.34°}$

$= 3.9\angle -51.34°$ A

Current through $Z_1$ is $I_1 = 3.9\angle -51.34° \times \dfrac{(3 - j4)}{5 + j8 + 3 - j4}$

$= 3.9\angle -51.34° \times \dfrac{5\angle -53.13°}{8.9\angle 26.56°}$

$= 2.19\angle -131.03°$ A

Current through $Z_2$ is $I_2 = 3.9\angle -51.34° \times \dfrac{(5 + j8)}{5 + j8 + 3 - j4}$

$= 3.9\angle -51.34° \times \dfrac{9.4\angle 58°}{8.9\angle 26.56°}$

$= 4.12\angle -19.9°$ A.

Phase angle between the two currents $(131.03° - 19.9°) = 111.13°$.

**A3.9** Prove that the impedance of a parallel ac circuit containing $R$ and $L$ in one branch and $R$ and $C$ in the other branch (Fig. 3.57) is equal to $R$ when $R^2 = \dfrac{L}{C}$.

If $L = 0.01$ $H$ and $C = 90$ μF, determine the impedance and current in each branch. Assume supply voltage to be 220 V, 50 Hz.

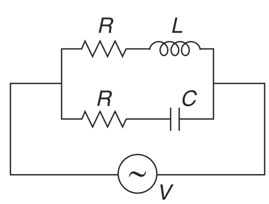



**Fig. 3.57** ***Circuit diagram for Ex. A3.9***

***Solution***

$$\text{Impedance } Z = \frac{(R + j\omega L)\left(R - \dfrac{j}{\omega C}\right)}{2R + j\left(\omega L - \dfrac{1}{\omega C}\right)} = \frac{R^2 + \dfrac{L}{C} + j\left(R\omega L - \dfrac{R}{\omega C}\right)}{2R + j\left(\omega L - \dfrac{1}{\omega C}\right)}$$

Since, $\dfrac{L}{C} = R^2$ we have

$$Z = \frac{R^2 + R^2 + jR\left(\omega L - \frac{1}{\omega C}\right)}{2R + j\left(\omega L - \frac{1}{\omega C}\right)} = R\,\frac{2R + j\left(\omega L - \frac{1}{\omega C}\right)}{2R + j\left(\omega L - \frac{1}{\omega C}\right)} = R.$$

When $L = 0.01\ H$ & $C = 90\ \mu$F,

$$Z = R = \frac{L}{C} = \frac{0.01}{90 \times 10^{-6}} = \frac{1}{9} \times 10^3 = 111.11\ \Omega$$

$$I_1 = \frac{220}{111.11 + j\omega\, 0.01} = \frac{220}{111.11 + j3.14} = 1.98\ \underline{|-1.62°}\ \text{A}$$

$$I_2 = \frac{220}{111.11 - j\,\frac{10^6}{90\omega}} = \frac{220}{111.11 - j35.385} = 1.98\ \underline{|17.66°}\ \text{A}.$$

**A3.10** A series circuit consists of a capacitor and a coil takes a maximum current of 0.314 A at 200 V, 50 Hz. If the voltage across the capacitor is 300 V at resonance determine the capacitance, inductance, resistance and $Q$ of the coil.

***Solution***

$$I_o = 0.314\ \text{A}$$

$$I_o R = 200 \quad \text{or,} \quad R = \frac{200}{0.314}\ \Omega = 636.943\ \Omega$$

$$I_o X_C = 300\ \text{V} \quad \text{or,} \quad X_C = \frac{300}{0.314}\ \Omega = 955.4\ \Omega = \frac{1}{\omega_o C}.$$

$$\therefore \quad C = \frac{1}{2\pi \times 50 \times 955.4}\ \text{F} = 3.33\ \mu\text{F}.$$

Again $\quad I_o X_L = 300 \quad \text{or} \quad X_L = \frac{300}{0.314} = 955.4 = \omega_o L$

$$\therefore \quad L = \frac{955.4}{2\pi \times 50}\ \text{H} = 3.04\ \text{H}.$$

$$\text{'}Q\text{' of the coil} = \frac{\omega_o L}{R} = \frac{2\pi \times 50 \times 3.04}{636.943} = 1.5.$$

**A3.11** Find the average and rms value of the waveform shown in Fig. 3.58.

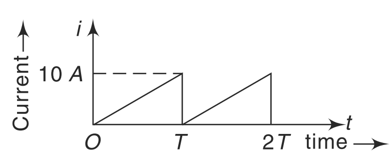



**Fig. 3.58** ***Circuit diagram for Ex. A3.11***

***Solution***

The slope of the curve is obtained as

$$i = \frac{10}{T}\,t$$

Average value of the waveform is given by

$$I_{\text{av}} = \frac{1}{T}\int_0^T i\,dt = \frac{1}{T}\int_0^T \frac{10}{T}\,t\,dt = \frac{1}{T}\left[\frac{10}{T}\,\frac{t^2}{2}\right]_0^T = \frac{10 \times T^2}{T^2 \times 2} = 5\ \text{A}.$$

rms value of the waveform is obtained as

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = \sqrt{\frac{1}{T}\int_0^T \frac{10}{T^2} t^2 dt} = \sqrt{\frac{100}{T^3}\left[\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{\frac{100 \times T^3}{T^3 \times 3}} = \frac{10}{\sqrt{3}} = 5.77 \text{ A.}$$

**A3.12** A coil having a resistance of 25 Ω and an inductance of 25 mH is connected in parallel with a variable capacitor. For what value of $C$ will the circuit the resonant if a 90 V, 400 Hz. source is applied? What will be the line current under this condition?

***Solution***

$$(f_o) = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}, \text{ where } (f_o) = \text{Resonant frequency}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.025C} - \left(\frac{25}{0.025}\right)^2} = 400 \text{ Hz}$$

or $$\frac{1}{0.025C} - 10^6 = (400 \times 2\pi)^2 = 6310144$$

∴ $$C = 5.47 \ \mu\text{F}$$

Line current $$(I_o) = \frac{V}{(L/CR)} = \frac{90}{\dfrac{0.025}{5.47 \times 10^{-6} \times 25}} = 0.492 \text{ A.}$$

**A3.13** A choke coil has a resistance of 40 Ω and $Q$ factor of 5 at 1000 Hz. It is connected in parallel with a variable capacitor to a 10 V, 1000 Hz ac supply. Find (a) $X_C$ for resonance (b) equivalent impedance as seen by the source and (c) current drawn from the supply.

***Solution***

$$Q \text{ factor} = \frac{\omega_o L}{R} = 5$$

or $$\frac{2\pi \times 1000L}{40} = 5$$

so $$L = \frac{40 \times 5}{2\pi \times 1000} = 0.0318 \text{ H}$$

(a) If $C$ is the capacitance then

$$\omega_o = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

or $$(2\pi \times 1000)^2 = \frac{1}{0.0318C} - \left(\frac{40}{0.0318}\right)^2$$

or $$\frac{1}{0.0318C} = 41020615.89$$

or $$C = 0.766 \ \mu\text{F}$$

or $$X_C = \frac{1}{\omega_o C} = (2\pi \times 1000 \times 0.766 \times 10^{-6})^{-1}$$

$$= 207.9 \ \Omega.$$

(b) Equivalent impedance as seen by the source is

$$\frac{L}{CR} = \frac{0.0318}{40 \times 0.766} \times 10^6 = 1037.86 \ \Omega.$$

(c) Current taken from supply $= \dfrac{10}{1037.86} = 9.6$ mA.

**A3.14** A pure capacitor of 60 μF is in parallel with another single circuit element. If the applied voltage and total current are $v = 100 \sin (2000\ t)$ and $i = 15 \sin (2000t + 75°)$ respectively find the other element.

***Solution***

$$V = 100 \sin 2000\ t$$

$$i = 15 \sin (2000\ t + 75°)$$

$$\text{Power factor} = \cos \theta = \cos 75° = 0.2588 \text{ leading}$$

As the total current is leading by an angle less than 90° (i.e. 75°) so the other element must be a resistor.

$$\therefore \qquad \tan \theta = \tan 75° = 3.73 = \frac{X_C}{R}$$

or $$R = \frac{X_C}{3.73} = \frac{1}{\omega C\ 3.73} = \frac{10^6}{2000 \times 50 \times 3.73}\ \Omega = 2.68\ \Omega.$$

**A3.15** A resistor and a capacitor are in series with a variable inductor across a 100 V, 50 Hz supply. The maximum current obtained by varying inductance is 5 A. The voltage across capacitance is 200 V. Find the circuit elements.

***Solution***

At resonance $$(I) = \frac{V}{R} = 5\text{ A}$$

or $$R = \frac{V}{5} = \frac{100}{5} = 20\ \Omega$$

Voltage across capacitor $(V_C) = I \times \dfrac{1}{\omega C} = 200$

or $$C = \frac{I}{200\omega} = \frac{5}{200 \times 2\pi \times 50}\text{ F} = 79.6\ \mu\text{F}$$

At resonance, $$\omega L = \frac{I}{\omega C}$$

So, $$L = \frac{I}{\omega^2 C} = \frac{10^6}{(2\pi \times 50)^2 \times 79.6}\text{ H}$$

$$= 0.127\text{ H.}$$

**A3.16** An inductor in series with a variable capacitor is connected across a constant voltage source of frequency 10 kHz. When the capacitor is 700 pF the current is maximum, when the capacitance is 900 pF the current is half of its maximum value. Find the resistance, inductance and $Q$ factor of the inductor.

***Solution***

Resonant freqency $f_o = \dfrac{1}{2\pi \sqrt{LC}}$

or $$10 \times 10^3 = \frac{1}{2\pi \sqrt{L \times 700 \times 10^{-12}}} = \frac{10^6}{2\pi \sqrt{700 L}}$$

or $$700\ L = \left(\frac{10^6}{10 \times 10^3 \times 2\pi}\right)^2 = 253.56$$

or $$L = 0.362\text{ H.}$$

The maximum current $= \dfrac{V}{R}$ where $V$ is the supply voltage and $R$ is the resistance.

When capacitor value = 900 pF.

$$\text{Current} = \frac{1}{2} \times \text{max. value of current}$$

$$\therefore \quad \text{current} = \frac{1}{2} \times \frac{V}{R} = \frac{V}{2R}.$$

Again $$\frac{V}{2R} = \frac{V}{\sqrt{R^2 + \left\{2\pi \times 10 \times 10^3 \times 0.362 - \dfrac{10^{12}}{2\pi \times 10 \times 10^3 \times 900}\right\}^2}}$$

or $$4R^2 = R^2 + 25409139.5 \quad \text{or} \quad R = 2910.3\ \Omega$$

$$Q_{\text{factor}} = \frac{\omega_o L}{R} = \frac{2\pi \times 10 \times 10^3 \times 0.362}{2910.3} = 7.8.$$

**A3.17** A sinusodial signal has a value of (–5) A at $t = 0$ and reached its first negative maximum of (–10) A at 2 ms. Write the equation for the current.

***Solution***

Let us consider the equation for the current is

$$i(t) = I_m \sin(\omega t + \theta)$$

At $t = 0$, $i(t) = I_m \sin\theta = -5$

At $t = 2 \times 10^{-3}$ s, $i(t) = I_m \sin(\omega \times 2 \times 10^{-3} + \theta) = -10$

Now, $I_m = 10$ A (negative and positive maximum values are same)

Hence $$\sin\theta = -\frac{5}{10} = -0.5 = \sin\left(\frac{-\pi}{6}\right) = \sin 210^\circ$$

i.e. $$\theta = 210^\circ$$

Again $$\sin(\omega \times 2 \times 10^{-3} + 210^\circ) = -\frac{10}{10} = -1 = \sin 270^\circ$$

Hence $$\omega \times 2 \times 10^{-3} = 270^\circ - 210^\circ = 60^\circ = \frac{\pi}{3}$$

i.e. $$\omega = \frac{\pi \times 10^3}{6} = 523.6$$

$$i = 10 \sin(523.6t + 210^\circ)\ \text{A}.$$

**A3.18** In a circuit the voltage and impedance are given by $v = (100 + j80)$ V and $Z = (10 + j8)\ \Omega$. Find the active and reactive power of the circuit.

***Solution***

It is given that,

$$Z = 10 + j\,8 = 12.8\angle 38.66^\circ\ \Omega$$

$$v = 100 + j\,80 = 128\angle 38.66^\circ\ \text{V}.$$

Hence current $$i = \frac{v}{Z} = \frac{128\angle 38.66^\circ}{12.8\angle 38.66^\circ} = 10\angle 0^\circ\ \text{A}.$$

Power factor angle = angle between $v$ and $i$ = 38.66°

Active power of the circuit

$$v.i \cos\theta = 128 \times 10 \cos(38.66^\circ) = 999.51\ \text{W}.$$

Reactive power of the circuit

$$v.i \sin\theta = 128 \times 10 \sin(38.66^\circ) = 799.6\ \text{VAR}.$$

**A3.19** Find an expression for the current and calculate the power when a voltage for $v = 283 \sin (100\ \pi t)$ is applied to a coil having $R = 50\ \Omega$ and $L = 0.159$ H.

***Solution***

Impedance $(Z) = R + j\omega L$

$\omega = 100\pi = 314$

Hence $Z = 50 + j \times 314 \times 0.159$

$= 50 + j\,49.95$

$= 70.67\angle 44.97°.$

Now $v = 283 \sin 100\ \pi t$

rms value of $v = \dfrac{283}{\sqrt{2}} = 200$ V.

$\therefore$ Rms value of current $(i_{rms}) = \dfrac{283\angle 0°}{\sqrt{2} \times 70.67\angle 44.97°} = 2.83\angle -44.97°$ A

Now $(i_{max}) = \sqrt{2} \times 2.83 = 4$ A.

The expression for current $i = 4 \sin (100\ \pi t - 44.97°)$ A

Power = $VI \cos \theta$, where $V$ and $I$ are the rms values of voltage, and current and $\theta$ is the power factor angle.

Hence Power $= 200 \times 2.83 \cos (44.97°) = 400$ W.

**A3.20** Find the average and rms value of the waveform shown in Fig. 3.59.

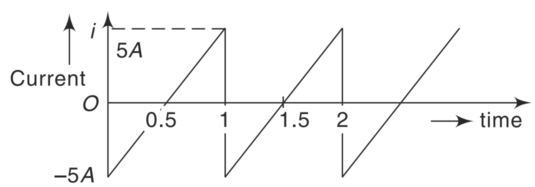



**Fig. 3.59** ***Waveform for Ex. A3.20***

***Solution*** The average value of the waveform over a full cycle ($t = 0$ to $t = 1$) is 0. Considering half cycle ($t = 0.5$ to $t = 1$)

$$I_{av} = \frac{1}{0.5}\int_{0.5}^{1} i\,dt$$

Now, $$i = \frac{5}{0.5}t - 5 = 10\,t - 5$$

Hence $$I_{av} = \frac{1}{0.5}\int_{0.5}^{1} (10t - 5)\,dt = 2\left[\frac{10t^2}{2} - 5t\right]_{0.5}^{1}$$

$$= 2[5 \times 1^2 - 5 \times 1 - 5 \times (0.5)^2 + 5 \times 0.5]$$

$$= 2[2.5 - 1.25]$$

$$= 2.5 \text{ A.}$$

$$I_{r.m.s.} = \sqrt{\frac{1}{0.5}\int_{0.5}^{1} (10t - 5)^2\,dt}$$

$$= \sqrt{\frac{1}{0.5}\int_{0.5}^{1} (100t^2 + 25 - 100t)\,dt}$$

$$= \sqrt{2\left[100\,\frac{t^3}{3} - 100\,\frac{t^2}{2} + 25t\right]_{0.5}^{1}}$$

$$= \sqrt{2\left\{\frac{100}{3}\times 1^3 - 100\times\frac{1^2}{2} + 25\times 1 - \frac{100}{3}(0.5)^3 + 100\times(0.5)^2\times\frac{1}{2} - 25\times(0.5)\right\}}$$

$$= 2.885\text{ A}$$

**A3.21** Find the average and rms value of the waveform shown in Fig. 3.60.

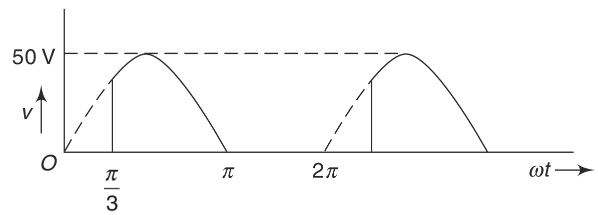



**Fig. 3.60** *Wave form for Ex. A3.21*

***Solution***

The average value of the waveform

$$I_{av} = \frac{1}{2\pi}\int_{\frac{\pi}{3}}^{\pi} 50\sin\omega t\, d(\omega t) = \frac{50}{2\pi}[-\cos\omega t]_{\pi/3}^{\pi}$$

$$= \frac{50}{2\pi}\left[\cos\frac{\pi}{3} - \cos\pi\right]$$

$$= \frac{25}{\pi}[0.5 + 1]$$

$$= 11.937\text{ V.}$$

The rms value of the waveform

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{\frac{\pi}{3}}^{\pi}(50)^2\sin^2\omega t\, d(\omega t)}$$

$$= \frac{50}{\sqrt{2\pi}}\sqrt{\int_{\frac{\pi}{3}}^{\pi}\frac{1}{2}(1-\cos 2\omega t)\, d(\omega t)}$$

$$= \frac{1}{\sqrt{2}}\times 19.95\sqrt{\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\pi/3}^{\pi}}$$

$$= 14.11\sqrt{\pi - \frac{\sin 2\pi}{2} - \frac{\pi}{3} + \frac{\sin\frac{2\pi}{3}}{2}}$$

$$= 14.11\sqrt{\pi - \frac{\pi}{3} + 0.433} = 22.42\text{ A.}$$

**A3.22** If the waveform of a current has form factor 1.2 and peak factor 1.7 find the average and rms value of the current if the maximum value of the current is 100 A.

***Solution***

$$\text{Form factor} = \frac{I_{\text{rms}}}{I_{\text{av}}} = 1.2$$

$$\text{Peak factor} = \frac{I_{\text{m}}}{I_{\text{rms}}} = \frac{100}{I_{\text{rms}}} = 1.7$$

Hence $$I_{\text{r.m.s.}} = \frac{100}{1.7} = 58.82 \text{ A}$$

and $$I_{\text{av}} = \frac{I_{\text{rms}}}{1.2} = \frac{58.82}{1.2} \text{ A} = 49 \text{ A.}$$

**A3.23** An emf given by 100 sin $\left(314t - \frac{\pi}{4}\right)$ V is applied to a circuit and the current is 20 sin (314$t$ – 1.5708) A. Find (a) frequency and (b) circuit elements.

***Solution***

$$V = 100 \sin\left(314t - \frac{\pi}{4}\right)$$

∴ $$\omega = 314 \text{ rad/s}$$

(a) Frequency $f = \dfrac{\omega}{2\pi} = \dfrac{314}{2 \times 3.14} = 50$ Hz.

(b) $i = 20 \sin (314\, t - 1.5708) \text{ A} = 20 \sin\left(314t - \dfrac{\pi}{4}\right) \text{ A}$

Current $i$ is lagging w.r.t $v$ by an angle of $\dfrac{\pi}{2} - \dfrac{\pi}{4}$ i.e. $\dfrac{\pi}{4}$.

Hence the circuit contains $R$ and $L$ only. If $Z$ is the impedance then

$$R = Z \cos\theta = \frac{100}{20} \cos\left(\frac{\pi}{4}\right) = 3.536\ \Omega$$

and $$\omega L = Z \sin\theta = \frac{100}{20} \sin\left(\frac{\pi}{4}\right) = 3.536\ \Omega$$

or, $$L = \frac{3.536}{314} = 11.26 \text{ mH.}$$

**A3.24** A two element series circuit consumes 700 W and has a power factor of 0.707 leading. If the applied voltage is $v = 141 \sin (314\, t + 30°)$, find the circuit elements.

***Solution***

$$\text{Active power} = VI \cos\theta = 700 \text{ W.}$$

Power factor $\cos\theta = 0.707$ (lead).

Hence $$VI = \frac{700}{0.707} = 990.$$

Now $$V = \text{rms value of voltage} = \frac{141}{\sqrt{2}} = 99.7 \text{ V.}$$

Hence, $$I = \frac{990}{99.7} = 9.93 \text{ A}$$

and $$Z = \frac{V}{I} = \frac{99.7}{9.93} = 10\ \Omega.$$

As the power factor is leading so the circuit contains resistance $R$ and capacitance $C$.

Hence $$R = Z\cos\theta = 10 \times 0.707 = 7.07\ \Omega$$

and $$\frac{1}{\omega C} = Z\sin\theta = 10 \times 0.707 = 7.07\ \Omega$$

or $$C = \frac{1}{314 \times 7.07}\text{ farad} = 450.45\ \mu\text{F}.$$

**A3.25** A cosine wave is represented by $v_{ab} = V_m \cos(\omega t + \theta)$, where the frequency is 50 Hz. In the expression of $v_{ab}$, the angle $\omega t$ and $\theta$ are usually expressed in radians. Express the angle in degrees and write down in the expression for $V_{ab}$.

***Solution***

$$v_{ab} = V_m \cos(\omega t + \theta)\ [\text{given, } \omega t \text{ and } \theta \text{ in radians}]$$

∴ In degrees we can represent as

$$v_{ab} = V_m \cos(2 \times 180 \times 50\, t + \theta)$$
$$= V_m \cos(18000\, t + \theta);\ \theta \text{ in degree.}$$

[Note that in radian the same expresion can be simplified as

$$v_{ab} = V_m \cos(2\pi f \cdot t + \theta)$$
$$= V_m \cos(2 \times 3.14 \times 50\, t + \theta)$$
$$= V_m \cos(314\, t + \theta),\ \theta \text{ in radians]}$$

**A3.26** A cosine wave is expressed as $v_{ab} = V_m \cos(360\, ft + \theta)$, where $\theta$ is in degree. Convert it to sine wave expression.

***Solution***

Given, $$v_{ab} = V_m \cos(360\, ft + \theta)$$

(Note that '360' stands for $2\pi$, where $\pi = 180°$) since we can convert a cosine function into a sine function by adding 90° to the angle ($\theta$), we now can write

$$v_{ab} = V_m \sin(360\, ft + \theta + 90°),\ \theta \text{ in degree.}$$

[Similarly, we can convert a sine function $V_{ab} = V_m \sin(360\, ft + \theta)$ to a cosine function by subtracting 90° from $\theta$.

i.e. $$v_{ab} = V_m \cos(360\, ft + \theta - 90°),\ \theta \text{ in degree.]}$$

**A3.27** A coil of resistance 10 Ω and inductance 10 mH is connected in parallel with a 25 μF capacitor. Calculate the frequency at resonance and the effective impedance of the circuit.

***Solution***

$$R = 10\ \Omega \quad L = 10\text{ mH} \quad C = 25 \times 10^{-6}\text{ F.}$$

$$\text{Resonant frequency } f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$
$$= \frac{1}{2\pi}\sqrt{\frac{1}{10 \times 10^{-3} \times 25 \times 10^{-6}} - \frac{(10)^2}{(10 \times 10^{-3})^2}}$$
$$= \frac{1}{2\pi}\sqrt{\frac{10^8}{25} - 10^6}$$
$$= 275.8\text{ Hz.}$$

The effective impedance of the circuit $\left(\frac{L}{CR}\right) = \frac{10 \times 10^{-3}}{25 \times 10^{-6} \times 10} = 40\ \Omega.$

**A3.28** A capacitor is placed in parallel with two inductive loads, one 20 A at 30° lag and another one of 40 A at 60° lag. What must be the current in the capacitor so that the current from the external circuit shall be at unity power factor?

***Solution***

$$I_1 = 20\angle -30°\text{A}$$

$$I_2 = 40\angle -60°\text{A}$$

Horizontral component of

$$I_1 = I_{1x} = 20 \cos(-30°) \text{ A} = 17.32 \text{ A.}$$

Vertical component of

$$I_1 = I_{1y} = 20 \sin(-30°) \text{ A} = -10 \text{ A.}$$

Horizontal component of

$$I_2 = I_{2x} = 40 \cos(-60°) \text{ A} = 20 \text{ A.}$$

Vertical component of

$$I_2 = I_{2y} = 40 \sin(-60°) \text{ A} = -34.64 \text{ A.}$$

If $I$ be the total current vertical component of

$$I = I_y = (-10 - 34.64) \text{ A} = -44.64 \text{ A.}$$

If the power factor is unity then vertical component of $I$ should be zero.

Hence if $I_c$ be the current from the capacitor then, $I_c + I_y = 0$, i.e. $I_c = -I_y = 44.64$ A.

**A3.29** Two circuits having the same numerical ohmic impedance are joined in parallel. The power factor of one circuit is 0.8 lag and that of the other is 0.6 lag. Find the power factor of the whole circuit.

***Solution***

Let the supply voltage be V and the numerical value of impedance of each circuit be $Z$.

Current in the Ist circuit $I_1 = \frac{V}{Z}\angle -\cos^{-1}(0.8) = \frac{V}{Z}\angle -36.87°$.

Current in the 2nd circuit $I_2 = \frac{V}{Z}\angle -\cos^{-1}(0.6) = \frac{V}{Z}\angle -53.13°$.

The resultant current is $I_1 + I_2$

$$= \frac{V}{Z}[\cos(-36.87°) + j\sin(-36.87°) + \cos(-53.13°) + j\sin(-53.13°)]$$

$$= \frac{V}{Z}(1.4 - j\,1.4) = 1.98\,\frac{V}{Z}\angle -45°$$

Hence power factor of the whole circuit is cos (45°) lag = 0.707 lag.

**A3.30** AC voltage ($v$) = $V_m \sin \omega t$ is applied across a load $R$. Obtain the expressions for instantaneous and average power.

***Solution***

$$\text{Instantaneous power } (p) = \frac{v^2}{R}$$

or

$$p = \frac{V_m^2}{R} \cdot \sin^2 \omega t = \frac{V_m^2}{2R}(1 - \cos 2\,\omega t)$$

In the expression of $p$, the constant part is $\left(\frac{V_m^2}{2R}\right)$ and hence it represents the magnitude of average power.

$$\therefore \qquad P_{av} = \frac{V_m^2}{2R}$$

**A3.31** A voltage and a current in a circuit are expressed as

$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t - \theta).$$

Find the instantaneous power.

***Solution***

Instantaneous power $p$ is given by

$$p = v.i = V_m \sin \omega t \times I_m \sin(\omega t - \theta)$$

$$= V_m \times I_m \times \frac{1}{2} [\cos (\omega t - \omega t + \theta) - \cos(\omega t + \omega t - \theta)]$$

$$= \frac{1}{2} V_m I_m [\cos \theta - \cos(2\omega t - \theta)].$$

[It can be observed that

$$P_{av} = \frac{1}{2} V_m I_m \cos \theta,$$

but, $$V_m = \sqrt{2} \times V_{rms}; I_m = \sqrt{2}\, I_{rms}$$

$$\therefore \qquad P_{av} = \frac{1}{2} \times 2\, V_{rms} I_{rms} \cos \theta = V_{rms} \times I_{rms} \times \cos \theta$$

'cos $\theta$' being known as power factor.]

**A3.32** Obtain the average value of the current waveform $i = I_m \sin \omega t$ for (a) one complete period $T$ and (b) half period $T/2$.

***Solution***

(a) $$I_{av} = \int_0^T I_m \sin \omega t\, dt = \frac{1}{T}\int_0^T I_m \sin \frac{2\pi}{T} \cdot t.\, dt = -\frac{I_m}{2\pi}\left[\cos \frac{2\pi}{T} \cdot t\right]_0^T = 0.$$

Thus average value of a sine wave (or even a cosine wave) for one complete period is zero.

(b) $$I_{av} = \frac{1}{T/2}\int_0^{T/2} I_m \sin \frac{2\pi}{T} \cdot t \cdot dt = -\frac{I_m}{\pi}\left[\cos \frac{2\pi}{T} \times t\right]_0^{T/2} = +\frac{2I_m}{\pi}.$$

It may be noted here that we have assumed the current waveform starting from the origin. As a general case we may think of the ac waveform starting from $t = t_o$ in the time scale on $x$-axis instead of assuming it starting from $t = 0$.

This gives us

$$I_{av} = \frac{1}{T}\int_{t_0}^{t_0 + T} I_m \sin \frac{2\pi}{T} \cdot t \cdot dt = -\frac{I_m}{2\pi}\left[\cos \frac{2\pi}{T} \cdot t\right]_{t_0}^{t_0 + t} = 0, \text{ for the complete cycle.}$$

For the half cycle,

$$I_{av} = \frac{1}{T/2}\int_{t_0}^{t_0 + T} I_m \sin \frac{2\pi}{T} \cdot t \cdot dt = -\frac{I_m}{\pi}\left[\cos \frac{2\pi}{T} \cdot t\right]_0^{t_0 + T/2} = 0,$$

$$= -\frac{I_m}{\pi}\left(-\cos \frac{2\pi\, t_0}{T} - \cos \frac{2\pi\, t_0}{T}\right)$$

$$= \frac{2I_m}{\pi} \cos \omega t_o.$$

*Check:* if we assume $t_o = 0$, $I_{av}$ for the half cycle of the ac wave becomes $\left(\frac{2I_m}{\pi}\right)$.

It may be noted here we could have written $I_{av}$ for full cycle as

$$I_{av} = \frac{1}{2\pi} \int_0^{2\pi} I_m \sin \omega t \, d(\omega t)$$

and for half cycle as

$$I_{av} = \frac{1}{\pi} \int_0^{\pi} I_m \sin \omega t \, d(\omega t)$$

**A3.33** Draw the power triangles for inductive and capacitive loads.

***Solution***

We start drawing with voltage and current phasors and in next step we draw power triangles (Fig. 3.61) and (Fig. 3.62). The steps are shown as follows:

***For inductive loads***

Note:

$$P = VI \cos \phi = (IZ)\, I \times \frac{R}{Z} = I^2 R$$

$$Q = VI \sin \phi = (IZ) \times I \times \frac{X}{Z} = I^2 X$$

$$|S| = \sqrt{P^2 + Q^2} \qquad \angle\phi = \tan^{-1} \frac{Q}{P}$$

$$\cos \phi = \frac{P}{\sqrt{P^2 + Q^2}} = P|S|.$$

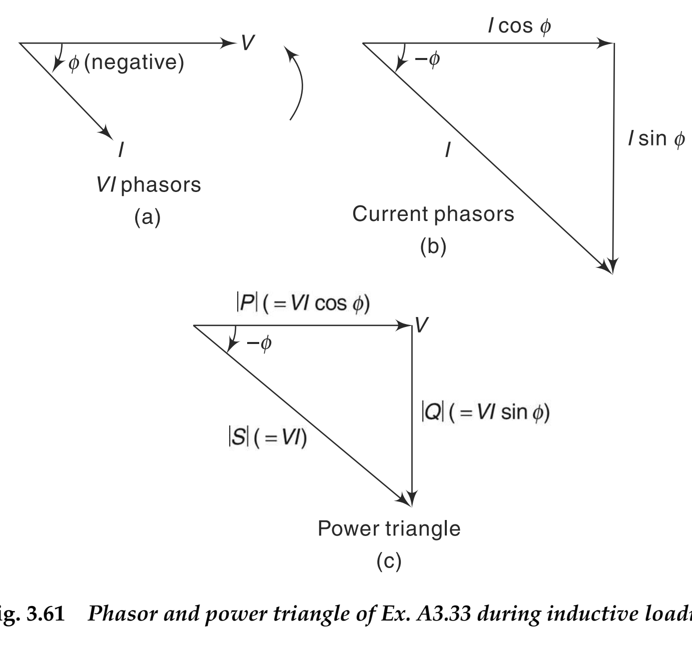



**Fig. 3.61** ***Phasor and power triangle of Ex. A3.33 during inductive loading***

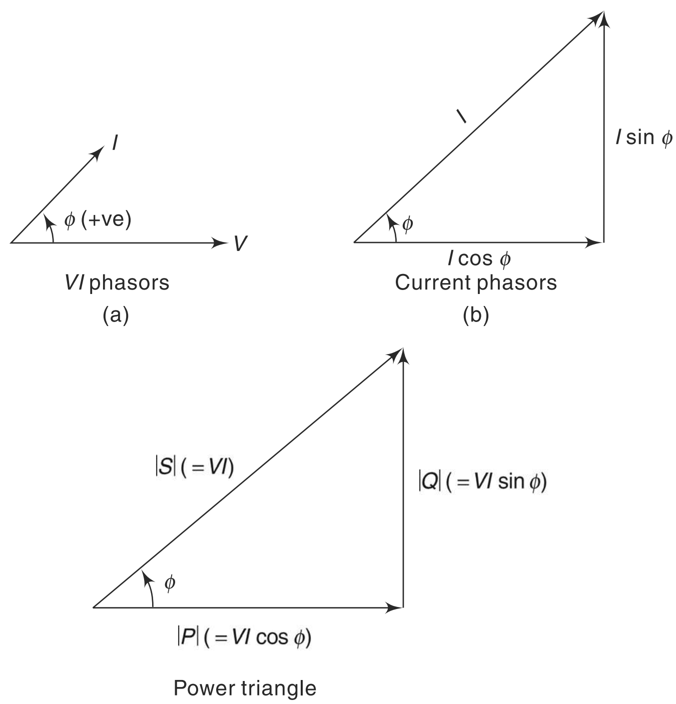



**Fig. 3.62** ***Phasor and power triangle of Ex. A3.33 during capacitive loading***

**A3.34** What is the reactance of a 10 μF capacitor at $f = 0$ Hz (dc) and $f = 50$ Hz?

***Solution***

At dc, $X_C = \dfrac{1}{2\pi \times 0 \times 10 \times 10^{-6}}$ = (infinite) Ω i.e. across a dc voltage, the capacitor would act as an open circuit at steady state.

When $f = 50$ Hz,

$$X_C = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = 318.47\ \Omega.$$

[If the element is an inductor, the reactance $X_L$ at $f = 0$ Hz would have been $X_L = 2\pi \times 0 \times L = 0\ \Omega$, indicating that the inductor acts as a short circuit across a dc voltage at steady state.]

**A3.35** At what frequency will a 100 μF capacitor offer a reactance of 100 Ω?

***Solution***

$$X_C = \frac{1}{\omega C}$$

$$\therefore \quad \omega = \frac{1}{X_C \times C} = \frac{1}{100 \times 100 \times 10^{-6}}$$

$$= 100 \text{ rad/sec.}$$

$\therefore \qquad f = \dfrac{100}{2\pi} = 15.9$ Hz.

At 15.9 Hz, the given capacitor will offer a reactance of 100 Ω.

**A3.36** An inductor draws a current $i = I_m \sin \omega t$. Obtain the expression of instantaneous voltage across it.

***Solution***

$v_L = L\dfrac{di}{dt}$ ($v$) being the voltage developed across the inductor opposing the supply voltage.

Here,
$$v_L = L \cdot \frac{d}{dt}(I_m \sin \omega t)$$
$$= \omega L\, I_m \cos \omega t = V_m \cos \omega t$$
$$= V_m \sin (\omega t + 90°).$$

$\therefore$ If ($v$) be the applied voltage
$$v = V_m \sin(\omega t - 90°)$$
$$i = I_m \sin \omega t$$
$$v_L = V_m \sin(\omega t + 90°).$$

[($v_L$) is in direct opposition to $v$ as phase angle between ($v$) and ($v_L$), is 180°].

**A3.37** An inductive coil consumes active power of 500 W and draws 10 A from a 60 Hz AC supply of 110 V. Obtain the values of resistance and inductance of the coil.

***Solution***

$\because \qquad I^2R = 500 \text{ W}; \quad I = 10 \text{ A}$

we find $\qquad R = 500/10^2 = 5\ \Omega.$

However, $\qquad Z = \dfrac{V}{I} = \dfrac{110}{10} = 11\ \Omega$

$\therefore \qquad Z = \sqrt{R^2 + (\omega L)^2}.$

Here $\qquad (\omega L)^2 = (11)^2 - (5)^2 = 96$

$\therefore \qquad L = \dfrac{\sqrt{96}}{\omega} = \dfrac{\sqrt{96}}{2 \times \pi \times 60} = 26$ mH.

Thus, the given coil has 5 Ω resistance and 26 mH inductance.

**A3.38** For the waveform shown in Fig. 3.63, find the average and rms values for full cycle.

***Solution***

$$V_{av} = \frac{1}{T}\int_0^{T/2} V_m \sin \omega t\, dt \qquad [\because \text{voltage between internal } T/2 \text{ and } T \text{ is zero}]$$
$$= \frac{V_m}{\omega T}[-\cos \omega t]_0^{T/2} = \frac{V_m}{\pi}$$
$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{T} V_m^2 \sin^2 \omega t\, dt}$$
$$= \sqrt{\frac{1}{2T}\int_0^{T/2} V_m^2 (1 - \cos 2\,\omega t)\, dt}$$

$$= \frac{V_m}{2}.$$

[The form factor can be calculated as $V_{rms}/V_{av}$ i.e, $(V_m/2)/(V_m/\pi)$ or, 1.57.]

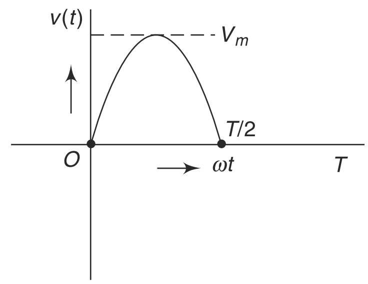



**Fig. 3.63** ***Waveform for Ex. 3.38 [T = 2π; T/2 = π]***

**A3.39** (a) Why the p.f. if ac circuit is always positive?

(b) Discuss when the reactive power is positive or negative.

***Solution***

(a) p.f. is represented by cos $\theta$. Since cos $\theta$ is an even function [$\cos\theta = \cos(-\theta)$] hence the power factor is always positive. Moreover, it is obviously always ≤ 1.0.

(b) $Q$ (reactive power) = $VI \sin\theta$

The sine wave is an odd function [$\sin\theta = -\sin(-\theta)$]. Hence $Q$ is +ve when $\theta$ is negative $Q$ is –ve when $\theta$ is +ve.

∴ Lagging current (inductive circuit) produces +ve and leading current (capacitive circuit) produces ($-Q$). In resistive circuit $\theta = 0$ giving $Q = 0$.

**A3.40** A saw-tooth current waveform is shown in Fig. 3.64. Determine the average value, rms value and form factor.

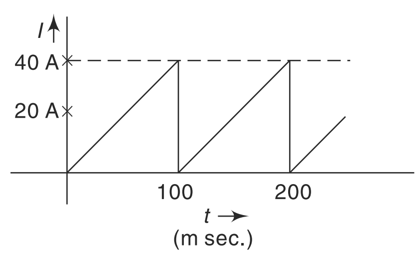



**Fig. 3.64** ***Waveform for Ex. 3.40***

***Solution***

The average value is obtained as

$$I_{av} = \frac{1}{T}\int_0^T i\, dt.$$

Here, $T = 100 \times 10^{-3}$ sec.

and $i = \dfrac{40 \times t}{100 \times 10^{-3}}$, a linear function

$$\therefore \quad I_{av} = \frac{1}{100 \times 10^{-3}} \int_0^{100 \times 10^{-3}} \frac{40}{100 \times 10^{-3}} \cdot t\, dt$$

$$= \frac{40}{(100 \times 10^{-3})^2} \cdot \left.\frac{t^2}{2}\right|_0^{100 \times 10^{-3}} = 20 \text{ A}.$$

rms value can be obtained as

$$I_{\text{rms}} = \left[\frac{1}{T}\int_0^T t^2\,dt\right]^{1/2}$$

$$= \left[\frac{1}{100\times10^{-3}}\int_0^{100\times10^{-3}}\left(\frac{40\times t}{100\times10^{-3}}\right)^2 dt\right]^{1/2}$$

$$= \left[\frac{1}{100\times10^{-3}}\int_0^{100\times10^{-3}}(400\,t)^2\,dt\right]^{1/2}$$

$$= \left[\frac{(400)^2}{100\times10^{-3}}\times\frac{t^3}{3}\Bigg|_0^{100\times10^{-3}}\right]^{1/2}$$

$$= 23.1 \text{ A.}$$

The form factor is given by $\left(\frac{I_{\text{rm}}}{I_{\text{av}}}\right)$. Here form factor $\left(=\frac{23.1}{20}\right) = 1.155.$ ▪▪▪▪▪▪▪

**A3.41** A resistor and a capacitor are connected in series across a 150 V AC 40 Hz supply. The current in the circuit is measured as 5 A. If the frequency of the supply be raised to 50 Hz, the current becomes 6 A. Find the values of the resistance and capacitance.

***Solution***

When $V = 150$ V, $f_1 = 40$ Hz, $I_1 = 5$ A

∴ $Z_1 = \frac{V}{I_1} = \frac{150}{5} = 30\ \Omega.$

[$Z_1$ is the circuit impedance of the *RC* series circuit at 40 Hz supply frequency.]

When $V = 150$ V, $f_2 = 50$ Hz, $I_2 = 6$ A.

∴ $Z_2 = \frac{V}{I_2} = \frac{150}{6} = 25\ \Omega.$

[$Z_2$ is the circuit impedance of the same $R_C$ series circuit at 50 Hz supply frequcy.]

∵ In a series *RC* circuit,

$$Z = \sqrt{R^2 + (1/2\pi\times f\times C)^2},$$

for the first case we can write

$$\sqrt{R^2 + \left(\frac{1}{2\pi\times 40\times C}\right)^2} = 30$$

and in the second case we can write

$$\sqrt{R^2 + \left(\frac{1}{2\pi\times 50\times C}\right)^2} = 25.$$

Solving these two equations for two unknown *R* and *C*, we get

$R = 30$ ohm and $C = 143.8\ \mu$F. ▪▪▪▪▪▪▪

**A3.42** An inductive circuit takes 50 A of current at of power factor 0.8 (lag) from a 250 V, 50 Hz supply. Calculate the value of the capacitance that is required to be connected across the inductive circuit to make the power factor unity.

***Solution***

When the capacitor is in parallel to the inductive circuit, the net power factor would be unity provided the capacitor's capacitive current cancel the inductive current of the inductive circuit i.e., $I_C = I_L \sin\phi$ [$I_C$ being the capacitor current while $I_L$ the inductive circuit current; $\phi$ is the p.f. of the inductive circuit]

or $$I_C = 50 \times 0.6 = 30 \text{ A}.$$

Again, $I_C = \dfrac{V}{X_C}$, $X_C$ being the capacitive reactance.

$$\therefore \quad X_C = \frac{V}{I_C} = \frac{250}{30} = 8.33\ \Omega.$$

Also, $$X_C = \frac{1}{2\pi f C}$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 8.33}$$

$$= 382.32\ \mu\text{F}.$$

**A3.43** Obtain average and effective value of waveform shown in Fig. 3.65.

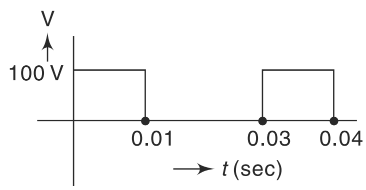



**Fig. 3.65** ***Waveform for Ex. 3.43***

***Solution***

$$V_{av} = \frac{1}{T}\int_0^{T/3} v\,dt$$

$$= \frac{1}{0.03}\int_0^{0.01} 100.\,dt$$

[$\because v(t) = 100$ for $0 \le t \le 0.01$; $v(t) = 0$ for $0.01 \le t \le 0.03$]

$$= \frac{1}{0.03} \times 100\,[t]_0^{0.01}$$

$$= 33.33 \text{ V}.$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{T/3} v^2\,dt}$$

$$= \sqrt{\frac{1}{0.03}\int_0^{0.01} 100^2\,dt}$$

$$= \sqrt{\frac{1}{0.03} \times 100^2 \times [t]_0^{0.01}}$$

$$= 57.74 \text{ V}.$$

**A3.44** The voltage and currents in a circuit element are given as $v = 141 \sin(314\,t + 30°)$ V and $i = 14.1 \sin(314\,t - 60°)$ A. Identify the element and find its value.

***Solution***

Since the angle associated with voltage is +ve while that for the current is –ve, hence the voltage leads the current. The angle of lead is 30–(–60), i.e. 90°. Hence the element is an inductor.

$$X_L = \frac{V_m}{I_m} = \frac{141}{14.1} = 10\ \Omega$$

or $$\omega L = 10, \text{ giving } L = \frac{10}{2\pi f} = \frac{10}{314} = 31.8 \text{ mH.}$$

**A3.45** The voltage and current in an element are

$$v = 100 \sin (314\, t - 20°) \text{ V}$$

$$i = 10 \sin (314\, t - 20°) \text{ A.}$$

Identify the element and find its values.

***Solution***

It may be observed that the voltage and current are in same phase. Thus the element is a resistor.

$$R = \frac{V_m}{I_m} = \frac{100}{10} = 10\ \Omega.$$

**A3.46** The voltage and current in a circuit element are $v$ = 100 cos (314 $t$ – 80°) V and $i$ = 100 cos (314 $t$ + 10°) A. Identify the element and find its value.

***Solution***

It may be observed that the current leads the voltage (angle associated with current is +ve while that associated with voltage is –ve).

The angle of lead for the current is 10 – (–80°) = 90°.

Thus the element is a capacitor.

$$X_C = \frac{V_m}{I_m} = \frac{100}{100} = 1\ \Omega = \frac{1}{\omega C}$$

$\therefore$ $$C = \frac{1}{314 \times 1} = 3184.7\ \mu\text{F.}$$

**A3.47** In an ac circuit

$$v = 200 \sin 314\, t \text{ V}$$

$$i = 20 \sin (314\, t - 30°)$$

Determine (a) the power factor

(b) True or active power

(c) Apparent or total power

(d) Reactive power.

***Solution***

The phase angle $\phi$ between the voltage and current is 30° while the current lags the voltage.

Here, $$V_{rms} (=V) = \frac{200}{\sqrt{2}} = 141.44 \text{ V.}$$

$$I_{rms} (=I) = \frac{20}{\sqrt{2}} = 14.14 \text{ A.}$$

$\therefore$ True power $= V_{rms} \times I_{rms} \times \cos \phi$ [cos $\phi$ being the power factor]

$$= 141.44 \times 14.14 \times \cos 30°$$

$$= 1732 \text{ W.}$$

Hence we have observed that p.f. of the circuit is 0.866 while the true or active power consumed is 1732 W. The apparent (or total power) being given by ($V \times I$), we find that its value in the given problem is (141.44 × 14.14), i.e. 2000 VA(2 kVA). The reactive power is obtained as $VI \sin \phi$, i.e. 141.44 × 14.14 × sin 30° or 1 kAR (app).

## HARDER PROBLEMS

**H3.1** A voltage of 100∠30° V is applied to a circuit having two parallel branches. If the currents are 20∠60° A and 10∠–45° A respectively find the kW, KVAR and kVA in each branch and in the whole circuit. What is the p.f. of the combined load?

***Solution***

For 1st branch

$$\text{kVA} = \frac{100 \times 20}{10^3} = 2$$

$$\text{kW} = \frac{100 \times 20}{10^3} \cos(60° - 30°) = 1.732$$

$$\text{KVAR} = \frac{100 \times 20}{10^3} \sin(60° - 30°) = 1 \text{ (lead).}$$

For the 2nd branch

$$\text{kVA} = \frac{100 \times 10}{10^3} = 1$$

$$\text{kW} = \frac{100 \times 10}{10^3} \cos(30° + 45°) = 0.2588$$

$$\text{KVAR} = \frac{100 \times 10}{10^3} \sin(30° + 45°) = 0.9659 \text{ (inductive)}$$

$$\begin{aligned} \text{Total current} &= 20\angle 60° + 10\angle -45° \\ &= 10 + j\,17.32 + 7.07 - j\,7.07 \\ &= 17.07 + j\,10.25 = 19.9\angle 30.98°. \end{aligned}$$

For the whole circuit

$$\text{kVA} = \frac{100 \times 19.9}{10^3} = 1.99$$

$$\text{kW} = 1.99 \cos(30° - 30.98°) = 1.989$$

$$\text{KVAR} = 1.99 \sin(30° - 30.98°) = 0.034 \text{ (capacitive)}$$

P.f. of combined load = cos (30° – 30.98°) = 10.98 (lead).

**H3.2** A coil having a resistance of 30 Ω and inductive reactance of 33.3 Ω is connected to a 125 V, 50 Hz. source. A series circuit consisting of 200 Ω resistor and a variable capacitor is then connected in parallel with the coil. For what value of capacitance will the circuit be in resonance? Given that resonant frequency is 60 Hz.

***Solution***

Here a series R-L circuit is in parallel with a series R-C circuit.

Resonant frequency $$f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC}\left[\frac{L - CR_L^2}{L - CR_C^2}\right]}.$$

Now at 50 Hz., $(Xs_L) = 33.3\ \Omega$

or $$L = \frac{33.3}{2\pi \times 50}\text{ H} = 0.106\text{ H.}$$

Again, resonant frequency $= 60 = \frac{1}{2\pi}\sqrt{\frac{1}{0.106C} - \frac{0.106 - C(30)^2}{0.106 - C(200)^2}}$

or $$\frac{1}{1.06C} - \frac{0.106 - 900C}{0.106 - 40000C} = (376.8)^2 = 141978.24$$

or $$\frac{0.106 - 40000C - 0.106C\,(0.106 - 900C)}{0.106C\,(0.106 - 40000C)} = 141978.24$$

$$0.106 - 40000C + 95.4C^2 = 1595.27C - 601987737C^2$$

or $C = 66\ \mu\text{F}$ (taking higher value).

**H3.3** Two coils of resistances 5 Ω and 10 Ω and inductance 0.01 H and 0.03 H respectively are connected in parallel. Calculate (a) the conductance, suseptance and admittance of each coil (b) the total current taken by the circuit, when it is connected to a 230 V, 50 Hz. supply (c) the characteristics of single coil which will take the same current and power as taken by the original circuit.

***Solution***

(a) Admittance of coil 1, $(y_1)$
$$= \frac{1}{Z_1} = \frac{1}{5 + j100\pi \times 0.01} = \frac{1}{5 + j3.14}$$
$$= \frac{1}{5.9\angle 32.13°}$$
$$= 0.169\angle{-32.13°} = 0.143 - j\,0.089.$$

Admittance of coil 2, $(y_2)$
$$= \frac{1}{Z_2} = \frac{1}{10 + j100\pi \times 0.03} = \frac{1}{10 + j9.42}$$
$$= \frac{1}{13.74\angle 43.3°}$$
$$= 0.073\angle{-43.3°} = 0.053 - j\,0.05.$$

Conductance of coils 1 and 2 are 0.143 S and 0.053 S respectively, while sesceptance of coils 1 and 2 are 0.089 S (inductive) and 0.05 S (inductive) respectively.

Admittance of coils 1 and 2 are 0.169 S and 0.073 S respectively.

(b) Total current taken by the circuit is

$$Vy = 230(y_1 + y_2)$$
$$= 230(0.143 - j\,0.089 + 0.053 - j\,0.05)$$
$$= 230(0.196 - j\,0.139)$$
$$= 55.26\text{ A.}$$

(c) $$Z = \frac{1}{y} = \frac{1}{0.196 - j0.139} = \frac{1}{0.24\angle{-35.34°}} = 4.167\angle 35.34° = 3.4 + j2.41.$$

The resistance of the single coil is 3.4 Ω and the inductive reactance is 2.41 Ω.

**H3.4** How a current of 50 A is shared among three parallel impedances of (5 + *j* 8), (6 – *j* 8) and (8 + *j* 9) Ω?

***Solution***

$$Z_1 = 5 + j\,8 = 9.43\angle 58°$$
$$Z_2 = 6 - j\,8 = 10\angle{-53.13°}$$

$$Z_3 = 8 + j\,9 = 12\angle 48.37°.$$

The admittances of the three branches

$$y_1 = \frac{1}{9.43\angle 58°} = 0.106\angle{-58°}$$

$$y_2 = \frac{1}{10\angle{-53.13°}} = 0.1\angle{+53.13°}$$

$$y_3 = \frac{1}{12\angle 48.37°} = 0.08\angle{-48.37°}.$$

Net admittance $y = y_1 + y_2 + y_3 = \frac{10}{V}$, where $V$ is the supply voltage.

$\therefore$ $$(0.056 - j\,0.09) + (0.06 + j\,0.08) + (0.053 - j\,0.06) = \frac{10}{V}$$

or $$0.17 - j\,0.07 = \frac{10}{V}$$

or $$V = \frac{10}{0.183} = 54.64 \text{ V}.$$

So the currents in the three impedances are

$$I_1 = y_1 V = 0.106 \times 54.64 = 5.792 \text{ A}$$

$$I_2 = y_2 V = 0.109 \times 54.64 = 5.464 \text{ A}$$

$$I_3 = y_3 V = 0.08 \times 54.64 = 7.00 \text{ A}.$$

**H3.5** A coil of resistance 10 Ω and inductance 0.5 H is connected in series with a capacitor across a voltage source. When the frequency is 50 Hz the current is maximum. Another capacitor is connected in parallel with the circuit. What capacitance must it have so that the combination acts as a pure resistor at 100 Hz?

***Solution***

The current is maximum at resonance. So 50 Hz is the resonant frequency.

$\because$ $$f_o = \frac{1}{2\pi\sqrt{LC}},$$

or $$50 = \frac{1}{2\pi\sqrt{0.5C}}$$

or $$C = \left(\frac{1}{50 \times 2\pi}\right)^2 \times \frac{1}{0.5} = 20.28\ \mu\text{F}.$$

At 100 Hz the impedance of the series branch is

$$Z_1 = 10 + j\left(2\pi \times 100 \times 0.5 - \frac{10^6}{2\pi \times 100 \times 20.28}\right).$$

$$= 10 + j\,235.48 = 235.69\angle 87.57°$$

If $C'$ is the capacitor connected in parallel with the circuit, impedance of the parallel branch is

$$= -jX_C$$

$$= -j\,\frac{1}{2\pi \times 100C'}.$$

Admittance of the combined circuit

$$= \frac{1}{235.69\angle 87.57°} + j200\pi C'$$

$= 0.0042\angle{-87.57°} + j628C'$

$= 0.000178 - j0.0042 + j628C'.$

Net susceptance should be zero if the circuit acts as a pure resistor.

So, $0.0042 = 628\,C'$ i.e., $C' = 6.68\ \mu F.$

**H3.6** In a series resonant circuit, the resistance is 6 Ω, the resonant frequency is $4.1 \times 10^6$ rad/s and the bandwidth is $10^5$ rad/s. Find $L$ and $C$ of the network, half power frequencies and $Q_{\text{factor}}$ of the circuit.

***Solution***

$$R = 6\ \Omega, \quad \omega_o = 4.1 \times 10^6 \text{ rad/s}$$

Bandwidth $(\omega_2 - \omega_1) = \Delta\omega = 10^5$ rad/s

$$\therefore \quad Q = \frac{\omega_o}{\Delta\omega} = \frac{4.1 \times 10^6}{10^5} = 41$$

Also,

$$Q = \frac{\omega_o L}{R} = 41$$

or

$$\frac{4.1 \times 10^6 \times L}{R} = 41$$

or

$$L = \frac{41 \times 6}{4.1 \times 10^6} = 6 \times 10^{-5}\text{ H.}$$

As at resonance, $X_L = X_C$,

$$C = \frac{1}{L\omega_o^2} = \frac{1}{6 \times 10^{-5} \times (4.1 \times 10^6)^2} = \frac{10^{-6}}{1008.6}$$

$$= 9.91 \times 10^{-10}\text{ F.}$$

Lower half power frequency $(\omega_1) = \left(\omega_o - \frac{R}{2\pi L}\right) = 4.1 \times 10^6 - \frac{6 \times 10^5}{2\pi \times 6} = 4.08 \times 10^6$ rad/s.

Upper half power frequency $(\omega_2) = \left(\omega_o + \frac{R}{2\pi L}\right)$

$$= 4.1 \times 10^6 + \frac{6 \times 10^5}{2\pi \times 6}$$

$$= 4.116 \times 10^6 \text{ rad/s.}$$

**H3.7** The total current $I$ in Fig. 3.66 is 15 A at lagging p.f. and the power consumed is 4 kW. The voltmeter reading is 300 V. Find the values of $R_1$, $X_1$, and $X_2$.

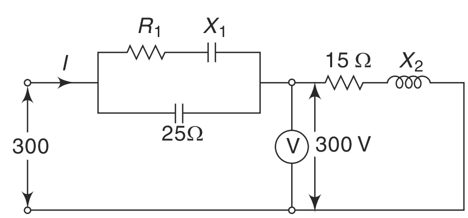



**Fig. 3.66** ***Circuit diagram for Ex. H3.7***

***Solution***

The voltmeter reading is the voltage across 15 Ω and $X_2$. $I$ is the current through that branch

$$\frac{300}{15} = \sqrt{(15)^2 + X_2^2}$$

or $$X_2 = \sqrt{\left(\frac{300}{15}\right)^2 - (15)^2} = 13.23\ \Omega.$$

Let $(R + jX)$ be the equivalent impedance of the parallel circuit

So net impedancce, $$Z = (R + 15) + j(X + 13.23)$$

∴ $$(R + 15)(15)^2 = 4000 \qquad \text{(i)}$$

and $$(R + 15)^2 + (X + 13.23)^2 = \left(\frac{300}{15}\right)^2 = 400 \qquad \text{(ii)}$$

From Eq. (i) $$R = \frac{4000}{225} - 15 = 2.78\ \Omega$$

From Eq. (ii) $$(X + 13.23)^2 = 400 - (2.78 + 15)^2 = 83.87$$

or $$X = -4.072\ \Omega.$$

Net admittance of the parallel branch

$$y = \frac{1}{2.78 - j4.072} = \frac{2.78 + j4.072}{24.31} = 0.114 + j\,0.1675.$$

So, $$\frac{1}{R_1 - jX_1} + \frac{1}{-j25} = 0.114 + j\,0.1675$$

or $$\frac{R_1 + jX_1}{R_1^2 + X_1^2} + j\,0.04 = 0.114 + j\,0.1675$$

or $$\frac{R_1}{R_1^2 + X_1^2} = 0.114 \qquad \text{(iii)}$$

and $$\frac{X_1}{R_1^2 + X_1^2} + 0.04 = 0.1675$$

or $$\frac{X_1}{R_1^2 + X_1^2} = 0.1275. \qquad \text{(iv)}$$

Now, $$\frac{R_1}{X_1} = \frac{0.114}{0.1275} = 0.89.$$

From Eq. (iii) $0.89X_1 = 0.114\ \{(0.89)^2 X_1^2 + X_1^2\}$

or $0.204\ X_1 = 0.89$ or $X_1 = 4.35$

and $R_1 = 0.89 \times 4.35 = 3.88\ \Omega.$

**H3.8** Calculate the real and reactive power to each load and the total complex power provided by the source (Fig. 3.67)

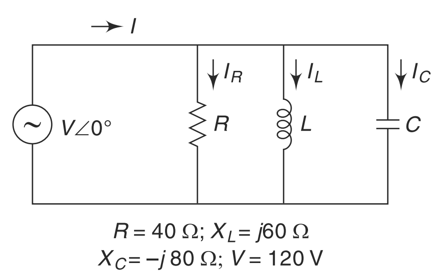



**Fig. 3.67** ***Circuit diagram for Ex. H3.8***

***Solution***

$$I_R = \frac{V}{R} = \frac{120\angle 0^\circ}{40} = 3\angle 0^\circ\ \text{A}$$

$$I_L = \frac{V}{X_L} = \frac{120\angle 0^\circ}{j60} = 2\angle -90^\circ\ \text{A}$$

$$I_C = \frac{V}{X_C} = \frac{120\angle 0°}{-j80} = 1.5\angle 90° \text{ A.}$$

$\therefore$ total current ($I$) $= I_R + I_L + I_C$ (vector sum)

$$= 3\angle 0° + 2\angle -90° + 1.5\angle 90°$$

$$= 3 - j0.5 = 3.041\angle -9.46° \text{ A.}$$

Active power in $R$ being given by $P$, we find

$$P = V \times I_R = 120 \times 3 = 360 \text{ W}$$

$$P = I_R^2 \times R = 3^2 \times 40 = 360 \text{ W}$$

$$P = \frac{V^2}{R} = \frac{(120)^2}{40} = 360 \text{ W.}$$

Since $\phi = 0$ in the expression of current $I_R$ i.e, as $I_R$ and $V$ are in phase, hence reactive power consumed in $R$ is zero.

Again for $L$ and $C$ elements, $\phi$ is either –90° or +90°, i.e cos $\phi = 0$ in both the cases. Hence reactive power consumption by $L$ or $C$ element is zero.

Reactive power for $L$ is obtained as

$$Q_L = V \times I_L = 120 \times 2 = 240 \text{ VAR}$$

$$Q_L = I_L^2 X_L = 2^2 \times 60 = 240 \text{ VAR}$$

$$Q_L = \frac{V^2}{X_L} = \frac{120^2}{60} = 240 \text{ VAR.}$$

Reactive power for $C$ element is obtained as

$$Q_C = V \times I_C = 120 \times 1.5 = 180 \text{ VAR}$$

$$Q_C = I_C^2 \times X_C = (1.5)^2 \times 80 = 180 \text{ VAR}$$

$$Q_C = (V^2/X_C) = 120^2/80 = 180 \text{ VAR.}$$

For total complex power we can write

$$S = P + j\,(Q_L - Q_C).$$

[$Q_L$ is +ve in inductive circuit while $Q_C$ is negative for capacitive circuit]

$$= 360 + j\,(240 - 180) = (360 + j60) \text{ VA}$$

[Also, $S = VI^* = 120\angle 0° \times 3.041\angle 9.46°$

$$= 364.9\angle 9.46° \text{ VA} = 360 + j60 \text{ VA.}]$$

The real power provided by the source is 360 W. The reactive power provided by the source is 60 VAR (inductive circuit requirement is actually 240 VAR but capacitor generates 180 VAR hence net requirement is only 60 VAR).

**H3.9** A 400 V single phase ac motor is tested in parallel with a 100 Ω resistor. The motor takes 5 A current at lagging p.f. and the total current is 7 A. Find the p.f. and power of the whole circuit and for the motor alone.

***Solution***

Current through resistor $(I_R) = \frac{400}{100} \text{ A} = 4 \text{ A.}$

Current through motor $(I_m) = 5$ A.

Total current $(I) = 7$ A.

Let the motor current be $(x - jy)$.

$$\therefore \quad x^2 + y^2 = 5^2 = 25.$$

Total current = $4 + x - jy$.

$$\therefore \quad (4 + x)^2 + y^2 = 7^2 = 49.$$

or $\quad 16 + x^2 + y^2 + 8x = 49$

or $\quad 25 + 8x = 49 - 16 = 33$

or $\quad 8x = 8$

or $\quad x = 1.$

Thus $\quad y = \sqrt{25 - 1^2} = \sqrt{24} = 4.9.$

p.f. of the motor $= \dfrac{x}{I_m} = \dfrac{1}{5} = 0.2$ lagging.

The complex part of the total current negative. So the p.f. of the whole circuit is lagging.

$\therefore$ the p.f. of the whole circuit $= \dfrac{4 + x}{7} = \dfrac{4 + 1}{7} = \dfrac{5}{7} = 0.714$ (lag).

Power of the whole circuit $= 400 \times$ real part of total current

$= 400 \times (4 + x) = 400 \times 5 = 2000$ W.

Power of motor alone $= 400 \times$ real part of motor current

$= 400 \times 1 = 400$ W.

**H3.10** A voltage of 400 V is applied across a pure resistor, a pure capacitor and an inductive coil which are in parallel. The resultant current is 6 A and the currents in the above components are 3 A, 4 A and 2 A respectively. Find the power factor of the inductive coil and the power factor of the whole circuit.

***Solution***

The current in the resistor $I_R = 3$ A.

The current in the capacitor $(I_C) = 4$ A.

The current in the inductive coil $(I_L) = 2$ A.

Let the current in the inductive coil be $(x - jy)$

$$\therefore \quad \sqrt{x^2 + y^2} = 2 \quad \text{or,} \quad x^2 + y^2 = 4. \qquad \text{(i)}$$

Total current $I = 3 + j\,4 + x - jy$

so $$\sqrt{(3 + x)^2 + (4 - y)^2} = 6 \qquad \text{(ii)}$$

or $\quad x^2 + y^2 + 6x - 8y + 9 + 16 = 36$

or $\quad 4 + 6x - 8y = 36 - 25 = 11$

or $\quad 6x - 8y = 7$

or $\quad x = \dfrac{7 + 8y}{6}.$

Substituting $x$ in Eq. (1)

$$\left(\frac{7 + 8y}{6}\right)^2 + y^2 = 4$$

or $\quad 49 + 112y + 64y^2 + 36y^2 = 144$

or $\quad 100y^2 + 112y - 95 = 0$

or $\quad y^2 + 1.12y - 0.95 = 0$

or $\quad y = 0.564 \quad$ (taking positive value of $y$)

so $$x = \frac{7 + 8 \times 0.564}{6} = 1.92.$$

$$\text{p.f. of the inductive coil} = \frac{x}{I_L} = \frac{1.92}{2} = 0.96 \text{ lag.}$$

$$\text{p.f. of the whole circuit} = \frac{\text{Real part of total current}}{I} = \frac{(3 + x)}{6} = \frac{(3 + 1.92)}{6} = 0.82.$$

As the $j$ component of total current is positive, i.e. (4 – 0.564) = 3.436 so the p.f. of the circuit is leading, i.e. p.f. of the whole circuit is 0.82 lead.

## EXERCISES

1. Explain with the help of a diagram how alternating current is generated.
2. Define the following:
   (a) Amplitude of an alternating quantity
   (b) Instantaneous value of an alternating quantity
   (c) Frequency (d) Phase
   (e) Phase difference (f) Time period
3. Define rms and average value of an alternating quantity. Explain how these value can be obtained.
4. Define form factor and peak factor of an alternating quantity.
5. Explain with the help of diagrams what you understand by in phase, lagging and leading as applied to simusoidal quantities.
6. Define power factor as applied to ac circuits. What do you mean by active power, reactive power and apparent power?
7. Explain the meaning of the following terms in connection with alternating current:
   (a) inductance (b) capacitance (c) reactance
   (d) impedance (e) admittance (f) susceptance
   (g) conductance.
8. Show that power consumed in a purely inductive circuit and purely capacitive circuit is zero when sinusoidal voltage is applied across it.
9. Explain with the help of a diagram the phenomenon of resonance in series *R-L-C* circuit.
10. Derive an expression for the resonant frequency of a parallel circuit, one branch consisting of a coil of inductance $L$ and resistance $R$ and the other branch of capacitance $C$.
11. Derive the quality factor of a series *R-L-C* circuit at resonance.
12. Define quality factor in a series *R-L-C* circuit. Determine the half power frequencies in terms of quality factor and the resonant frequency for series *R-L-C* circuit.
13. Why is a series resonant circuit called an acceptor circuit and parallel resonant circuit a rejector circuit?

14. Explain dynamic impedance in connection with parallel resonant circuit.
15. Define bandwidth in a series *R-L-C* circuit. Prove that in a series *R-L-C* circuit $(Q_0) = \dfrac{\omega_o L}{R} = \dfrac{f_o}{\text{Bandwidth}}$.
16. Find the average value, rms value, form factor and peak factor of the waveform shown in Fig. 3.68

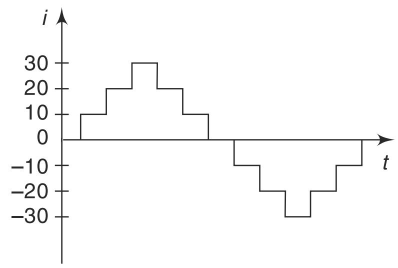



**Fig. 3.68** ***Waveform for Ex. 16***

[*Ans:* 15 A, 17.8 A, 1.187, 1.685]

17. The maximum value of a sinusoidally alternating voltage is 100 volts. Find the instantaneous value at $\frac{1}{9}$ cycle and $\frac{1}{18}$ cycle. [*Ans:* 64.28 V, 34.2 V]
18. The voltage given by $(v_1) = 50 \sin(377t - 30°)$ and $(v_2) = 20 \sin(377t + 45°)$ act in the series circuit. Determine the frequency and rms value of the resultant voltage. [*Ans:* 60 Hz, 41.34 V]
19. Define phasors as used in the study of ac circuits. Use phasors to find the sum of the sinusoids 40 sin 314*t* and 30 cos $\left(314t - \frac{\pi}{4}\right)$.

[*Ans:* 64.78 sin (314 *t* + 19.11°)]

[*Hint:*
$$a = 40 \sin 314t$$
$$b = 30 \cos\left(\frac{\pi}{4} - 314t\right)$$
$$= 30 \sin\left(\frac{\pi}{2} - \frac{\pi}{4} + 314t\right)$$
$$= 30 \sin\left(314t + \frac{\pi}{4}\right)$$

Using the method described in Art 5.3.3, we get the required sum is 64.78 sin (314*t* + 19.11°)].

20. The equation of an alternating current is $i = 62.35 \sin 323t$ A.
Determine its
(a) maximum value (b) frequency (c) r.m.s. value
(d) average value and (e) form factor

[*Ans:* $I_{max}$ = 62.35 A; *f* = 51.41 Hz; $I_{rms}$ = 44.1 A; $I_{av}$ = 39.69 A, FF = 1.11]

[*Hint:*

(a) Maximum value 62.35 A

(b) Frequency = $\frac{323}{2\pi}$ Hz = 51.41 Hz

(c) r.m.s. value = $\frac{62.35}{\sqrt{2}}$ = 44.1 A

(d) Average value = $\frac{62.35}{\frac{\pi}{2}}$ = 39.69 A

(e) Form factor = $\frac{\text{r.m.s. value}}{\text{average value}} = \frac{44.1}{39.69}$ = 1.11]

21. An ac series circuit consisting of a pure resistance of 25 Ω, inductance of 0.15 H and capacitance of 80 μF is supplied from a 230 V, 50 Hz ac source.
    (a) Find the impedance of the circuit, the current, the power drawn by the circuit and the power factor.
    (b) Draw the phasor diagram. [*Ans:* $Z = 26.03/16.55°\Omega$ $|I| = 8.83$ A; $P = 1948.77$ W; cos $\phi = 0.96$ (lag)]

[Hint: (a) $Z = 25 + j\,100\pi \times 0.15 - j\,\frac{1}{100\pi \times 80 \times 10^{-6}}$

$= 25 + j\,7.29 = 26.05\,\angle 16.25°\ \Omega$

$I = \frac{230}{26.05}$ A = 8.83 A.

Power factor cos $\theta$ = cos 16.25° = 0.96 lagging

Power = 230 × 8.832 × 0.96 = 1948.77 W]

22. Figure 3.69 shows a circuit in which a coil having resistance $R$ and inductance $L$ is connected in series with a resistance of 80 Ω. The combination is fed from a sinusolidal source.

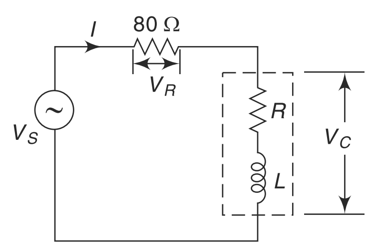



**Fig. 3.69** ***Circuit of Ex. 22***

The following measurements (rms value) are taken at 50 Hz. When the circuit is in steady state.

$|V_S| = 145$ V, $|V_R| = 50$ V

and $|V_C| = 110$ V

Find the value of $R$ and $L$. [*Ans:* $R = 102.8$ Ω; $L = 0.455$ H]

[*Hint:* $I = \frac{50}{80}\text{ A} = \frac{5}{8}\text{ A}$

$$Z^2 = R^2 + X_L^2 = \left(\frac{110}{\frac{5}{8}}\right)^2 (176)^2 \quad \text{(i)}$$

$$\text{Again } \left(\frac{145}{\frac{5}{8}}\right)^2 = (80 + R)^2 + X_L^2 \quad \text{(ii)}$$

Solving (i) and (ii)

$R = 102.8\ \Omega$ and $X_L = 142.85$ or $L = \frac{142.85}{2\pi \times 50}$ H = 0.455 H].

23. A coil of resistance 10 Ω and inductance 0.02 H is connected in series with another coil of resistance 6 Ω and inductance 15 mH across a 230 V, 50 Hz supply. Calculate (i) impedance of the circuit, (ii) voltage drop across each coil, (iii) the toal power consumed by the circuit.

[*Ans:* $Z = 19.41\angle 34.49°\ \Omega$; $V_{\text{drop(1)}} = 19139.92\angle -2.35°$; $V_{\text{drop(2)}} = 90.3\angle 3.65°$V; $P = 2.25$ kW]

[Hints: (i) Impedance $= 10 + j2\pi \times 50 \times 0.02 + 6 + j2\pi \times 50 \times 0.015$

$= 10 + j6.28 + 6 + j\,4.71 = 16 + j\,10.99$

$= 19.41 \underline{|34.48°}\ \Omega$.

(ii) Voltage across the 1st coil

$$(10 + j6.28) \times \frac{230}{19.41 \underline{|34.48°}} = 139.94 \underline{|-2.35°} \text{ V}$$

Voltage across the 2nd coil is

$$(6 + j\,4.71) \times \frac{230}{19.41 \underline{|34.48°}} = 90.39 \underline{|3.65°} \text{ V}$$

(iii) Total power $= \left(\frac{230}{19.41}\right)^2 \times 16 = 2246.6$ W]

24. A circuit consists of two parallel branches. The branch currents are represented by $i_1 = 10 \sin \omega t$ and $i_2 = 20 \sin (\omega t + 60°)$. If the supply frequency is 50 Hz, calculate the resultant current at $t = 0$ and at $t = 1$ ms.

[*Ans:* 17.31 A and 22.65 A]

25. A 100 V, 80 W lamp is to be operated on a 240 V, 50 Hz supply. Calculate the value of (a) non-inductive resistor, (b) pure inductor to be connected in series with the lamp so that it can be used at its rated voltage.

[*Ans:* 175 Ω, 0.868 H]

## MULTIPLE CHOICE QUESTIONS

1. Peak value being the same, which of the following will have the highest rms value?
   (a) sine wave (b) half wave rectified sine wave
   (c) triangular wave (d) square wave
2. The peak value of a sine wave is 400 V. The average value is
   (a) 254.8 V (b) 5656 V
   (c) 282.8 V (d) 400 V
3. An inductor supplied with 100 V ac with a frequency of 10 KHz passes a current of 1 mA. The value of inductor is
   (a) 1.7 H (b) 16 mH
   (c) 1 mH (d) 160 mH
4. When two sinusoidal quantities are said to be in phase quadrature, their phase difference is
   (a) 90° (b) 0°
   (c) 45° (d) 30°
5. Two sinusoidal currents are given by $i_1 = 10 \sin (\pi t + \pi/3)$ and $i_2 = 15 \sin (\omega t - \pi/4)$. Phase difference between them is
   (a) 150° (b) 50°
   (c) 105° (d) 60°
6. The period of a sinusoidal wave is
   (a) the same as frequency
   (b) time period to complete one cycle
   (c) expressed in ampere
   (d) none of these
7. In a generator maximum value of emf is generated within the coil axis is at
   (a) zero degree with field axis
   (b) 45° with field flux
   (c) 180° with field axis
   (d) 90° with field flux
8. For which of the following sinusoids, the rms value and mean value is the same?
   (a) sine wave (b) triangular wave
   (c) square wave (d) halfwave rectified sine wave
9. The time period of a sinusoid with 200 Hz frequency will be
   (a) 0.05 s (b) 0.005 s
   (c) 0.53 (d) 0.00053

10. Admittance is the reciprocal of
    (a) inductive reactance (b) reactive power
    (c) capacitive reactance (d) impedance
11. In any ac circuit always
    (a) apparent power is more than the actual power
    (b) actual power is more than the reactive power
    (c) reactive power is more than the apparent power
    (d) reactive power is more than the actual power
12. Zero degree phase difference exists between voltage and current in an ac current
    (a) when current is maximum and voltage is zero
    (b) when voltage and current both reach zero and maximum at the same time
    (c) when voltage is maximum and current is zero or vice versa
    (d) when voltage is maximum, current is neither zero or maximum and vice versa
13. The period of a sine wave is 1/5 m seconds, its frequency is
    (a) 5000 Hz (b) 40 Hz
    (c) 20 Hz (d) 30 Hz
14. When the current and voltage in a circuit are out of phase by 90°, the power is
    (a) zero (b) undefined
    (c) maximum (d) minimum
15. The positive maximum of a sine wave occurs at
    (a) 180° (b) 90°
    (c) 45° (d) 0°
16. In purely inductive circuit
    (a) reactive power is zero (b) apparent power is zero
    (c) actual power is zero (d) none of above
17. Form factor of a sine wave is
    (a) 0.637 (b) 0.707
    (c) 1.11 (d) 1.414
18. What is the periodic time of a system with a frequency of 50 Hz?
    (a) 0.2 s (b) 2 s
    (c) 0.02 s (d) 20 s
19. For the same peak value, which wave will have the least rms value?
    (a) square wave (b) triangular wave
    (c) sine wave (d) full wave rectified sine wave
20. Inductive reactance of a circuit is more when
    (a) inductance is more and frequency of supply is more
    (b) inductance is more and frequency is less

(c) inductance is less and frequency is less
(d) inductance is less and frequency is more

21. Inductance affects the direct current flow at the time of
    (a) turning on and off (b) operation
    (c) turning on (d) turning off
22. In a series RC circuit as frequency increases
    (a) current decreases (b) current remains unaltered
    (c) current increases (d)
23. Which of the following waves has unity form factor?
    (a) triangular (b) square
    (c) sine wave (d) square wave
24. The current in a circuit is given by $i = 50 \sin \omega t$. If the frequency be 25 Hz, how long will it take for the current to rise to 25 amp?
    (a) 0.02 sec (b) 0.05 sec
    (c) $3.33 \times 10^{-3}$ (d) 0.033 sec
25. In a parallel *RC* circuit, the supply current always _____ the applied voltage
    (a) lags (b) leads
    (c) remains on phase with (d) none of the above
26. For a given power factor of the load, if the p.f. of the load decreases, it will draw from the supply
    (a) less current (b) more current
    (c) same current
27. In a series circuit on resonance, the following will occur:
    (a) $X_L = X_C$ (b) $V_L = V_C$
    (c) $Z = R$ and $V = V_R$ (d) all above
28. P.F. of following circuit will be zero when the circuit contains
    (a) capacitance only (b) resistance only
    (c) inductance only (d) capacitance and inductance
29. In ac circuit the power curve is a sine wave having
    (a) half the frequency of voltage
    (b) double the frequency of voltage
    (c) same frequency of voltage
    (d) three times the frequency of voltage
30. In an ac circuit, a low value of KVAR compared with kW indicates
    (a) maximum load current (b) low efficiency
    (c) high p.f. (d) unity p.f.
31. A certain waveform has a form factor of 1.2 and a peak factor of 1.5. If the maximum value is 100 then the r.m.s. value is
    (a) 55.5 (b) 66.6
    (c) 39.8 (d) 25.4

32. The periodic time of a system with a frequency of 60 Hz is
    (a) 0.02 second (b) 0.011 second
    (c) 0.0166 second (d) 0.0139 second
33. An alternating voltage is given by the equation $U = 100 \sin\left(300\,t + \frac{\pi}{6}\right)$. The frequency of the voltage is
    (a) 47.77 Hz (b) 61.29 Hz
    (c) 300 Hz (d) 50 Hz
34. In a pure Inductive circuit,
    (a) the current is in phase with the voltage
    (b) the current logs behind the voltage by $\pi/2$
    (c) the current leads the applied voltage by $\pi/2$
    (d) none of these
35. A coil of resistance 50Ω and inductive reactance of 150Ω is connected across a supply voltage of 240 V. The current flowing through the circuit is
    (a) 1.52 A (b) 1.028 A
    (c) 1.24 A (d) 0.98 A
36. A series *RL* circuit having $R = 25\Omega$ and $X_L = 9.42\Omega$ is connected across a 230 V, 50 Hz supply. The power factor of the circuit is
    (a) 0.847 lag (b) 0.847 lead
    (c) 0.94 lead (d) 0.94 lag
37. 2 kW load takes a current of 10 A from 240 V ac supply. The KVA of the load is
    (a) 4.8 KVA (b) 2.4 KVA
    (c) 1.2 KVA (d) 20 KVA
38. *Q*-factor of a series *RLC* circuit is given by
    (a) $\frac{L}{R}\sqrt{\frac{1}{C}}$ (b) $\frac{1}{R}\sqrt{\frac{1}{LC}}$
    (c) $\frac{1}{R}\sqrt{\frac{L}{C}}$ (d) $\frac{1}{L}\sqrt{\frac{R}{C}}$
39. A circuit consists of a coil of 70Ω resistance and 2 H inductance in series with a capacitor of 0.5$\mu$F capacitance. The resonant frequency is
    (a) 147 Hz (b) 159 Hz
    (c) 171 Hz (d) 135 Hz
40. Rejector circuit is a
    (a) *RLC* Parallel resonant circuit
    (b) *RLC* series resonant circuit
    (c) *RC*-*RL* Parallel resonant circuit
    (d) none of these

41. Determine the Norton's current from the Thevenin's equivalent circuit as shown in Fig. 3.70.

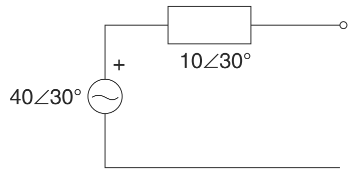



**Fig. 3.70**

(a) 4∠0° A (b) 0.25 ∠ 60°

(c) 4.37∠ 30° (d) 0.783 ∠ 37.62° A

42. Find the maximum power delivered to the load $R_L$ of the network as shown in Fig. 3.71.

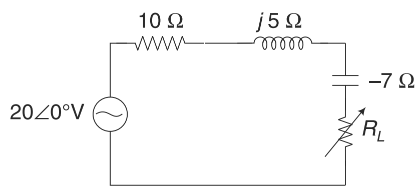



**Fig. 3.71**

(a) 0.5 W (b) 10 W

(c) 40 W (d) 20 W

43. Find the Thevenin equivalent impedance between Terminals 'a' and 'b' of the network as shown in Fig. 3.72.

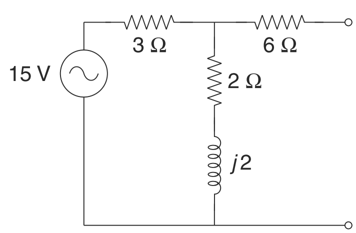



**Fig. 3.72**

(a) $7.38 + j0.62\Omega$ (b) $6.25 + j1.06\Omega$

(c) $8.1 + j0.59\Omega$ (d) $4.57 + j1.21\Omega$

44. Find the current in the resistor $R_L$ of the network shown in Fig. 3.73 using the principle of superposition

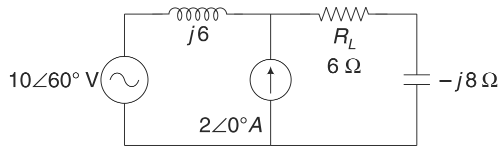



**Fig. 3.73**

(a) 2.13 ∠ 66.93° A (b) 1.27 ∠ 89.82° A
(c) 1.58 ∠ 78.43° A (d) 2.69 ∠ 82.36° A

45. In a practical $L - C$ parallel circuit, the dynamic impedance at resonance is
    (a) zero (b) infinity
    (c) $\frac{L}{C}$ (d) $\frac{L}{CR}$
46. Thevenin's Theorem is applicable to
    (a) active network (b) non-linear network
    (c) network with particular frequency
    (d) both (a) and (c)
47. Find the Norton's equivalent current for the network as shown in Fig. 3.74.

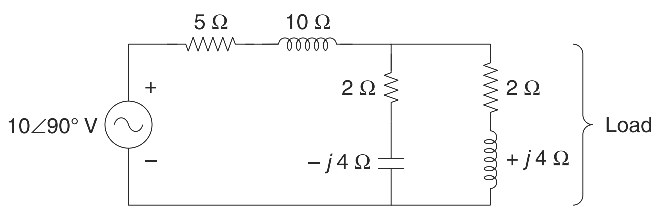



**Fig. 3.74**

(a) 0.815 ∠143.43° (b) 0.894 ∠26.56°
(c) 0.927 ∠43.14° (d) 0.853 ∠56.26°

48. In Norton's Theorem, a equivalent network is represented by
    (a) a voltage source with an impedance in series
    (b) a current source with an impedance in series
    (c) a voltage source with an impedance in parallel
    (d) a current source with an impedance in parallel
49. If the load is such that both its resistance and reactance are variable, maximum power would be delivered when
    (a) load impedance is less than the network impedance
    (b) load impedance is greater than the network impedance
    (c) load impedance is the complex conjugate of the network impedance
    (d) none of these

50. Find the value of $R_L$ for which power transfer is maximum of the network as shown in Fig. 3.75.

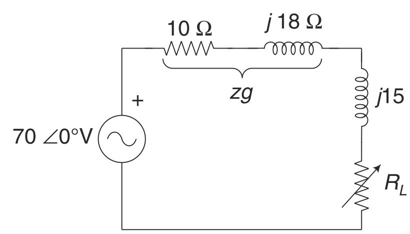



**Fig. 3.75**

(a) 25.47 (b) 14.106
(c) 37.62 Ω (d) 40.09 Ω

51. The form factor of a rectangular wave is equal to
(a) 1.0 (b) 1.11
(c) 1.15 (d) 1.414

52. The equivalent admittance $y$ of three branches connected in parallel having the admittance $y_1$, $y_2$ and $y_3$ respectively is

(a) $\frac{1}{y} = \frac{1}{y_1} + \frac{1}{y_2} + \frac{1}{y_3}$ (b) $\frac{1}{y} = y_1 + y_2 + y_3$

(c) $y = \frac{1}{y_1} + \frac{1}{y_2} + \frac{1}{y_3}$ (d) $y = y_1 + y_2 + y_3$

53. Bandwidth is defined as the range of frequencies for which the power delivered to the resistance $R$ is
(a) equal to the power delivered at resonance
(b) equal to or greater than half the power delivered at resonance
(c) equal to or less than half the power delivered at resonance
(d) less than half the power delivered at resonance

54. In a circuit having a resistance, reactance and power factor angle $\phi$, the power absorbed by the circuit is maximum when $\phi$ is equal to
(a) 90° (b) 45°
(c) 0° (d) none of these

55. In a circuit, $R$, $L$ and $C$ are connected in parallel across a sinusoidal voltage source of $V$ volts. The circuit current will lead the applied voltage if
(a) $I_C < I_L$ (b) $I_C = I_L$
(c) $I_C > I_L$ (d) none of these

56. The product of $V$, $I$ and sin $\theta$ is called
(a) active power (b) true power
(c) apparent power (d) reactive power

57. Which of the following will lead to high power factor?
    (a) High kVA, high kW (b) High kVA, low kW
    (c) Low kVA, low kW (d) Low kVA, high kW
58. $I$ versus $\omega$ curve in a pure inductor is
    (a) a straight line (b) parabolic
    (c) hyperbolic (d) exponential
59. If the capacitive reactance at 50 Hz is 32 Ω then at 100 Hz, it is
    (a) 78.5 Ω (b) 314 Ω (c) $\frac{1}{314}$ Ω (d) 16 Ω

## ANSWERS

| | | | | | |
|---|---|---|---|---|---|
| 1. (d) | 2. (a) | 3. (d) | 4. (a) | 5. (c) | 6. (b) |
| 7. (d) | 8. (c) | 9. (b) | 10. (d) | 11. (a) | 12. (b) |
| 13. (a) | 14. (a) | 15. (b) | 16. (c) | 17. (c) | 18. (c) |
| 19. (b) | 20. (a) | 21. (a) | 22. (c) | 23. (b) | 24. (c) |
| 25. (b) | 26. (b) | 27. (d) | 28. (d) | 29. (b) | 30. (c) |
| 31. (b) | 32. (c) | 33. (a) | 34. (b) | 35. (a) | 36. (d) |
| 37. (b) | 38. (c) | 39. (b) | 40. (a) | 41. (a) | 42. (b) |
| 43. (a) | 44. (a) | 45. (d) | 46. (d) | 47. (b) | 48. (d) |
| 49. (c) | 50. (b) | 51. (b) | 52. (d) | 53. (b) | 54. (c) |
| 55. (c) | 56. (d) | 57. (d) | 58. (c) | 59. (d) | |

## UNIVERSITY QUESTIONS WITH ANSWERS

1. Answer *True* or *False* with justification: [2004]
   (a) A series *R-L-C* circuit is fed from a sinusoidal voltage source. Voltage across any element can never exceed the applied rms voltage.

   *Answer*: False

   At resonance in series *R-L-C* circuit the net reactance of the circuit is 0 i.e. $X_L - X_C = 0$ or, $X_L = X_C$.

   The current $I_0$ is maximum at resonance. Hence $I_0 X_L = T_0 X_C$ or, $V_L = V_C$. Therefore voltage across inductance or the voltage across capacitance is more than the applied r.m.s voltage which is equal to $(I_0R)$ at resonance.

   (b) The impedance of an electric circuit is given by $\overline{Z} = (4 + j3)$ Ω. The value of the conductance (in mho) of this circuit is 0.2.

   *Answer*: False

   $\because \quad \overline{Z} = (4 + j3)\ \Omega$

   $\therefore \quad \text{Admittance } Y = \frac{1}{\overline{Z}} = \frac{1}{4 + j3} = \frac{4 - j3}{4^2 + 3^2} = \frac{4}{25} - j\,\frac{3}{25} = (0.16 - j\,0.12)$

   Hence the value of the conductance is 0.16 mho.

2. (a) Define phasor as used in the study of ac circuits. Use phasors to find the sum of the sinusoids 40 sin 314 $t$ and 30 cos $\left(314t - \frac{\pi}{4}\right)$ [2004]

*Answer:* A phasor is a vector rotating at a constant angular velocity.

[Refer Article 3.3]

Let $a = 40 \sin 314\, t$

$$b = 30 \cos\left(314t - \frac{\pi}{4}\right)$$

$$= 30 \sin\left(314t - \frac{\pi}{4}\right)$$

The horizontal component of the sum of sinusoids

$$V_x = 40 + 30 \cos\left(\frac{\pi}{4}\right) = 61.21.$$

The vertical component $V_y = 0 + 30 \sin\left(\frac{\pi}{4}\right) = 21.21.$

$\therefore$ Resultant $= \sqrt{(61.21)^2 + (21.21)^2} = 64.78.$

$\therefore$ $\theta = \tan^{-1}\dfrac{21.21}{61.21} = 19.11°.$

$\therefore$ Sum of the sinusoids is 64.78 sin (314$t$ + 19.11°).

(b) Figure 1 below shows a circuit in which a coil having resistance $R$ and inductance $L$ is connected in series with a resistance of 80 Ω. The combination is fed from a sinusoidal source. [2004]

The following measurements (rms value) are taken at 50 Hz when the circuit is in steady state.

$$|\overline{V}_S| = 145 \text{ V}, |\overline{V}_R| = 50 \text{ V and } |\overline{V}_C| = 110 \text{ V}$$

Find value of $R$ and $L$.

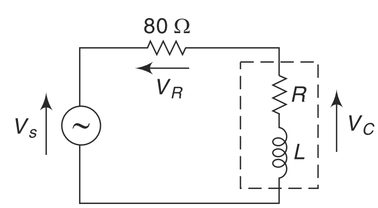



**Fig. 1**

*Answer*: $V_R = 50$ V

$\therefore$ Current $(I) = \dfrac{50}{80} = 0.625$ A.

$$V_C = I\sqrt{R^2 + X_L^2} = 110$$

$\therefore$ $$R^2 + X_L^2 = \left(\frac{110}{0.625}\right)^2 = 30976 \qquad (1)$$

$$V_S = I\sqrt{(R+80)^2 + X_L^2}$$

$\therefore \quad (R + 80)^2 + X_L^2 = \left(\frac{145}{0.625}\right)^2 = 53824$

or, $\quad R^2 + X_L^2 + 160\,R + 6400 = 53824 \qquad (2)$

Substituting the value of $(R^2 + X_L^2)$ from equation (1) in equation (2) we have

$$30976 + 160\,R + 6400 = 53824$$

or, $\quad R = 102.8\ \Omega$

From equation (1),

$$X_L = \sqrt{30976 - (102.8)^2} = 142.857\ \Omega$$

3. What do you mean by (i) r.m.s value (ii) form factor (iii) peak factor? [2005]

*Answer:* Refer Article 3.2.

4. (a) Explain with the help of phasor diagram the phenomenon of resonance in a circuit containing an inductance, a capacitor and a series resistor. [2005]

*Answer*: Refer Article 3.14.

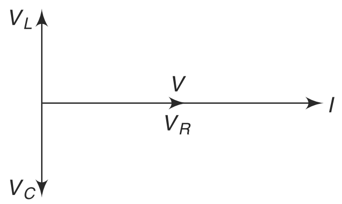



**Fig. 2** ***Phasor diagram at resonance***

(b) A coil of resistance 10 Ω and inductance 0.02 H is connected in series with another coil of resistance 6 Ω and inductance 15 mH across a 230 V, 50 Hz supply. Calculate: [WBUT 2013]

(i) impedance of the circuit

(ii) the voltage drop across each coil

(iii) the total power consumed by the circuit.

*Answer:*

*First coil*

Resistance $(R_1) = 10\ \Omega$

Inductance $(L_1) = 0.02$ H

$\therefore$ Inductive reactance $(X_{L1}) = wL_1 = 2\pi \times 50 \times 0.02 = 6.283\ \Omega$

*Second coil*

Resistance $(R_2) = 6\ \Omega$

Inductance $(L_2) = 15$ mH $= 0.015$ H

$\therefore$ Inductive reactance $(X_{L2}) = wL_2 = 2\pi \times 50 \times 0.015 = 4.712\ \Omega$

(i) Impedance of the circuit

$(z) = (10 + 6) + j\,(6.283 + 4.712) = 16 + j10.995$

$= 19.41\angle 34.5°\ \Omega$

(ii) Current in each coil $(I) = \frac{V}{Z} = \frac{230}{19.4} = 11.85$ A

$\therefore$ Voltage drop in first coil $(V_1) = 11.85 \times \sqrt{(10)^2 + (6.283)^2}$

$= 139.95$ V

Voltage drop in second coil $(V_2) = 11.85 \times \sqrt{(6)^2 + (4.712)^2}$

$= 90.4$ V

(iii) Total power consumed by the circuit

$(P) = I^2(R_1 + R^2) = (11.85)^2 \times (10 + 6) = 2246.76$ W

5. Write short note on active and reactive power. [2005]

   *Answer*: Refer Article 3.11.

6. In a series *R-L-C* circuit, the condition for resonance is [2006]

   (i) the current $I_C$ and $I_L$ are in phase opposition

   (ii) resultant current is zero

   (iii) the voltage drop across $X_L$ and $X_C$ are equal

   (iv) power factor is zero.

   *Answer*: (iii) the voltage drop across $X_L$ and $X_C$ are equal.

7. Explain why power loss in a pure inductance/pure capacitance is equal to zero in an a.c. circuit. [2006, 2014]

   *Answer*: Refer Article 3.11.2 and 3.11.3.

8. The equation of an alternating current is $i = 62.35 \sin 323t$ A. Determine its [2006]

   (i) maximum value (ii) frequency

   (iii) rms value (iv) average value (v) form factor.

   *Answer*: $i = 62.35 \sin 323t$

   (i) maximum value 62.35 A

   (ii) Frequency $f = \frac{323}{2\pi} = 51.4$ Hz

   (iii) Rms value $= \frac{62.35}{\sqrt{2}} = 44.09$ A

   (iv) Average value $= \frac{2 \times 62.35}{\pi} = 39.69$ A

   (v) Form factor $= \frac{\text{rms value}}{\text{Average value}} = 1.11$

9. An ac series circuit consisting of a pure resistance of 25 Ω, inductance of 0.15 H and capacitance of 80 μF is supplied from a 230 V, 50 Hz ac.

   [2006]

(a) Find (i) the impedance of the circuit, (ii) the current (iii) the power drawn by the circuit and (iv) the power factor.

(b) Draw the phasor diagram.

*Answer*: $R = 25\ \Omega$

$L = 0.15$ H

$C = 80\ \mu$F

$V = 230$ V and $f = 50$ Hz

$\therefore$ Inductive reactance $(X_L) = 2\pi fL$

$= 2\pi \times 50 \times 0.15$

$= 47.124\ \Omega$

Capacitive reactance $(X_C) = \dfrac{1}{2\pi fC}$

$= \dfrac{1}{2\pi \times 50 \times 80 \times 10^{-6}}$

$= 39.8\ \Omega$

(a) (i) The impedance of the circuit $(Z) = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{(25)^2 + (47.124 - 39.8)^2}$

$= 26.05\ \Omega$

(ii) The current $(I) = \dfrac{V}{Z} = \dfrac{230}{26.05} = 8.829$ A

(iii) The power drawn by the circuit (P) $= I^2R = (8.829)^2 \times 25$

$= 1948.78$ W

(iv) Power factor $(\cos\theta) = \dfrac{R}{Z} = \dfrac{25}{26.05}$

$= 0.9597$ (lag)

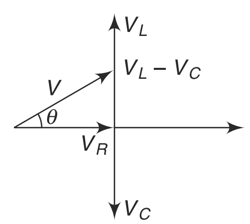


**Fig. 3**

(b) Phasor diagram (Fig. 3)

$V_R = IR = 8.829 \times 25 = 220.725$ V

$V_L = IX_L = 8.829 \times 47.124 = 416.06$ V

$V_C = IX_C = 8.829 \times 39.8 = 351.39$ V

10. In a series *R-L-C* resonant circuit, during resonance, which one of the following is maximum? [2007]

(a) Impedance (b) Voltage (c) Current (d) None of these

*Answer*: (c) Current.

11. If the peak value of a sine wave is 100 volts, then its rms value will be [2007]

(a) 70.7 V (b) 63.6 V (c) 10.0 V (d) 88 V.

*Answer*: (a) 70.7 V.

12. Derive an expression of [2007]
    (a) average
    (b) r.m.s value of a half wave rectified voltage wave.

    *Answer*: A half wave rectified voltage wave is shown in Fig. 4.

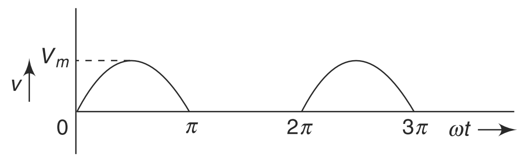



**Fig. 4**

(a) $$V_{av} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin \omega t \, d(\omega t)$$
$$= \frac{V_m}{2\pi}[-\cos \omega t]_0^{\pi}$$
$$= \frac{V_m}{2\pi}[1 + 1] = \frac{V_m}{\pi} = 0.318\ V_m$$

(b) $$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2 \omega t \, d(\omega t)}$$
$$= \frac{V_m}{2\sqrt{\pi}}\sqrt{\int_0^{\pi} 2\sin^2 \omega t \, d(\omega t)}$$
$$= \frac{V_m}{2\sqrt{\pi}}\sqrt{\int_0^{\pi} (1 - \cos 2\ \omega t)\, d(\omega t)}$$
$$= \frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\omega t - \frac{\sin 2\ \omega t}{2}\right]_0^{\pi}}$$
$$= \frac{V_m}{2\sqrt{\pi}}\sqrt{\pi} = \frac{V_m}{2} = 0.5\ V_m$$

13. A resistance of 100 ohms is connected with an inductance of 1.2 H and capacitance of 10 microfarad in series. The combination is connected across 100 V, 50 Hz supply. Find
    (a) current in the resistance
    (b) voltage across the capacitance
    (c) power consumed.

    Draw the phasor diagram. [2007]

    *Answer*: Resistance $R = 100\ \Omega$

*Answer:* Inductive reactance $X_L = wL = 2\,\pi \times 50 \times 1.2 = 377\ \Omega$

Capacitive reactance $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = 318\ \Omega$

Net reactance $X = X_L - X_C = 59\ \Omega$

$\therefore$ Impedance $Z = \sqrt{R^2 + X^2} = \sqrt{(100)^2 + (59)^2}$

$= 116.11\ \Omega$

(a) Current in the resistance $I = \dfrac{V}{Z} = \dfrac{100}{116.11}$

$= 0.86$ A

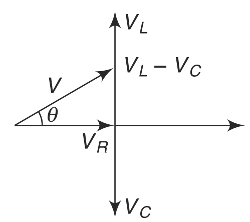



**Fig. 5** ***Phasor diagram***

(b) Voltage across the capacitance

$V_C = IX_C$

$= 0.86 \times 318$

$= 273.48$ V

(c) Power consumed $P = I^2R = (0.86)^2 \times 100$

$= 73.96$ watts

14. For series *R-L-C* circuit, the impedance at resonance is [June 2008]

(a) maximum (b) minimum (c) infinity (d) zero

*Answer*: (b) minimum.

15. A resistance of 20 Ω, an inductance of 0.2 H and a capacitance of 100 μF are connected is series across 220 V, 50 Hz. Determine the following: [June 2008]

(a) Impedance
(b) Current
(c) Voltage across *R*, *L*, *C*
(d) Power factor and angle of lag
(e) Power in Watts and VA

*Answer:*

$R = 20\ \Omega$

$L = 0.2$ H

$C = 100\ \mu$F

$\therefore$ Inductive reactance $X_L = wL = 2\pi \times 50 \times 0.2$

$= 62.83\ \Omega$

Capacitance reactance $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2p \times 50 \times 100 \times 10^{-6}} = 31.847\ \Omega$

(a) Impedance $Z = \sqrt{R^2 + (X_L \sim X_C)^2}$

$= \sqrt{(20)^2 + (62.83 - 31.847)^2}$

$= 36.89\ \Omega$

(b) Current $I = \dfrac{V}{Z} = \dfrac{220}{36.89}$ A $= 5.96$ A

(c) Voltage across $R$ is $V_R = IR = 5.96 \times 20 = 119.2$ V

Voltage across $X_L$ is $V_L = I\,X_L = 5.96 \times 62.83 = 374.46$ V

Voltage cross $X_C$ is $V_C = I\,X_C = 5.96 \times 31.847 = 190$ V

(d) Power factor $\cos\theta = \dfrac{R}{Z} = \dfrac{20}{36.89} = 0.542$ lag (as $X_L > X_C$)

Angle of lag $\theta = \cos^{-1} 0.542 = 57.16°$

(e) Power in watts is $VI\cos\theta = 220 \times 5.96 \times 0.542 = 711$

Power in $VA$ is $VI = 220 \times 5.96 = 1311$

16. (a) A full wave retified sinusoidal is clipped at $\dfrac{1}{\sqrt{2}}$ of its maximum value. Calculate the average and rms value of such a voltage waveform. Also calculate the form factor and peak factor.

*Answer:* The full wave rectified sinusoidal voltage waveform clipped at $\dfrac{1}{\sqrt{2}}$ of its maximum value is shown in Fig. 6. [June 2008]

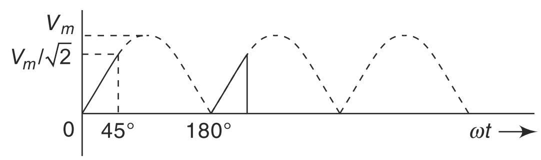



**Fig. 6**

$$\text{Average value } V_{av} = \frac{1}{\pi}\int_0^{\pi/4} V_m \sin\omega t\, d(\omega t)$$

$$= \frac{V_m}{\pi}[-\cos\omega t]_0^{\pi/4}$$

$$= \frac{V_m}{\pi}[-0.707 + 1]$$

$$= 0.0932\, V_m$$

$$\text{Rms value } V_{rms} = V_m\sqrt{\frac{1}{\pi}\int_0^{\pi/4} V_m^2 \sin^2 \omega t\, d(t)}$$

$$= V_m\sqrt{\frac{1}{2\pi}\int_0^{\pi/4}(1 - \cos 2\,\omega t)\, d(\omega t)}$$

$$= \frac{V_m}{\sqrt{2\pi}}\sqrt{\left[\omega t - \frac{\sin 2\,\omega t}{2}\right]_0^{\pi/4}}$$

$$= \frac{V_m}{\sqrt{2\pi}}\sqrt{\frac{\pi}{4} - \frac{1}{2}}$$

$$= \frac{V_m}{\sqrt{2\pi}} \times 0.534 = 0.213\ V_m.$$

$$\text{Form factor} = \frac{V_{\text{rms}}}{V_{\text{avg}}} = \frac{0.213V_m}{0.932V_m} = 2.285$$

$$\text{Peak factor} = \frac{V_{\text{rms}}}{V_{\text{rms}}} = \frac{V_m}{0.213V_m} = 4.695$$

17. The power factor of a purely inductive circuit is [December 2008, 2014]
    (a) zero (b) one
    (b) infinity (d) 0.5
    *Answer*: (b) zero
18. For a series R-L-C circuit, the impedance at resonance is [December 2008]
    (a) maximum (b) minimum
    (b) zero (d) infinty
    *Answer*: (b) minimum.
19. An alternating voltage is represented by

$$v = 62.35 \sin 323t$$

Determine (a) the maximum value, (b) rms value, (c) average value, (d) frequency of the wave, and (e) form factor. [December 2008]

*Answer:* (a) Maximum value = 62.35 V

(b) rms value $= \frac{62.35}{\sqrt{2}}$ V = 44.1 V

(c) Average value $= \frac{2 \times 62.35}{\pi} = 39.71$ V

(d) Frequency of the wave $f = \frac{323}{2\pi} = 51.4$ Hz

(e) Form factor $K_f = \frac{44.1}{39.71} = 1.11$

20. (a) A resistance of 20 Ω an inductance of 0.2 H and a capacitance of 100 μF are connected in series across a 220-V, 50-Hz supply. Determine (i) impedance, (ii) current, (iii) power factor, and (iv) power consumed [December 2008]

*Answer:* $R = 20$ W $L = 0.2$ H $C = 100\ \mu$F

$\therefore$ $X_L = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$

$$X_c = \frac{10^6}{2\pi \times 50 \times 100} = 31.847\ \Omega$$

(i) Impedance $Z = \sqrt{(20)^2 + (62.83 - 31.847)^2} = 36.877\ \Omega$

(ii) Current $I = \frac{220}{36.877} = 5.96$ A

(iii) Power factor $\cos\theta = \frac{R}{Z} = \frac{20}{36.877}$ 0.54 lag

(iv) Power consumed $VI \cos\theta = 220 \times 5.96 \times 0.54 = 711$ W

(b) Draw the phasor diagram.

*Answer:* The phasor diagram is shown in Fig. 7.

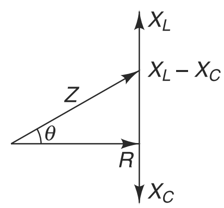



**Fig. 7**

21. A sinusoidal voltage is represented by [June 2009]
$v = 141.4 \sin(314t - \pi/2)$. The frequency is

(a) 60 Hz (b) 50 Hz
(c) 100 Hz (d) none of these

*Answer*: (b) 50 Hz

22. A resistance of 8.0 Ω and an inductive rectance of 6.0 Ω will offer an impedance of [June 2009]

(a) 14 Ω (b) 10 Ω
(c) 11 Ω (d) none of these

*Answer*: (b) 10 Ω

23. In a series R-L-C circuit, the power factor at resonance is

(a) unity (b) zero
(c) 0.5 (d) none of these

*Answer*: (a) unity

24. The conductance $G$ of a series circuit having a resistance $R$ and inductive reactance, $X_L$ is given by

(a) $G = \frac{1}{R}$ (b) $G = \frac{R}{X_L}$

(c) $G = \frac{X_L}{R^2 + X_L^2}$ (d) $G = \frac{R}{R^2 + X_L^2}$

*Answer:* (d) $G = \frac{R}{R^2 + X_L^2}$

25. A circuit takes a current of 3 A at a power factor of 0.6 lagging when connected to a 115 V, 50 Hz supply. Another one circuit takes a current of 5 A at a power factor of 0.707 leading when connected to the same supply after the first circuit is removed. If the two circuits are connected in series across a 230 V, 50 Hz supply, calculte.

(a) the current drawn from the source, (b) the power consumed, and (c) the power factor of the circuit.

*Answer:*

$I_1 = 3$ A  $\cos\theta_1 = 0.6$ lag  $V = 115$V

$I_1 = 5$ A  $\cos\theta_2 = 0.707$ lead  $V = 115$V

Impedance of the first circuit $Z_1 = \dfrac{115}{3\,\angle -\cos^{-1} 0.6}$

$= 38.33\,\angle 53.13°\ \Omega$

$= 23 + j\,30.664$

Impedance of the second circuit

$$Z_2 = \frac{115}{5\,\angle 0\cos^{-1} 0.707} = 23\,\angle -45°\ \Omega = 16.26 - j16.2$$

Now supply voltage is 230 V.

(a) Current drawn from the source

$$I = \frac{230}{(23 + 16.26) + j\,(30.664 - 16.26)} = \frac{230}{39.26 + j14.4}$$

$= 5.5\,\angle -20.14°$

(b) Power consumed $P = (5.5)^2 \times 39.26 = 1187.615$ W

(c) Power factor $\cos\theta = \cos = 0.14° = 0.9388$ lag.

26. The form factor of a wave is 1. Its shape is [WBUT 2011, 2014]

(a) sinusoidal (b) triangular

(c) square (d) sawtooth

*Answer:* (c)

27. The admittance of a parallel circuit is $0.5 \angle -30°$. The circuit is [WBUT 2011]

(a) inductive (b) capacitive

(c) resistive (d) in resonance

*Answer:* (a)

28. What is resonance? Deduce the expression of frequency in a series *RLC* circuit at resonance. [WBUT 2011]

*Answer:* Refer Article 3.14

29. At $t = 0$, the instantaneous value of a 50 Hz sinusoidal current is 5 rmp and increases in magnitude further. Its rms value is 10 Amp. [WBUT 2011]

(a) Write the expression for its instantaneous value

(b) Find the current at $t = 0.01$ and $t = 0.015$ second.

(c) Sketch the waveform indicating these values.

*Answer:* $i = 10\sqrt{2}\sin(2\pi \times 50t + \alpha)$ or $i = 14.14\sin(100\,\pi t + \alpha)$

At $t = 0$, $i = 5$ A

$\therefore\ 5 = 10\sqrt{2}\sin\alpha$ or $\alpha = 20.7°$

(a) $i = 14.14 \sin (100\,\pi t + 20.7°)$

(b) At $t = 0.01$ s

$i = 14.14 \sin (\pi + 20.7°) = -4.998$ A

At $t = 0.015$ s

$i = 14.14 \sin (1.5\pi + 20.7°) = -13.227$ A

(c)

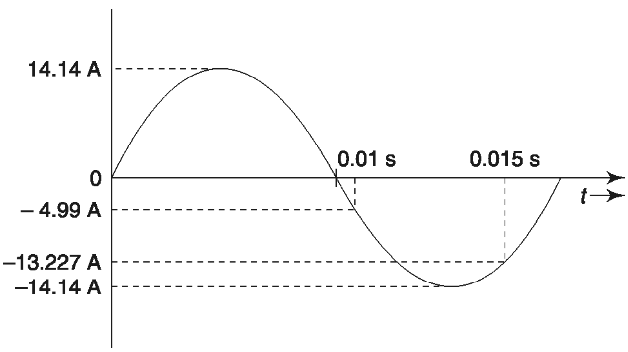



**Fig. 8**

30. (a) Explain what are meant by phase and phase difference of sinusoidal waves. [WBUT 2011]

*Answer:* Refer Article 3.2 and 3.3.

(b) A coil of resistance 30 Ω and inductance 320 mH is connected in parallel to a circuit consisting of 75 Ω in series with 150 μF capacitor. The circuit is connected to a 200 volt, 50 Hz supply. Determine supply current and circuit power factor.

*Answer:* $R_1 = 30\ \Omega \quad L = 0.32$ H

$$Z_1 = 30 + j\,100\,\pi \times 0.32 = 30 + j\,100.53$$

$$R_2 = 75\ \Omega, \quad C = 150\ \mu\text{F}$$

$$Z_2 = 75 - j\frac{10^6}{100\,\pi \times 150} = 75 - j\,21.23$$

$$\therefore \quad I_1 = \frac{200}{30 + j\,100.53} = 1.906\,\underline{|-73.38°}\text{A}$$

$$\text{and } I_2 = \frac{200}{75 - j\,21.23} = 2.566\,\underline{|15.8°}\text{A}$$

$\therefore$ Supply current $I = I_1 + I_2$

$$= 1.906\,\underline{|-73.38°} + 2.566\,\underline{|15.8°}$$

$$= 3.014 - j\,1.1277$$

$$= 3.218 \text{ A}$$

$$\text{Power factor} = \cos\theta = \cos\left(\tan^{-1}\frac{1.1277}{3.014}\right) \text{log}$$

$$= 0.936 \text{ log}$$

31. (a) Prove that current in purely resistive circuit is in phase with applied ac voltage and current in purely capacitive circuit leads applied voltage by 90° and draw their waveforms. [WBUT 2011]

*Answer:* Refer Article 3.4.1 and 3.4.3

(b) A circuit consists of series combination of elements as reistance of 6 Ω, inductance of 0.4 H and a variable capacitor across 100 V, 50 Hz supply. Calculate (i) value of capacitance at resonance, (ii) voltage drop, across capacitor, and (iii) $Q$ factor of coil. [WBUT 2013]

*Answer:* $R = 6\ \Omega$, $L = 0.4$ A, V = 100 V

(i) At resonance $\omega_o = \frac{1}{\sqrt{LC}} \quad \Rightarrow C = \frac{1}{L\omega_o^2}$

or, $$C = \frac{1}{0.4 \times (2\,\pi \times 50)^2} F = 25.35\ \mu\text{F}$$

(ii) Voltage drop across capacitor $= I\,X_C = \frac{VX_C}{R} = \frac{VX_L}{R}$

$$= \frac{100 \times 2\,\pi \times 50 \times 0.4}{6}\ \text{V}$$

$$= 2093.33\ \text{V}$$

(iii) $Q$ factor of coil $= \frac{\omega_o L}{R} = \frac{2\,\pi \times 50 \times 0.4}{6} = 20.9$

32. In an electrical circuit, if the current lags the voltage by 60°, the circuit nature is

(a) *R–C* (b) *R–L*

(c) *LC* (d) none of these

*Answer:* (b) *R–L*

33. If $E_1 = \text{A} \sin \omega t$ and $E_2 = \text{A} \sin (\omega t - \theta)$ then

(a) $E_1$ lags $E_2$ (b) $E_2$ lags $E_1$

(c) $E_1$ and $E_2$ are in phase (d) none of these

*Answer:* (b) $E_2$ lags $E_1$

34. The bandwidth of a series resonant ac circuit is equal to

(a) $\frac{\text{R}}{(2\pi\text{L})}$ (b) $\frac{1}{(\text{RLC})}$

(c) $\frac{1}{(2\pi\text{R})}$ (d) $\frac{1}{(\text{wc})}$

*Answer:* (a) $\frac{\text{R}}{(2\pi\text{L})}$

35. Derive an expression for the resonant frequency of a parallel circuit, one branch consisting of a coil of inductance L and a resistance R and the other branch of capacitance C.

*Answer:* Refer Article 3.17.

36. Define power factor of an ac circuit. State the disadvantages associated with having a low power factor. [WBUT 2014]

    *Answer:* Refer Article 3.11.7.

37. Prove that the current in a purely resistive circuit is in phase with applied ac voltage and current in a purely capacitive circuit leads applied voltage by 90° and also draw their waveforms.

    *Answer:* Refer Article 3.4.

38. A sinusoidal voltage is represented by $v = 14.4 \sin (314.18t - 90°)$.

    The r.m.s value of the voltage, its frequency and phase angle are respectively

    (a) 141.42 V, 314.16 Hz 90° (b) 100 V, 50 Hz, −90°

    (c) 87.92, 60 Hz. 90° (d) 200 V, 56 Hz, −90°

    *Answer:* (b) [WBUT 2013]

39. What is resonance? Deduce the expression of frequency in a parallel RLC circuit at resonance. [WBUT 2013]

    *Answer:* Refer Article 3.17.

40. A two element series circuit consumes 700 W of power and has power factor of 0.707 leading when energized by a voltage source of waveform $v = 141 \sin (314t + 30°)$, Find out the circuit elements [WBUT 2013]

    *Answer:* Refer Example 3.30

41. Derive the expression of quality factor of a series RLC circuit at resonance [WBUT 2013]

    *Answer:* Refer Article 3.15

42. Two impedances $Z_1 = (47.92 + j76.73)\ \Omega$ and $Z_2 = (10 - j5)\ \Omega$ are connected in parallel across a 200 V, 50 Hz. supply

    Find the current through each impedance and total current. What is the phase difference angle of each branch current with respect to the applied voltage? [WBUT 2014]

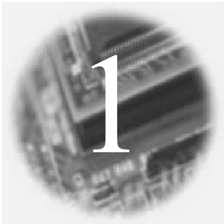
    *Answer:* Refer Example A 3.6

43. A coil of resistance 10 Ω and inductance 0.02 H is connected in series with another coil of resistance 6 Ω and inductance 15 mH across a 230 V, 50 Hz supply calculate (i) impedance of the circuit (b) the voltage drop across each coil and (c) the total power consumed by the circuit. [WBUT 2014]

    *Answer:* Refer Q no. 23 in the exercise

# PART-II

# BASIC ELECTRONICS ENGINEERING-I



# 1 BASIC CONCEPTS IN SEMICONDUCTOR PHYSICS

## 1.1 INTRODUCTION

The electrical conductivity of semiconductors is in between that of conductors and insulators. Semiconductors are extensively used in electronic circuits. In this chapter the fundamental aspects of semiconductor physics and its basic application have been discussed.

## 1.2 BOHR'S ATOMIC STRUCTURE

Neils Bohr, a Danish physicist, had developed a clear picture of the atomic structure. He stated that

(i) An atom consists of a nucleus which contains neutrons (neutral particles) and protons (positively charged particles).

(ii) The electrons (negatively charged particles) revolve around the nucleus in different orbits.

(iii) The nucleus has a positive charge which attracts the electrons. The electrons would collapse into the nucleus if they do not have the required centrifugal force of their motion. Therefore, an electron travels in a stable orbit and moves at a right velocity for centrifugal force to balance the attraction of the nucleus.

(iv) The electrons in each permitted orbit have a certain fixed amount of energy. The larger the radius of orbit; the greater is the energy of electron.

(v) An electron is projected to the higher orbit if some additional energy in form of heat, light or other radiations are *given* to it. Then the atom is said to be in a state of *excitation*. This state does not last long since the electron soon returns back to the original (lower) orbit. As it falls back, it radiates the acquired energy in the form of heat, light, etc.

A simple two-dimensional structure of a silicon atom is shown in Fig.1.1(a). It has 14 electrons, equal to number of protons in the nucleus. The 14 electrons are revolving around the nucleus only in their permitted orbits i.e. first, second and third having radii $r_1$, $r_2$ and $r_3$ respectively. Two electrons revolve in the first orbit, 8 in the second and 4 in the third orbit. The silicon atom contains 4 electrons in the outermost orbit and these electrons are called valence electrons. Therefore, silicon atom is a *tetravalent atom*.

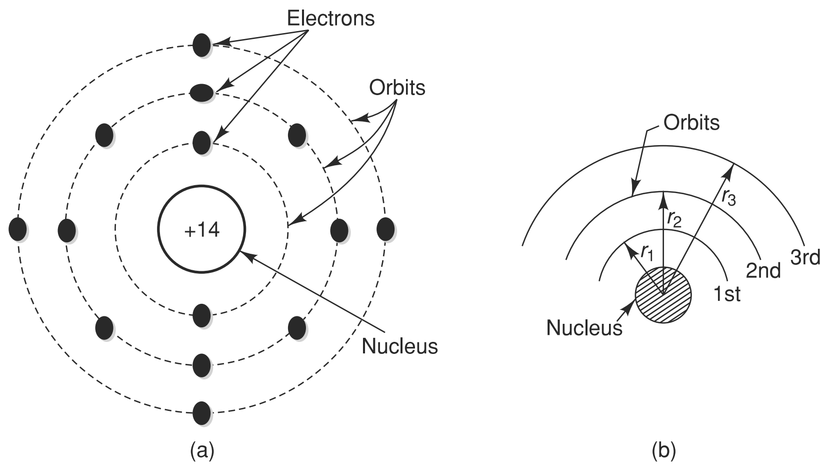



**Fig. 1.1** *Bohr's atomic structure*

## 1.3 ENERGY LEVELS IN AN ATOM

An atom consists of a positively charged nucleus which contains almost all its mass. Surrounding this nucleus are negatively charged electrons moving about in closed orbits. Each orbit is characterized by a discrete energy level and do not emit radiation and are said to be in *stationary* or *non-radiating* state. A stationary state is determined by the condition that the angular momentum of the electron in this state is quantized and must be an integral multiple of $h/2\pi$, i.e.,

$$mvr = nh/2\pi \tag{1.1}$$

where $m$ = mass of electrons, kg
$v$ = speed of electron in its circular path, m/s
$r$ = radius of orbit, m
$h$ = Planck's constant (= $6.63 \times 10^{-34}$ Js)
$n$ = integer

As we substitute $n = 1, 2, 3$ ... we get stable orbits with radii $r_1, r_2, r_3$ ... as shown in Fig. 1.1(b). For $n = 1$, we get an orbit closest to the nucleus and possessing minimum energy. This lowest energy level is the innermost or $K$ shell of the atom and can accommodate only 2 electrons. The next higher energy level, known as $L$ shell, can accommodate 8 electrons and so on. The maximum number of electrons that an orbit can have is $(2n^2)$. The outermost orbit is generally not completely filled and the electrons in this orbit are known as *valence electrons*.

The lower the energy level, the more tightly bound is the electron. In a transition from one stationary state (corresponding to energy $W_2$) to another stationary state corresponding energy radiation is emitted. The frequency of this radiant energy is

$$f = (W_2 - W_1)/h \tag{1.2}$$

The electrons can revolve only in the permitted orbits (i.e. orbits of radii $r_1$, $r_2$ and $r_3$) and not in any intermediate orbit [Fig. 1.2(a)]. Thus, all the radii between $r_1$ and $r_2$ or between $r_2$ and $r_3$ are forbidden. Each orbit has fixed amount of energy associated with it and an electron in the outer orbit possesses more energy than the electron in the inner orbit.

The level of energy obtained by different orbits is conveniently represented by the energy level diagram (horizontal lines) shown in Fig. 1.2(b). The ***first orbit*** represents the ***first energy level***, the ***second orbit*** represents the ***second energy level***, and so on. Thus, energy level is just another way of representing the orbital radius.

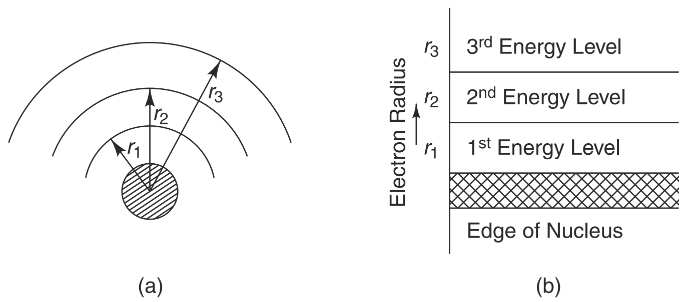



**Fig. 1.2** *Energy levels*

## 1.4 ENERGY BANDS

As discussed earlier, in case of a single isolated atom, the electrons revolving in any orbit possess a definite energy. However, in a solid, an atom is greatly influenced by the closely-packed neighbouring atoms. Because of this the electrons in the same orbit have a range of energies rather than a single energy. This is known as *energy band*.

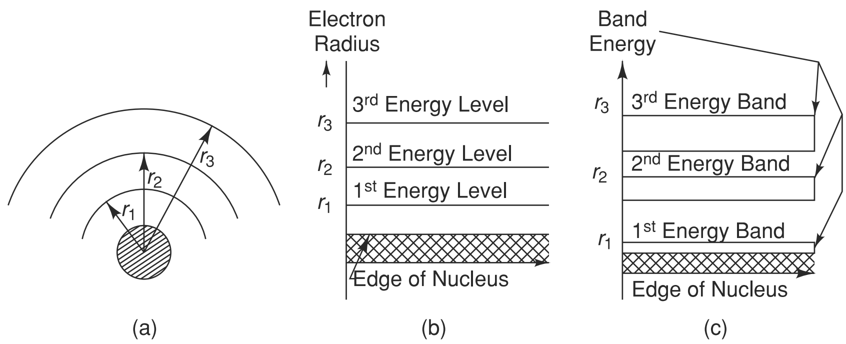



**Fig. 1.3** *Energy level and energy band*

*The range of energies possessed by electrons of the same orbit in a solid is known as energy band*. Figure 1.3 shows how energy levels are changed into energy bands. All the electrons moving in the first orbit have slightly different energy levels as no two electrons have the same charge environment. The electrons with slightly different energy levels form a cluster or band called energy band.

## 1.5 DIFFERENT ENERGY BANDS IN SOLIDS

The individual energy levels (*K*, *L*, *M*, etc.) of an isolated atom can be converted into corresponding energy bands. Although there are a number of energy bands in solids, we are more concerned with the following:

***(i) Valence Band*** The electrons in the outermost orbit of an atom are known as valence electrons. Under normal condition of the atom, valence band contains the electrons of highest energy. This band may be filled completely or partially.

The energy band which possesses the valence electrons is called *valence band* (Fig.1.4.)

***(ii) Conduction Band*** In some of the materials (e.g., metals), the valence electrons are loosely attached to the nucleus and can be detached very easily. These electrons are known as free electrons and are responsible for the conduction of current. For this reason, these electrons are also known as *conduction electrons*.

The energy band which possesses the conduction (or free) electrons is called *conduction band* (Fig.1.4.)

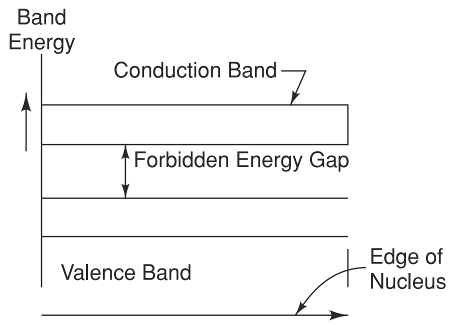



**Fig. 1.4** ***Energy band diagram in solids***

If a substance has *empty conduction band*, it means current conduction is not possible in that substance and is known as *insulator*.

***(iii) Forbidden Energy Gap*** The energy gap between the valence band and conduction band is known as *forbidden energy gap* (Fig. 1.4.).

Forbidden energy gap is a region in which no electron can stay. The width of the forbidden energy gap represents the bondage of valence electrons to the atom. The greater the forbidden energy gap, more tightly the valence electrons are bound to the nucleus. To make the valence electron free, external energy equal to the forbidden energy gap must be supplied which lifts the electron from valence band to conduction band.

## 1.6 FERMI LEVEL

The distribution of free electrons in a solid can be written as

$$dn_e = p_e \, dE \tag{1.3}$$

where $dn_e$ is the number of free electrons per m$^3$ whose energies lie between $E$ and $E + dE$ (i.e. in the energy interval $dE$) and $p_e$ is the density of electrons in this interval.

The function $p_e$ can be written as

$$P_e = f(E)\, N(E) \tag{1.4}$$

where $N(E)$ is the density of states (i.e., in the conduction band) and $f(E)$ is the probability that a quantum state with energy $E$ is occupied by the electron. This probability is represented by the following expression known as the Fermi Dirac function:

$$f(E) = \frac{1}{1 + Exp^{[(E-E_f)KT]}} \tag{1.5}$$

where, $k$ = Boltzmann constant = $0.8625 \times 10^{-4}$ eV/ $^0$K
$T$ = temperature, °K
$E_f$ = Fermi level for the crystal, eV

It is seen from Eq (1.5) that if $E = E_F, f(E) = 0.5$ for any value of $T$. Thus Fermi level is the energy state with 50 per cent probability of being filled if no forbidden band exists. At $T = 0$°K, two possible conditions exist: (a) if $E > E_F$, the exponential term in Eq (1.5) is infinite and $f(E) = 0$, i.e. there is zero probability of finding an occupied quantum state of energy higher than $E_F$, at absolute zero; (b) if $E < E_F$, the exponential term in Eqn (1.5) is zero and $f(E) = 1$, i.e. all quantum states with energies less than $E_F$ will be occupied at absolute zero temperature. For most of the metals, $E_F$ is less then 10 eV.

## 1.7 CLASSIFICATION OF SOLIDS AND ENERGY BANDS

On the basis of electrical conductivity, the solids may be classified as *insulators*, *conductors* and *semiconductors* whose electrical behaviour can be explained with the help of energy bands.

*(i)* ***Insulators*** The substances (like wood, glass, mica, etc.) which do not allow the passage of current through them are known as *insulators*. The valence band of these substances is full, whereas the conduction band is completely empty. Moreover, the forbidden energy gap between valence band and conduction band is very large (15 eV nearly) as shown in Fig. 1.5(a). Therefore, a large amount of energy, i.e., a very high electric field is

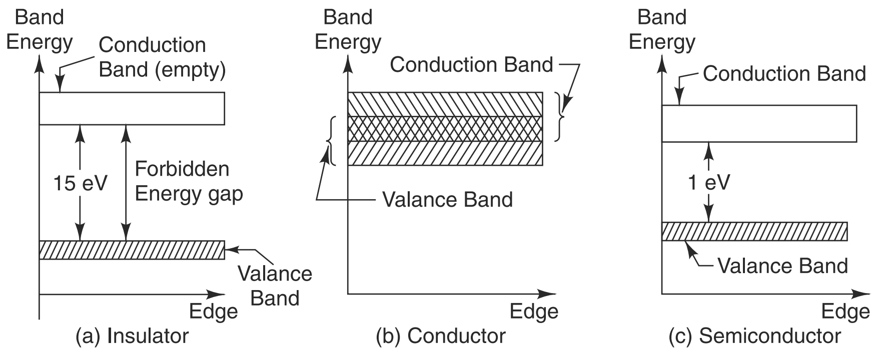



**Fig. 1.5** ***Energy Band in solids***

required to push the valence electrons to the conduction band. Hence such materials, under ordinary conditions, do not conduct at all and are designated as insulators.

*(ii)* ***Conductors*** The substances (like copper, aluminium, silver, etc.) which allow the passage of current through them are known as *conductors*. The valence band of these substances overlap the conduction band as shown in Fig. 1.5(b). Due to this overlapping, a large number of free electrons are available for conduction. This is the reason, why a slight potential difference applied across such substances causes a heavy flow of current through them.

*(iii)* ***Semiconductors*** The substances (like carbon, silicon, germanium, etc.) whose electrical conductivity lies in between the conductors and insulators are known as *semiconductors*. Although, the valence band of these substances is almost filled and conduction band is almost empty as in case of insulators. But the forbidden energy gap between valence band and conduction band is very small (nearly 1 eV) as shown in Fig. 1.5(c). Therefore, a comparatively smaller electric field (much smaller than insulators but much greater than conductors) is required to push the valence electrons to the conduction band. Such materials, under ordinary condition, do not conduct current and behave as an insulator, but even at room temperature if some heat energy is imparted to the valence electron, a few of them cross over to the conduction band imparting minor conductivity to the semiconductors. As the temperature is increased, more valence electrons cross over to the conduction band and the conductivity of the material increases. These materials have negative temperature co-efficient of resistance.

## 1.8 IMPORTANT PROPERTIES OF SEMICONDUCTORS

The substances (like carbon, silicon, germanium, selenium, sulphur, etc.) which have resistivity ($10^2$ to 0.5 ohm-m) in between conductors and insulators are known as *semiconductors*.

However, it is not the resistivity alone that decides whether a substance is semiconductor or not. In fact, semiconductors have a number of peculiar properties (mentioned below) which distinguish them from insulators, conductors and resistance materials:

(i) *The resistivity of a semiconductor is less than an insulator but more than a conductor.*

(ii) *Semiconductors have negative temperature co-efficient of resistance.* The resistance decreases with the rise in temperature and vice-versa. (According to this property, the semiconductors behave like an insulator at very low temperatures but act as a conductor at high temperatures).

(iii) *When a suitable metallic impurity* (*like arsenic, gallium etc.*) *is added to a semiconductor, it changes the current conducting properties of the semiconductor appreciably.*

## 1.9 BONDS IN SEMICONDUCTOR MATERIALS

In every element, the atoms are held together by the bonding action of valence electrons. This bonding action is due to the fact that each atom needs to complete its last orbit by acquiring electrons in it. In most of the elements, the last orbit is incomplete. Therefore, to complete it, the atoms become active and in this process, the atom may lose, gain or share valence electrons with other atoms.

In semiconductors, bonds are formed by sharing of valance electrons. Such bonds are called *covalent* bonds. In this case, each atom contributes equal number of valence electrons and the contributed electrons are shared by the atoms engaged in the formation of the bond.

Figure 1.6 shows the covalent bonds among semiconductor (Ge or Si) atoms. A silicon (or germanium) atom has 4 valence electrons in outermost orbit. It is the tendency of each silicon atom to have 8 electrons in the last orbit. To do so, the silicon atom positions itself between four other silicon atoms. Each neighbour then shares an electron with the central atom. In this way, the central atom picks up four electrons, giving it a total of eight in its valence orbit. The bonding structure of silicon is shown in Fig. 1.7.

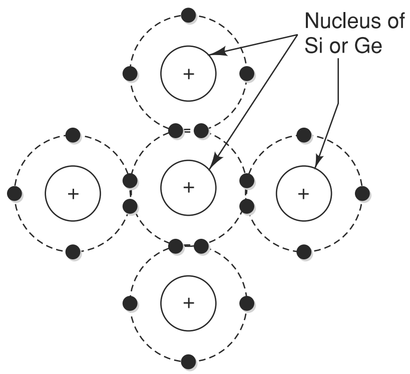



**Fig. 1.6** ***Silicon structure***

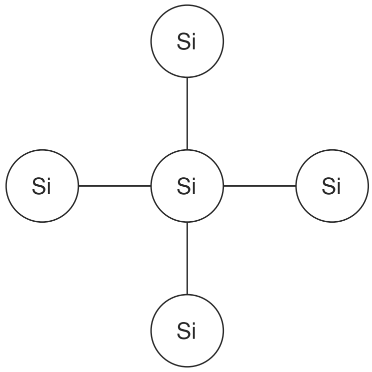



**Fig. 1.7** ***Shared bonding***

## 1.10 CRYSTALLINE STRUCTURE OF SEMICONDUCTORS

In semiconductors, the atoms are arranged themselves in an orderly pattern to form a solid. This orderly pattern is known as a *crystal*. Figure 1.8 shows the crystalline structure of silicon (semiconductor). It is clear that each atom is surrounded by neighbouring atoms in a repetitive manner. Therefore, a piece of silicon is generally called a silicon crystal.

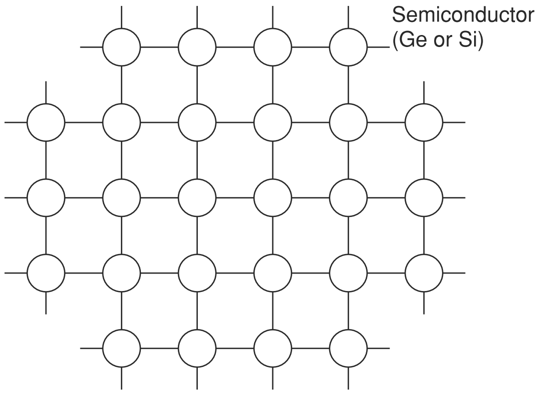



**Fig. 1.8** *A silicon crystal*

## 1.11 SEMICONDUCTOR MATERIALS USED IN ELECTRONICS

A large number of tetravalent materials are available such as carbon in diamond state, silicon, germanium and gray tin. The minimum energy required for breaking the covalent bond in these materials is 7, 1.12, 0.75 and 0.1 electron volt respectively. Carbon in diamond state, having forbidden energy gap of 7 eV, behaves more or less as an insulator and tin having forbidden energy gap of 0.1 eV behaves as a conductor. The energies of 0.72 eV and 1.1 eV are required to break a covalent bond in germanium and silicon respectively and therefore, *germanium* and *silicon* are considered to be the most suitable semiconductor materials.

*(i)* ***Germanium*** It was discovered in 1886. It is a natural element recovered from the ash of certain coals or from the flue dust of zinc smelters. The recovered germanium is reduced to pure germanium. For the applications in electronics, it has become the ideal substance since it can be purified well and crystallised easily than any other semiconductor material.

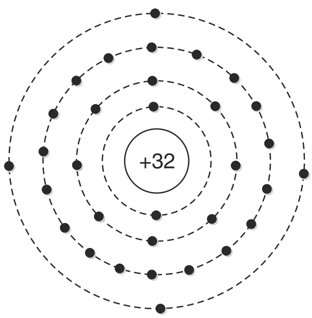



**Fig. 1.9** *Germanium atom*

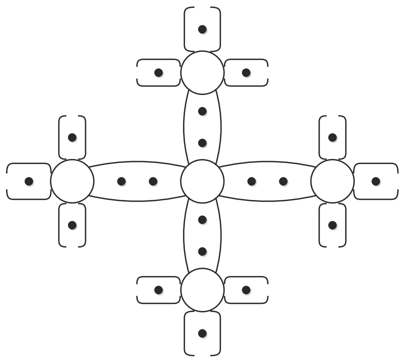


**Fig. 1.10** *Covalent bond*

The atomic structure of germanium is shown in Fig. 1.9. Its atomic number is 32. Therefore, it has 32 protons in the nucleus and 32 electrons distributed in the four orbits around the nucleus. The number of electrons in the first, second, third and fourth orbit are 2, 8, 18 and 4 respectively. It is clear that germanium atom has four valence electrons i.e. the electrons in the outermost (valence) orbit. Since germanium has 4 valence electrons, it is known as tetravalent element. The various germanium atoms are held together through covalent bonds as shown in Fig. 1.10. The atoms of germanium are arranged in an orderly pattern and form a crystalline structure as shown in Fig. 1.11.

Figure 1.12 shows the energy band diagram of germanium. The forbidden energy gap (i.e. gap between valence band and conduction band) in this material is very small, i.e., 0.7 eV. Therefore, a very small energy is sufficient to lift the electrons from the valence band to conduction band. Even at room temperature, a minor quantity of electrons are lifted to the conduction band. This is the reason, that even at room temperature germanium does not behave as an insulator but at the same time it does not behave as a conductor because electrons available in the conduction band are very minor and hence called a semiconductor.

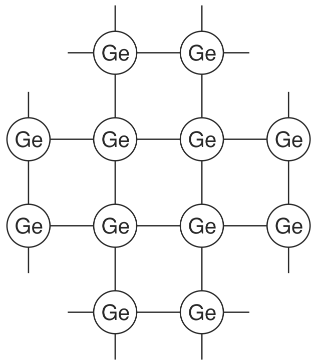



**Fig. 1.11** *Orderly pattern of germanium atoms*

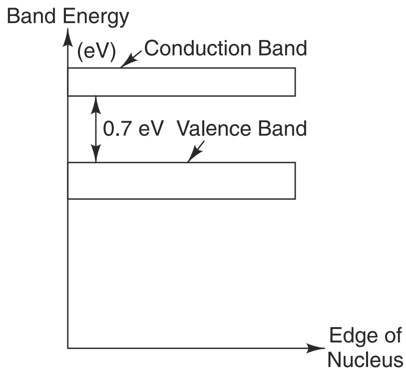



**Fig. 1.12** *Energy band diagram in germanium*

***(ii) Silicon*** It is an element available in most of the common rocks of silicon dioxide. The silicon dioxide is treated chemically and reduced to pure silicon which can be used for the preparation of electronic devices.

Figure 1.13 shows the atomic structure of silicon. Its atomic number is 14. Therefore, it has 14 protons in the nucleus and 14 electrons distributed in the three orbits around the nucleus. The number of electrons in the first, second and third orbit are 2, 8 and 4 respectively. The silicon atom also has four valence electrons and is known as tetravalent element. The various silicon atoms are held together through covalent bonds as shown in Fig. 1.14. The atoms of silicon are arranged in an orderly pattern and form a crystalline structure as shown in Fig. 1.15. Figure 1.16 shows the energy band diagram of silicon. The forbidden energy gap in this material is quite small i.e. 1.1 eV. It also needs a small energy to lift the electrons from valence band to conduction band.

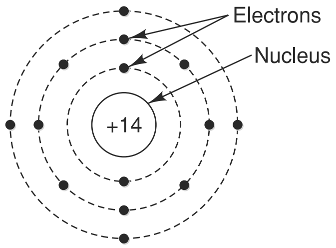



**Fig. 1.13** ***Electron structure of Si***

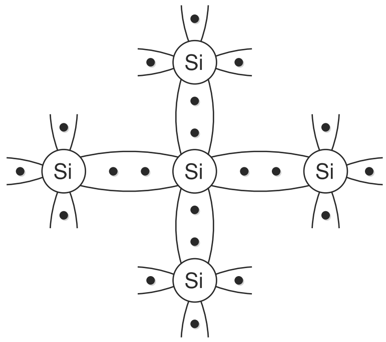



**Fig. 1.14** ***Covalent bond of Si***

Therefore, even at room temperature, a minute quantity of valence electrons are lifted to conduction band and constitute current conduction if a high electric field is applied. However, at room temperature, the number of electrons lifted to the conduction band in case of silicon are quite less than germanium. This is the reason why silicon semiconductor devices are preferred over germanium devices.

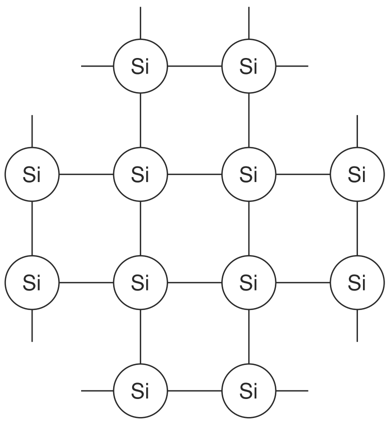



**Fig. 1.15** ***Orderly pattern of silicon atoms***

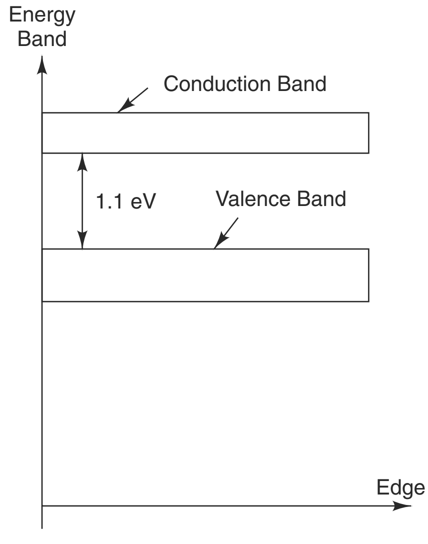



**Fig. 1.16** ***Energy band diagram of silicon***

## 1.12 EFFECT OF TEMPERATURE ON SEMICONDUCTORS

*(i)* ***At Absolute Zero (–273°C)*** In a pure semiconductor material, no free electrons are available at absolute zero temperature as the valence electrons

are tightly bound in through covalent bonding. In other words, the valance band is full and conduction band is empty (Fig. 1.17). If in this state, a semiconductor crystal is connected with some external battery, no current flows (Fig. 1.18).

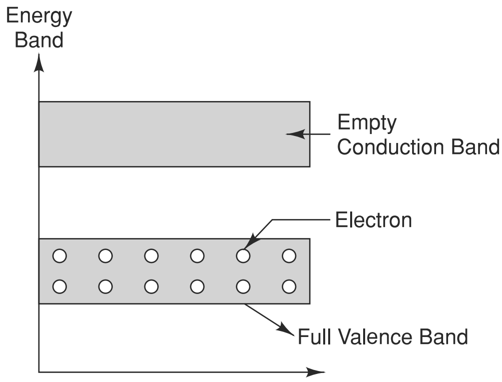



**Fig. 1.17** ***Energy band diagram of semiconductor at absolute zero***

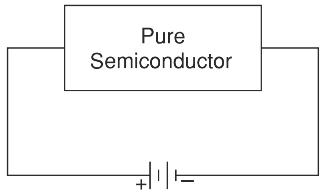



**Fig. 1.18** ***Semiconductor crystal connected to battery at absolute zero***

***(ii) Above Absolute Zero*** If the temperature of the crystal is raised above absolute zero, few covalent bonds within the crystal are broken due to the thermal energy supplied, thus few free electrons are available and they jump from valance band to conduction band crossing the forbidden gap. An electron leaving the valance band leaves behind an empty space which can accept another electron. This empty space is called a 'hole' (see Fig.1.19) and is regarded as equivalent positive charge.

If such a crystal is connected with a battery, free electrons will constitute small current (see Fig. 1.20) called *electron current*.

*Note*: We shall see in the following pages, that holes can also constitute a current called 'hole current'.

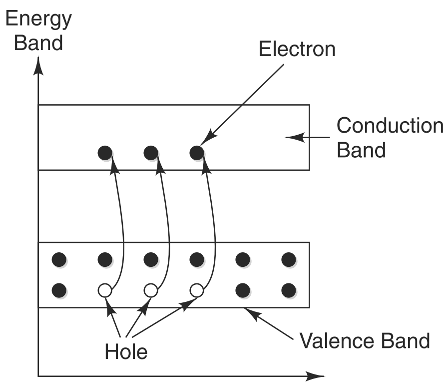



**Fig. 1.19** ***Semiconductor crystal bands at a temperature above absolute zero***

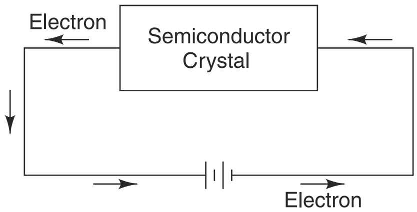



**Fig. 1.20** ***Circuit connection of semiconductor crystal above absolute zero***

## 1.13 HOLE

When energy is supplied to a semiconductor, a valence electron is lifted to a higher energy level, the departing electron leaves a vacancy in the valence band. This vacancy is called a *hole*.

Thus, a vacancy left in the valence band because of lifting of an electron from valence band to conduction band is known as *hole*.

## 1.14 CREATION OF ELECTRON-HOLE PAIRS

Whenever some external (heat) energy is supplied to a semiconductor, the valence electrons are projected up to the conduction band one after the other leaving behind a *vacancy* in the valence band called *hole*. The number of electrons to be lifted from valence band to the conduction band depends upon the quantity of external energy supplied to the semiconductor. If only one electron is lifted to the conduction band, then one hole is created in the valence band. Thus, each time an *electron-hole* pair is formed.

It is important to note the *hole* {i.e. vacancy created by the electron in the valence band) acts as a positive charge. It has strong tendency to attract the electrons from the nearby co-valent bonds.

## 1.15 RECOMBINATION OF ELECTRON-HOLE

When some external energy is supplied to a semiconductor, the electrons of the valence band are lifted to the conduction band and become free leaving behind holes (vacancies) in the valence band. The conduction band orbits in which free electrons are moving are larger than valence band orbits in which holes are formed. Occasionally, the conduction-band orbit of one atom may intersect the hole orbit of another. Because of this, generally conduction-band electron falls into a hole. *This merging of a free electron and a hole is called recombination.* When recombination takes place, the hole does not move elsewhere, it just disappears.

The recombination occurs continuously in a semiconductor. This would eventually fill every hole. However, incoming heat energy keeps producing new holes by lifting valence electrons up to the conduction band forming electron-hole pair. This creation of electron-hole pair and their recombination goes on continuously. *The average time between creation and the disappearance of an electron-hole pair is known as lifetime* which varies from a nanoseconds to several microseconds, depending upon various factors like crystal structure of the semiconductor etc.

## 1.16 HOLE CURRENT

When some external energy is supplied to a pure semiconductor, the co-valent bonds are broken forming electron-hole pairs. The electrons are lifted-up to conduction band leaving behind vacancies (holes) in the valence band. Under the influence of electric field, the free electrons constitute current. At the same time another current, the hole current, also flows in the semiconductor as explained below:

Figure 1.21 shows the phenomenon of hole-current in semiconductors (say Silicon). Suppose the valence electron at *A* has become free due to thermal energy, creating a hole in the covalent bond at *A*. The hole, having positive charge, is a strong centre of attraction for the electron. Since some electric field is applied across the semiconductor, a valence electron from nearby covalent bond (say at *B*) comes to fill in the hole at *A*. This creates a hole at *B*. Then, another valence electron (say at *C*) in turn may leave its bond to fill the hole at *B*, thus creating a hole at *C* which is further filled by the electron (say at *D*) from the nearby covalent bond.

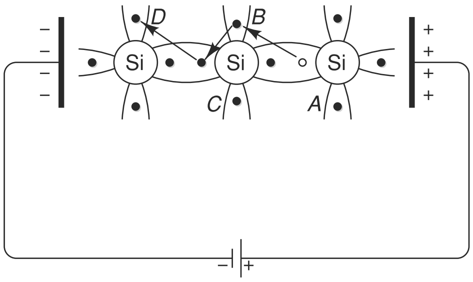



**Fig. 1.21** ***Hole current of semiconductor***

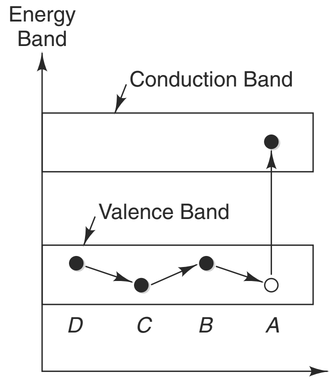



**Fig. 1.22** ***Energy band diagram***

This movement of the hole (positively charged vacancy in the valence band) from positive terminal of the supply to the negative terminal through semiconductor, constitutes *hole-current*.

Although in this case electron moves from one covalent band to the other to fill the hole in the valence band, but it is still called as hole-current. The hole-current is constituted by the flow of valence electrons in the valence band. As these electrons are rigidly attached in the covalent bonds, their movement is very slow. Hence the hole-current is very small as compared to the current constituted by the free electrons which jump into the conduction band. It is due to the movement of the electron is only due to the existance of hole in the valence band causing current.

The same phenomenon can be explained with the help of energy band diagram shown in Fig. 1.22.

## 1.17 INTRINSIC SEMICONDUCTOR

An extremely pure semiconductor is called *intrinsic semiconductor*.

On the basis of energy band phenomenon, an intrinsic semiconductor at absolute zero temperature is shown in Fig. 1.23. Its valence band is completely filled and the conduction band is completely empty.

When some heat energy is supplied to it, some of the valence electrons are lifted to conduction band leaving behind holes in the valence band as shown in Fig. 1.24. The electrons reaching at the conduction band are free to move at random. The holes created in the crystal also move at random in the crystal. This behaviour of semiconductors shows that they have negative temperature co-efficient of resistance i.e., the resistivity decreases or conductivity increases with the rise in temperature.

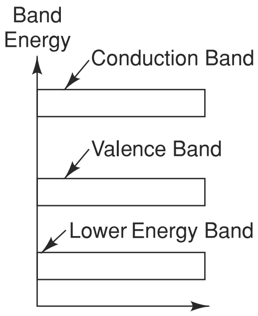



**Fig. 1.23** ***Energy band diagram of intrinsic semiconductor***

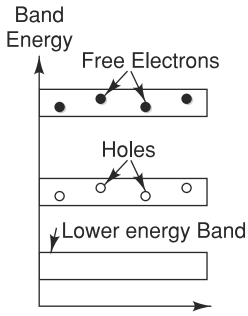



**Fig. 1.24** ***Energy band when external energy is supplied***

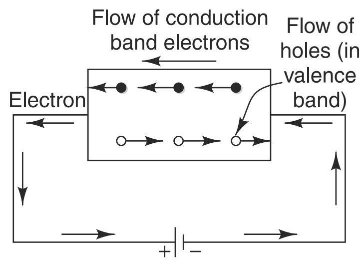



**Fig. 1.25** ***Conduction in holes of semiconductor***

## 1.18 CONDUCTION THROUGH INTRINSIC SEMICONDUCTORS AT ROOM TEMPERATURE

When an electric potential difference is applied across a pure semiconductor kept at room temperature, the current conduction takes place by two kinds of charge carriers, namely, electrons and holes. The free electrons available in the conduction band start drifting towards the positive terminal, whereas, the positive charge carriers (holes) start drifting towards the negative terminal, as shown in Fig.1.25. Therefore, the total current inside the semiconductor is the sum of currents due to free electrons and holes. Actually, in the semiconductors, holes are not moving. In fact, the negative terminal pushes an electron in valence band which occupies the hole (vacant space). At the same instant, one electron leaves the semiconductor at the other end keeping same number of holes available in valence band. However, it looks that holes are drifting from one end to the other.

***Drift Current*** The flow of current in the semiconductors, constituted by the drift of free electrons available in the conduction band and holes available in the valence band, which are formed due to external (i.e. heat) energy supplied to them, is known as *drift current*.

Although in an intrinsic semiconductor the concentration of electrons and holes is equal, nearly two-third of the current flowing through it is constituted by the free electrons and the remaining one-third by holes. This is because the electrons are much more mobile than the holes.

**Diffusion of carriers** Apart from the drifting phenomenon, another transport phenomenon occurs in a semiconductor in the absence of electric field when there is a concentration gradient of mobile carriers. The carriers move from the high concentration region to low concentration region. This phenomenon of random motion of charge carriers is called diffusion.

## 1.19 CONDUCTIVITY OF SEMICONDUCTORS

The resultant conductivity of a semiconductor depends upon:

1. the concentration of the mobile charge carriers (i.e. electrons and holes) and
2. the mobility of the charge carriers.

When an electric field of $E$ volts/metre is applied, the current densities ($J$) contributed due to the motion of electrons and holes are given by the following expressions:

$J_n = en\ v_n$ A/m$^2$ and

$J_p = ep\ v_p$ A/m respectively

where, $e$ = charge of an electron (or a hole) = $1.602 \times 10^{-19}$ C

$n$ = density of free electrons i.e. no. of free electrons/m$^3$

$p$ = density of holes i.e. no. of holes/m$^3$

$\mu_n$ = drift velocity of free electrons in m/s

$\mu_p$ = drift velocity of holes in m/s

Conductivity due to electrons

$$\sigma_n = J_n/E = en\ v_n/E = en\ \mu_n$$

Conductivity due to holes

$$\sigma_p = J_p lE = en\ V_p lE = en\ \mu_p$$

where $\mu_n$ and $\mu_p$ = mobility of free electrons and holes respectively. It is the drift velocity per unit field and is given in m/s per V/m or m$^2$/Vs.

Total conductivity of a semiconductor

$$\sigma = \text{conductivity due to electrons + conductivity due to holes}$$
$$= \sigma_n + \sigma_p = en\ \mu_n + ep\ \mu_p = e\ (n\mu_n + p\mu_p)$$

In case of intrinsic semiconductors, the electrons and holes are always present in equal concentration; Therefore, conductivity of intrinsic semiconductors

$$\sigma_i = en_i\ (\mu_n + \mu_p)$$

where $n_i = n = p$ and is called intrinsic concentration.

## 1.20 SILICON VERSUS GERMANIUM

In the early days of semiconductor devices, germanium was considered to be the best semiconductor material. But nowadays, it is rarely used in new designs of semiconductor devices. Silicon is considered to be the best for the preparation of semiconductor devices. It is, because at room temperature, a silicon crystal has almost no free electron compared with a germanium crystal. Because of this property, the semiconductor devices made from silicon material give far better performance than the semiconductor devices made from germanium.

Thus, silicon has totally overshadowed germanium in the fabrication of semiconductor devices e.g. diodes, transistors, thyristors, etc.

## 1.21 MASS-ACTION LAW AND CONTINUITY EQUATION

In an impure semiconductor whose impurity concentration is either spatially constant or changing very slowly with position, the mass-action law must be used to obtain the two unknowns, the electron and the hole concentration.

The mass-action law is the electron analogue of the chemical reaction in which the reacting species are the electrons and holes and the reactions are the electron-hole recombination and generation processes. The electronic 'chemical' equilibrium equation is then given by

$$\text{Electron} + \text{hole} \leftrightarrow \text{perfect covalent bond.}$$

Under thermal equilibrium, the product of the free electron concentration $n$ and hole concentration $p$ is a constant $n_i^2$ where $n_i$ is the intrinsic carrier concentration. The mass-action law is expressed by $np = n_i^2$. This $n_i$ is a function of temperature. When $n$ (or $p$) is increased by addition of impurities, $p$ (or $n$) must decrease in order to keep $n_i$ constant.

One of the basic relationships that govern the generation, recombination, trapping and flow (drift and diffusion) of electrons or holes in a solid is the continuity equation of charge and current. It is the macroscopic particle conservation law that accounts for the appearance and disappearance of particles of one species in a volume element $\Delta x \Delta y \Delta z$ and time interval $\Delta t$.

The continuity equation for holes is

$$\frac{\partial p_n}{\partial t} = \frac{p_n - p_{no}}{T_p} + D_p \frac{\partial^2 p_n}{\partial x^2} - \mu_p E \frac{\partial p_n}{\partial x}$$

where $p_n$ is the minority carrier hole, $T_p$ is the lifetime for holes, $p_{no} = p_n$ at $\frac{dp_n}{dt} = 0$, $D_p$ is the diffusion constant for holes, $E$ is the electric field and $\mu_p$ is the mobility for the holes.

Similarly continuity equation for electrons is

$$\frac{\partial n_p}{\partial t} = -\frac{n_p - n_{po}}{T_p} + D_n \frac{\partial^2 n_p}{\partial x^2} - \mu_n E \frac{\partial n_p}{\partial x}$$

## 1.22 EXTRINSIC SEMICONDUCTOR

Although an intrinsic semiconductor is capable to conduct a little current even at room temperature, it is not useful for the preparation of various electronic devices. To make it conductive, a small amount of suitable impurity is added. It is then called *extrinsic* (*impure*) *semiconductor*.

***Doping*** The process by which an impurity is added to a semiconductor is known as *doping*.

The amount and type of such impurities have to be closely controlled during the preparation of extrinsic semiconductors. Generally, one impurity atom is added to $10^8$ atoms of a semiconductor.

Thus, a semiconductor to which an impurity at controlled rate is added to make it conductive is known as an *extrinsic semiconductor*.

As discussed earlier, the purpose of adding impurity in the semiconductor crystal is to increase the number of free electrons or holes to make it conductive. If a pentavalent impurity (having 5 valence electrons) is added to a pure semiconductor a large number of free electrons will exist in it whereas, if trivalent impurity (having 3 valence electrons) is added, a large number of holes will exist in the semiconductor. Depending upon the type of impurity added, extrinsic semiconductor may be classified as:

(i) *n*-type semiconductor
(ii) *p*-type semiconductor

### 1.22.1 *n*-type Semiconductor

To obtain this semiconductor, an atom of pentavalent (which has 5 electrons in its last orbit) material (e.g. arsenic, antimony, etc) is added into the intrinsic semiconductor. The four electrons of impurity atom make covalent bonds with the four electrons of the semiconductor and the fifth electron of impurity atom becomes free (Fig. 1.26). Thus, one atom of impurity atom produces one 'free electron' which can constitute current. By adding calculated quantity of impurity, required number of free electrons may be obtained.

Such type of semiconductor is called N(negative) type or electron type, as it has more free electrons than holes, i.e., the number of free electrons obtained from pentavalent impurity exceeds number of holes in the material.

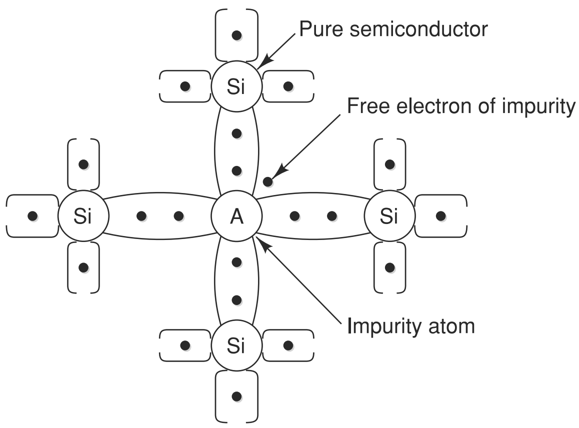



**Fig. 1.26** ***n-type semiconductor***

The reader however, should not think that an *n*-type semiconductor is negatively charged. The negative charge of free electrons is balanced by the positive charge of holes, but remember it has additional free electrons created by doping. It is electrically neutral.

If an *n*-type material is connected with a battery, the current conduction will be due to free electrons. This is called *n*-type conductivity. The conduction is just as in copper, aluminium and other metals (see Fig. 1.27).

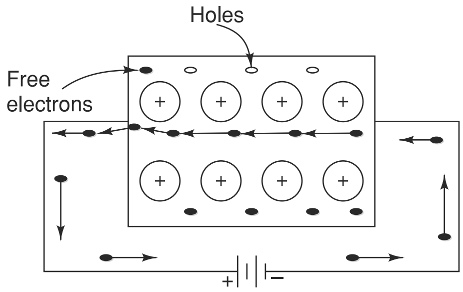



**Fig. 1.27** ***n-type semiconductor crystal in electric circuit***

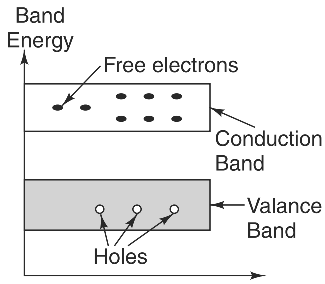



**Fig. 1.28** ***Energy band diagram of n-type material***

### 1.22.2 *p*-type Semiconductor

To obtain this semiconductor one atom of trivalent (which has 3 electrons in the last orbit) material (e.g., gallium, indium. bismuth, etc.) is added into $10^8$ atoms of an intrinsic semiconductor. Each impurity atom makes bond with germanium thus 3 bonds are complete. In the fourth covalent bond gallium has no electron to contribute. Thus the fourth bond is 'incomplete' and is short of one electron. The missing electron is called a hole. Each gallium atom generates one hole. Calculated number of gallium atoms, when added can generate required number of holes in the crystal (Fig. 1.29).

Such type of semiconductor is called *p* (positive) type or hole type as it has more holes than electrons, i.e., the number of holes exceeds the number of free electrons. It also contains immobile ions (negative charge) as well as electrons but as a whole, like an *n*-type material, a *p*-type semiconductor is also neutral. It has additional holes created by doping.

If a *p*-type material is connected with a battery the current conduction will be due to holes called *p*-type conductivity. Figure 1.30 shows representation of such a *p*-type material during conduction. Note that in the material holes are moving but conduction in external circuit is by electrons.

Figure 1.31 shows energy band diagram of a *p*-type material.

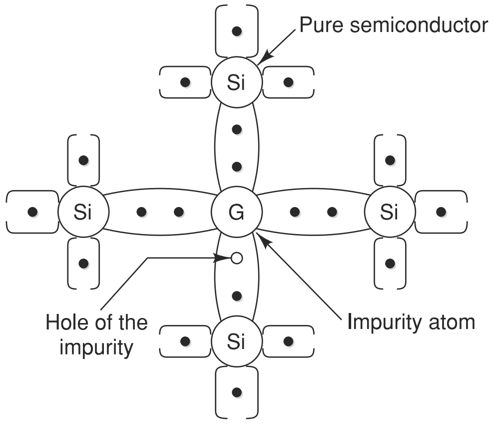



**Fig. 1.29** *p-type semiconductor*

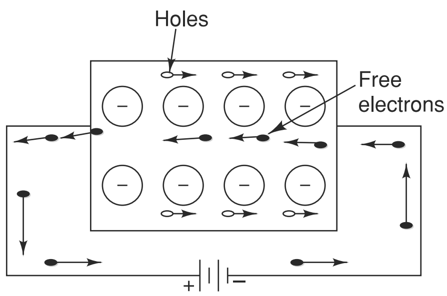



**Fig. 1.30** *p-type semiconductor in electric circuit*

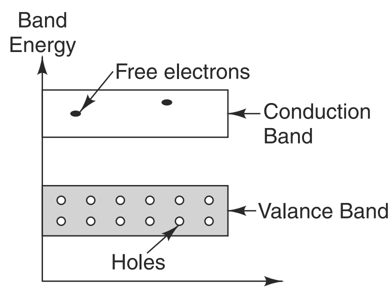



**Fig. 1.31** *Energy band diagram of p-type material*

## 1.23 EFFECT OF TEMPERATURE ON SEMICONDUCTORS

As discussed already, a pure or intrinsic semiconductor behaves more or less as an insulator at room temperature. The 'carrier concentration', i.e., density of electron/holes in an intrinsic material at room temperature is about $2*10^9$ per $m^3$. If we raise its temperature its carrier concentration is increased, in other words, conductivity of intrinsic semiconductors increases with the rise of temperature.

An extrinsic semiconductor behaves as a very good conductor even at room temperature due to extra electrons (or holes) denoted by impurity atoms.

If the temperature of an extrinsic semiconductor is raised there is no increase in the majority charge carriers as the impurity has already donated its all electrons (or holes). However, due to thermal energy supplied, concentration of minority carriers increases and at a temperature called (80°C for Ge and 200°C for Si) the extrinsic semiconductor behaves more or less as an intrinsic semiconductor.

## 1.24 CHARGE ON *n*-TYPE AND *p*-TYPE SEMICONDUCTORS

It has been discussed above that in *n*-type semiconductors, conduction is due to free electrons donated by the pentavalent impurity atoms. These electrons are the excess electrons with regard to the number of electrons needed to fill the covalent bonds in the semiconductor crystal. These excess electrons does not create any charge on the *n*-type semiconductor, since impurity atoms as well as germanium atoms all are electrically neutral themselves.

Similarly, in *p*-type semiconductors, conduction is due to holes created by the trivalent impurity atoms. These holes are the positively charged vacant spaces which can accept the electrons. The holes just represent the deficit of electrons with regard to the number of electrons needed to fill the covalent bonds in the semiconductor crystal. This deficit of electrons do not create any charge on the *p*-type semiconductor, since impurity atoms as well as germanium atoms are electrically neutral themselves.

*Thus, it follows that n-type as well as p-type semiconductors are electrically neutral.*

## 1.25 MAJORITY AND MINORITY CARRIERS

When a small amount of pentavalent impurity is added to a pure semiconductor, it provides a large number of free electrons in the crystal forming *n*-type semiconductor. However, it may be recalled that even at room temperature, some of the covalent bonds break, releasing a small number of hole-electron pairs. Thus, an *n*-type semiconductor contains a large number of free electrons (i.e., the electrons provided by pentavalent impurity added and a share of hole-electron pairs) but only a few number of holes. Therefore, in *n*-type semiconductor, the most of the current conduction is due to free electrons available in the semiconductor.

Thus, in *n*-type semiconductor, the electrons are the majority carriers, whereas, the holes are the minority carriers [See Fig. 1.32(a)].

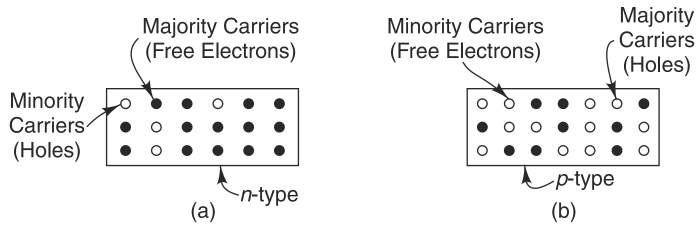



**Fig. 1.32** ***Majority and minority carriers in n-type and p-type semiconductor***

Giving the similar explanation, it can be concluded that in *p*-type semiconductor, the holes are the majority carriers, whereas,the electrons are the minority carriers. [See Fig. 1.32(b)]

## 1.26 *pn* JUNCTION

When a *p*-type semiconductor is suitably joined to an *n*-type semiconductor, the contact surface so formed is called *pn* junction.

All the semiconductor (solid state) devices contain one or more *pn* junctions. The *pn*-junction is the *control element* for semiconductor devices. To understand the working of various semiconductor devices, it is essential for the readers to have a thorough knowledge of formation of a *pn*-junction and its properties.

## 1.27 PROPERTIES OF *pn* JUNCTIONS

To understand the properties of a *pn* junction, consider two types of extrinsic semiconductors—one *p*-type and the other *n*-type as shown in Fig. 1.33. The *p*-type semiconductor is having negative acceptor ions and positively charged holes. Whereas, the *n*-type semiconductor is having positive donor ions and negatively charged electrons.

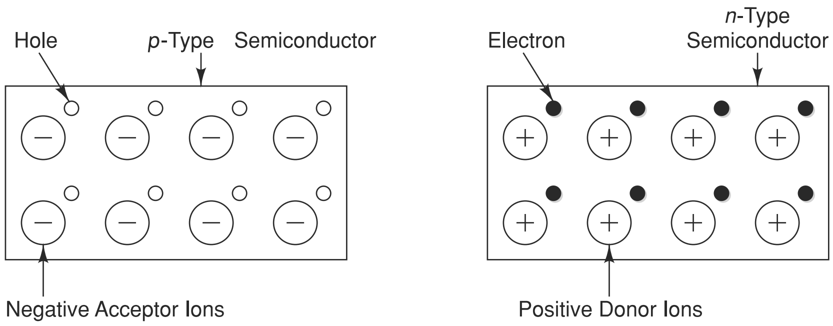



**Fig. 1.33** ***p and n type semiconductors***

When the two pieces are joined together and suitably treated, they form a *pn* junction. The moment they form a *pn* junction, some of the conduction electrons from *n*-type material diffuse over to the *p*-type material and undergo electron-hole recombination with the holes available in the valence band. Simultaneously holes from *p*-type material diffuse over to the *n*-type material and undergo hole-electron combination with the electrons available in the conduction band. This process is called *diffusion*. In this process, some of the free electrons move across the junction from *n*-type to *p*-type, leaving behind positive donor ions as they are robbed of the free electrons. This establishes a positive charge on the *n*-side of the junction.

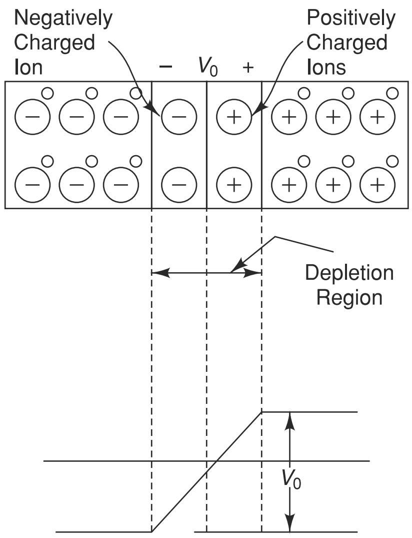



**Fig. 1.34** ***Diffusion process***

Simultaneously, the free electrons which cross over the junction recombine with the holes of *p*-type and uncover some of the negative acceptor ions as shown in Fig. 1.34. This establishes a negative charge on the *p*-side of the junction. This process of diffusion continues till a sufficient number of donor and acceptor impurity ions are uncovered and establish a requisite potential difference (i.e., nearly 0.3 V in case the *pn* junction for germanium semiconductor and 0.7 V for silicon *pn* junction). After this, further diffusion is prevented because now positive charge on *n*-side repels holes to cross from *p*-type to *n*-type and negative charge on *p*-side repels free electrons to enter from *n*-type to *p*-type. Thus, a potential difference created across the junction acts as a barrier which restricts further movement of charge carriers i.e., holes and electrons. This is called *a potential barrier* or *junction barrier* $V_0$.

***Potential Barrier*** A potential difference built up across the *pn* junction which restricts further movement of charge carriers across the junction is known as *potential barrier*.

It may be noted that on the both sides of the junction a layer is formed, this layer is depleted of free electrons and holes. This region is called *depletion layer*.

***Depletion Layer*** A region around the junction from which the charge carriers (free electrons and holes) are depleted is called *depletion layer*.

## 1.28 BEHAVIOURS OF A *pn* JUNCTION UNDER BIASING

When a *pn* junction is connected across an electric supply (potential difference), the junction is said to be under *biasing*.

The potential difference across the *pn* junction can be applied in two ways, namely—*forward biasing* and *reverse biasing*.

***Forward Biasing*** When the positive terminal of a *dc* source or battery is connected to *p*-type and negative terminal is connected to *n*-type semiconductor of a *pn* junction, as shown in Fig. 1.35, the junction is said to be in *forward biasing*. In this case, the applied forward potential acts in such a way that it establishes an electric field which reduces the field due to potential barrier. Thus, the barrier potential at the junction is reduced. Since the potential barrier voltage is very small (nearly 0.7 V for silicon and 0.3 V for germanium junction), a small forward voltage is sufficient to completely eliminate the barrier. Once the potential barrier is

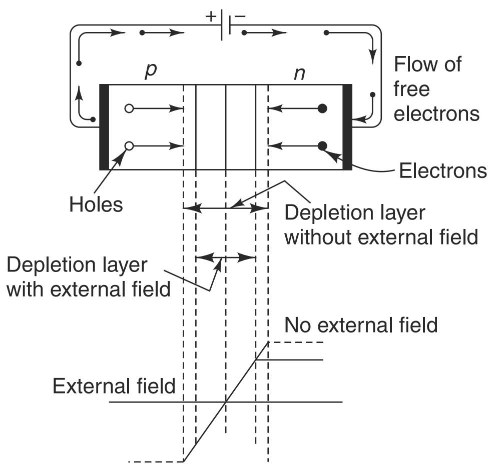



**Fig. 1.35** ***Forward biasing of a pn junction***

eliminated by the forward voltage, a conducting path is established for the flow of current. Thus, a large current starts flowing through the junction. This current is called *forward current*.

Hence, the external voltage applied to a *pn* junction that cancels the potential barrier to constitute easy flow of current through it is called *forward biasing*.

The following points are worth noting, when a *pn* junction is forward biased:

(i) The junction potential barrier is reduced and at some forward voltage (0.3 V for germanium and 0.7 V for silicon) it is eliminated altogether.

(ii) The junction offers low resistance (called forward resistance, $R_f$) to the flow of current through it.

(iii) The magnitude of flow of current through the circuit depends upon the applied forward voltage.

***Reverse Biasing*** When the positive terminal of a dc source or battery is connected to *n*-type and negative terminal is connected to *p*-type semiconductor of a *pn* junction, as shown in Fig. 1.36. the junction is said to be in *reverse* biasing.

In the case, the applied reverse potential acts in such a way that it establishes an electric field which increases the field due to potential barrier. Thus, the barrier potential at the junction is strengthened. The increased potential barrier prevents the flow of majority charge carriers across the junction. Thus, a high resistive path is established by the junction and practically no current flows through the circuit.

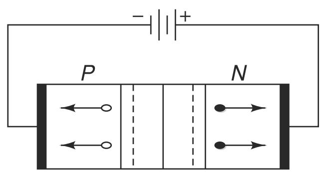



**Fig. 1.36** ***Reverse biasing of a pn junction***

Hence, the external voltage applied to a *pn* junction that strengthens the potential barrier and prevents the flow of current through it is called *reverse biasing*.

The following points are important to note, when a *pn* junction is reverse biased:

(i) The junction potential barrier is strengthened.

(ii) The junction offers a high resistance (called reverse resistance, $R_r$) to current flow.

(iii) Practically, no current flows in the circuit due to establishment of very high resistance path. It may be noted here that at reverse biasing, practically no current flows through the junction, but a little current (in $\mu A$) flows through the junction because of minority carriers available in the semiconductors even at room temperature. However, this current is neglected for all practical purposes.

From the above discussion, it is revealed that with forward bias, a low resistance path is set up by the *pn* junction and hence current flows through the circuit. On the other hand, with reverse bias, a high resistance path is established by *pn* junction and hence no current flows through the circuit. Because of this property, *pn* junction (i.e., crystal diode) is best suited for rectification of ac into dc.

***Effect of Short Circuit*** When a *pn* junction is short-circuited, no current can flow in the circuit; the electrostatic potential remains at the same value as that under open circuit conditions. If a current exists, it would cause heating of wires in the circuit. This heat energy would have to be supplied by the *p-n* layer which would have to cool down. This simultaneous heating of wires and cooling of *p-n* layar cannot take place under thermal equilibrium conditions. Therefore current is zero.

## 1.29 VOLT-AMPERE CHARACTERISTICS OF SEMICONDUCTOR *pn* JUNCTION DIODE

Figure 1.37 shows a circuit for determining the volt-ampere characteristics of a semiconductor diode. By the use of a potential divider the applied voltage can be varied both in the forward and reverse directions. The characteristic is shown in Fig. 1.38.

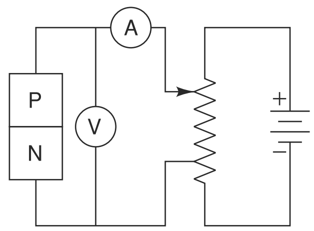



**Fig. 1.37** ***Circuit for determination of V–I characteristic***

When the applied voltage is zero, a small equilibrium potential $V_0$ exists across the depletion region. As the applied voltage is increased in the forward direction, this potential barrier $V_0$ is overcome and current starts flowing in the circuit. As the voltage is increased more and more, the current goes on increasing. The current is bipolar in character since it consists of both negative and positive carriers (i.e., electrons and holes). The total current (at a particular value of applied voltage) is constant throughout the circuit but the proportion of current due to holes and electrons varies with distance along the *p-n* layer. Since a *p-n* junction has very little heat storage capacity, a current higher than rated current should not be allowed to flow through it.

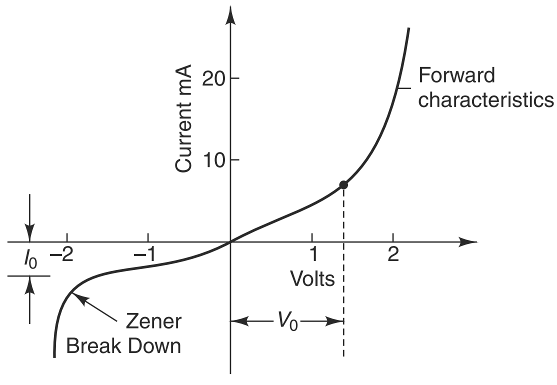



***Fig. 1.38 V–A characteristics of semiconductor diode***

When a reverse voltage is applied only a small current flows. This small reverse current is because of the reason that holes (produced due to thermal agitation) in the *n*-region and electrons (produced due to thermal agitation) in the *p*-region cross the junction in the easy current direction. However the rate at which these charge carriers are produced is a function of only temperature. When all the charge carriers are being drawn across the junction before they can recombine, an increase in reverse voltage does not increase the reverse current. This small current is the reverse saturation current denoted by $I_0$ in Fig. 1.38. If the operating temperature is increased, $I_0$ increases.

When the reverse voltage reaches a critical value, the reverse current through the diode increases abruptly and a relatively large current can flow with little increase in voltage. This phenomenon, known as reverse breakdown, occurs because electrons gain enough energy so that ionization by collision occurs and the covalent bonds are disrupted. This releases a large number of electrons and a large reverse current occurs. This breakdown occurs at the *zener breakdown voltage* and the diode is destroyed. However some diodes are built to operate specifically in the zener breakdown region and are known as *zener diodes*.

The current $i$ through the junction, under both forward and reverse bias conditions, is given by the Boltzmann diode equation

$$i = I_0(e^{V/\eta V_T} - 1)$$

where $I_0$ is the saturation current, $V$ is the applied voltage, $\eta$ equal 1 for germanium and is approximately 2 for silicon and $V_T$ is the volt equivalent of temperature and equals $kT/e = T/11600$ where $T$ is the absolute temperature, $k$ is Boltzmann's constant and $e$ is the charge on an electron. $V$ is positive for forward bias and negative for reverse bias. This equation assumes that the *p*- and *n*-regions have high conductivities so that voltage drops across these regions are zero and the whole of the applied voltage appears across the *p-n* junction.

A semiconductor diode has a very low resistance (only a few ohms) in the forward direction and very high (thousands of ohms) in the reverse direction. At any operating point, in the forward direction the slope of the *i-v* characteristic is known as *incremental conductance* of the diode.

A semiconductor diode has very little heat storage capacity and is burnt out very quickly due to overloads or excessive reverse voltage. The actual peak inverse voltage across the *p-n* junction should not exceed one-half of zener breakdown voltage. When a number of such diodes are connected in series, the peak inverse voltage per junction is usually kept less than one-third of breakdown value because the voltage distribution across the different junctions may not be uniform.

## 1.30 AVALANCHE-MULTIPLICATION

The forward current flowing in a *pn* junction consists mainly the majority carriers and smaller number of minority carriers. The current contributed by minority carriers saturates after sometime when there are no more minority carriers left. But if the reverse voltage across the diode is increased to a large value, a stage may come when a number of charge carriers from valence band break up their covalent bond and reach the conduction band to constant current. This breaking up of bonds is so violant that it knocks out more charge carriers for adjacent atoms. These free charge carriers further knock out more of them, the process being a cumulative one. A stage subsequently comes when the flow of minority carriers exceed the majority carriers. There is a sudden rise in magnitude of current. If the diode is not able to sustain this heavy current, it may break down. The voltage at which the current starts rising all of a sudden is called *avanche breakdown* voltage and the process is known as *avanche multiplication*.

## 1.31 CAPACITANCE ATTRIBUTED TO A *pn* JUNCTION

A semiconductor *p*-*n* junction in associated with capacitive effects when forward biased on reverse biased. However, the two capacitive effects are different. When the *p*-*n* junction diode is reverse biased, the capacitance of the diode originates from the concentration of positive and negative charge carriers on the *p* and *n* sides consisting the two parallel plates separated by the depletion region acting as dielectric. This is known as *transition capacitance* or *space charge capacitance* (the value being around 20 pF for no external bias and 5 pF for reverse bias of about 5 V). When the diode is forward biased, the predominant capacitance effect is due to minority carriers outside the depletion region. This is known as *diffusion capacitance* and is much larger (around few hundred pF and may be in the order of $\mu$F for heavy forward bias). Both transition and diffusion capacitances affect the switching performance of a diode.

## 1.32 TUNNEL DIODE AND TUNNELING PHENOMENON

A *tunnel diode* is also a *pn* junction diode but with the difference in the impurity concentration in *p* and *n* type materials. In an ordinary semiconductor diodes, the impurity concentration may be about 1 part in $10^8$ parts. The depletion width which is inversely proportional to square root of impurity concentration, is about 5 microns (= 5* $10^{-6}$ ). In tunnel diode the impurity concentration is 1 part of $10^3$ parts and depletion width is about $10^{-6}$ cm.

In an ordinary diode (*pn* junction diode), in order to make the diode conduct in the forward direction, a voltage sufficient to overcome the barrier potential is needed to be applied. This voltage is above 0.6 volt in case of silicon diodes. It has been found out that if the depletion region is too narrow, the charge carriers instead of climbing up the potential barrier may pierce through the junction barrier with little or no energy applied. This phenomenon is called *tunneling*. Thus a tunnel diode may act as a very good conductor in the forward direction for a very small applied voltage for the order of 0.1 volt or less.

The advantages of tunnel diodes are

(i) Low noise, low cost, simplicity.
(ii) It is immune to environmental changes.
(iii) It can be used as a high speed switch, switching times of the order of a nano sec are common. Switching times as small as 50 psec could be achieved.
(iv) Power consumption is low.

## 1.33 OTHER TYPES OF DIODES

*Schottky Barrier Diodes* are designed for high efficiency rectification essential for applications like switch mode power supply (SMPS), switching regulators etc. Unlike the conventional *p–n* junction diode rectifier, this rectifier functions by majority carrier conduction and therefore do not generate any reverse recovery transients from stored charge and minority carrier injection. Their electrical equivalent circuit is an ideal diode in parallel with a known capacitance equivalent of junction capacitance resulting in a very fast recovery performance and low conduction voltage. Their use substantially improves the efficiency of the switching circuit.

*Point Contact Diodes* are primarily intended for *RF* applications due to their extremely small internal capacitance, considerably less than that of a junction diode designed for the same circuit applications.

*Pin Diodes* are formed by diffusing heavily doped *p* and *n* regions into an almost intrinsically pure silicon. In practice, it is impossible to obtain intrinsically pure material, the *I*-layer may be considered to be a lightly doped *n*-region. Characteristics of pin diodes are primarily determined by the semiconductor nature of the cheap especially that of *I*-layer. Pin diodes are used as *RF* switches, attenuators and various type of phase shifting devices.

*Hot Carrier Diode* is a metal to semiconductor majority carrier conducting device with a single rectifying junction. The carriers are high mobility electrons in an *n* type semiconductor material. These are particularly useful as mixers and detectors at VHF and high frequency ranges. Notable features include its high operating frequency and lower conduction voltage. It is both mechanically as well as electrically superior when compared to a point contact diode.

When the diode is forward biased, there is always a minimum voltage that must be exceeded before there is sufficient conduction of current through the diode. This is known as the CUT-IN VOLTAGE and is 0.6 V for silicon and 0.2 V for germanium diodes. For all forward voltages greater than the cut-in voltage, there is a sharp rise in forward current i.e. a very small change in forward voltage causes a very large change in forward current. In fact, for all practical purposes, the voltage across the diode can be considered to be equal to the cut-in voltage irrespective of the forward current flowing through the diode.

In the reverse biased condition, the current that flows through the diode is the reverse leakage current. It may be a few nanoamperes in case of silicon diodes and typically 1 $\mu$A for germanium diodes. As the reverse bias voltage is increased further, a point comes where the junction breaks down and there is a steep rise in current which ultimately culminates in device getting damaged. This reverse bias voltage at which the break down occurs is known as PEAK INVERSE VOLTAGE (PIV) of the diode.

## 1.34 VARACTOR DIODE

Varactor diodes are used as voltage variable capacitors. They depend upon change in capacitance that occurs across their depletion layers when the reverse bias voltage is varied. When the reverse bias voltage is increased the depletion layer width increases which in turn increases the dielectric thickness i.e. the distance between the plates of the parallel plate capacitor increases. Hence the junction capacitance is decreased. Similarly when the reverse bias voltage is decreased the junction capacitance is increased. When the reverse bias is zero the capacitance becomes maximum.

The capacitance can be controlled using different doping profiles such as abrupt, hyper abrupt and linearly graded doping profiles. It can also be controlled by the size or geometry of the diode. The depletion region capacitance of an abrupt p-n junction is inversely proportional to the square root of the applied reverse voltage. In linearly graded junction the capacitance varies inversely as one third of reverse voltage. Hence the voltage sensitivity of the junction capacitance is higher for an abrupt junction than for linearly graded junction. It is also possible to vary the junction capacitance so as to make it to vary inversely as the square of the reverse voltage by special growth techniques.

The varactor diodes are known as VARICAPS and are designed to provide various capacitances ranging from few pF to 100pF or so. The maximum to minimum capacitance ratio is typically 2.5 to 3 in commercially available varactors.

Varactor diode finds its application in the tuning stage of a radio receiver or in a TV tuner in place of bulky variable capacitors. They are also used in harmonic generator, microwave frequency multipliers, active filters and self balancing bridge circuits.

**1.1** Determine the intrinsic carrier concentration of a germanium bar having 1 cm length and 1 mm$^2$ cross section given that the resistance is 2.2 kΩ at 300 K. Given that $\mu_n = 0.39$ m$^2$/Vs and $\mu_P = 0.19$ m$^2$/Vs at 300 K.

***Solution***

Area of Ge bar $A = 1 \text{ mm}^2$

Length of Ge bar $l = 1$ cm

Resistance $R = 2200\ \Omega$

resistivity $$\rho = \frac{RA}{l} = \frac{2200 \times 1 \times 10^{-6}}{1 \times 10^{-2}} = 0.22\ \Omega\text{m}$$

∴ Condectivity $$\delta = \frac{1}{p} = 4.545 \text{ S/m}$$

Now $$\delta = en_i = (\mu_n + \mu_P) = 1.6 \times 10^{-19} \times n_i (0.39 + 0.19)$$

where $n_i$ is the intrinsic carrier concentration of Ge.

∴ $$n_i = 4.897 \times 10^{19} \text{ per m}^3$$

**1.2** Determine the current flowing through an *N*-type silicon bar, 1 cm long, 20 $\mu$m wide and 5 $\mu$m thick, doped with a concentration of $10^{22}$ per m$^3$ phosphorous atoms when the applied voltage is 15 V across the bar. Given that $\mu_n = 0.13$ m$^2$/Vs

***Solution***

Conductivity due to electrons only

$$\delta = em\ \mu_n = 1.6 \times 10^{-19} \times 10^{22} \times 0.13 = 208 \text{ S}/m$$

∴ resistivity $$\rho = \frac{1}{\delta} = 4.8 \times 10^{-3}\ \Omega\text{m}$$

Hence, resistivity of the silicon bar is

$$R = \frac{\rho l}{A} = \frac{4.8\times10^{-3}\times1\times10^{-2}}{20\times5\times10^{-6}\times10^{-6}} = 480\ \text{K}\Omega$$

$\therefore$ current flowing through the bar is $I = \frac{V}{R} = \frac{15}{480}\times10^{-3} = 0.03125$ mA

**1.3** Find the concentration of donor atoms to be added to an intrinsic germanium bar to produce an N-type material of conductivity 550 s/m $\mu_n$ for Ge is 0.39 m$^2$/Vs.

***Solution***

The conductivity for *N*-type material is

$$\delta = ne\ \mu_n \text{ where } n \text{ is the concentration of electrons.}$$

$$550 = n \times 1.6 \times 10^{-19} \times 0.39$$

or $$n = 881 \times 10^{19} \text{ per m}^3$$

Hence, concentration of do nor atoms is $881 \times 10^{19}$ per m$^3$.

**1.4** If do nor-type impurity is added to the extent of one atom per $10^7$ silicon atom, calculate the resistivity of *N*-type silicon. What is the percentage increase in conductivity compared to the intrinsic silicon at 300 K? Given that $n_i = 1.5 \times 10^{16}$ per m$^3$,
$\mu_n = 0.15$ m$^2$/Vs, $\mu_P = 0.04$ m$^2$/Vs. Atomic density of Si at 300 K is $5 \times 10^{28}$ per m$^3$.

***Solution***

For intrinsic silicon, the conductivity

$$\delta_i = e\ n_i\ (\mu_n + \mu_P) = 1.6 \times 10^{-19} \times 1.5 \times 10^{16}\ (0.15 + 0.02)$$
$$= 0.456 \times 10^{-3}\ S_2$$
$$= 0.456 \times 10^{-2}\ S$$

Hence the resistivity of intrinsic silicon is

$$\rho_i = \frac{1}{\delta_i} = 2.193 \times 10^3\ \Omega\text{m}$$

Density of do nor impurity

$$n = N_D = \frac{5\times10^{28}}{10^7} = 5\times10^{21} \text{ per m}^3$$

The density of charge carries $n_i = 1.5 \times 10^{16}$ per m$^3$

The hole (minority carrier) density

$$P = \frac{n_i^2}{N_D} = \frac{(1.5\times10^{16}}{5\times10^{21}} = 0.45\times10^{11} \text{ per m}^3$$

As $n >> P$, $P$ can be neglected

$\therefore$ Conductivity of doped silicon semiconductor is

$$\delta = e_n\ \mu_n + e_P\ \mu_P = e_n\ \mu_m$$
$$= 1.6 \times 10^{-19} \times 5 \times 10^{21} \times 0.15$$

$= 1.2 \times 10^2$ S

$\therefore$ resistivity of *N*-type silicon is $\frac{1}{\delta} = \frac{1}{1.2 \times 10^2}$

$= 0.83 \times 10^{-2}$ W*m

Percentage increase in conductivity

$= (0.83 - 0.0456)\ 10^{-2}/0.0456 \times 10^{-2} \times 100 = 1720\%$

## EXERCISES

1. Write and explain Fermi-Dirac function. What is Fermi level?
2. What do you mean by (i) Valence band, (ii) conduction band and (iii) Forbidden energy gap?
3. Explain the behaviour of insulators, conductors and semiconductors with the help of energy bands.
4. Explain the bonds in semiconductors with suitable diagram.
5. What is meant by intrinsic and extrinsic semiconductor? Explain the majority and minority carriers in semiconductors.
6. Describe the properties of *P*-type and *N*-type semiconductors.
7. Define the following terms: (a) Diffusion (b) Potential barrier (c) Depletion layer.
8. At high temperature an extrinsic semiconductor behaves like an intrinsic semiconductor—Why?
9. Explain the mechanism of current flow in biased (a) *P*-type, and (b) *N*-type semiconductor.
10. Describe the behaviour of *PN* junction under (a) Forward biasing (b) Reverse biasing.
11. What is *p-n* junction diode? Explain how a barrier field appears across a *p-n* junction.
12. State and explain Boltzman diode equation.
13. What do you mean by capacitance of a *pn* junction diode?
14. Explain the principle of avalanche breakdown.
15. What is a tunnel diode? What are the advantages of a tunnel diode?
16. Write notes on: (a) Hot Carrier diodes (b) Point contact diodes (c) Schottky Barier diodes (d) Pin diodes and (e) Varactors.

## MULTIPLE CHOICE QUESTIONS

1. If the temperature of an *N*-type semiconductor is increased then it becomes [WBUT 2007] [WBUT – 2010]

   (a) more *n*-type (b) *p*-type
   (c) intrinsic (d) none of these

2. *SI* unit of mobility of charge carriers is [WBUT 2007]

   (a) ** (b) $m^2V^{-1}s^{-2}$
   (c) $m^2s^{-1}$ (d) $m^2V^{-1}s^{-1}$

3. The density of majority carriers in a *P*-type semiconductor depends on the [WBUT 2007]
   (a) donor impurity atom concentration
   (b) intrinsic atom concentration
   (c) doping technique
   (d) acceptor impurity atom concentration
4. Barrier potential of Ge diode is [WBUT 2007, 2011]
   (a) 0.3 V (b) 0.7 V
   (c) 0.4 V (d) 0 V
5. Avalanche breakdown primarily depends on the phenomena of [WBUT 2007, 2013]
   (a) ionization (b) particle collision
   (c) impurity doping (d) direct rupture of covalent bond
6. Fermi level of an N-type semiconductor lies [WBUT 2008]
   (a) near the conduction band edge
   (b) near the valence band edge
   (c) at the middle of the band gap
   (d) none of these
7. Band gap energy in eV for a semiconductor is around [WBUT 2008]
   (a) 0.5 (b) 1
   (c) 10 (d) none of these.
8. In an idea1 voltage controlled voltage source, the value of $R_1$ and $R_o$ tend to [WBUT 2008]
   (a) 0, 0 (b) 0, ∞
   (c) ∞, 0 (d) ∞ ∞
9. If a resistor has the colour code red–red–brown, the value of the resistor equals [WBUT 2008]
   (a) 22 ohms (b) 2.2 ohms
   (c) 220 ohms (d) 2.2 kohms
10. Band gaps of silicon and germanium are [WBUT 2009, 2012]
    (a) 0.67 eV and 1.1 eV (b) 5.89 eV and 4.6 eV
    (c) 0.87 eV and 6.78 eV (d) 0.54 eV and 0.7861 eV
11. Semiconductors have [WBUT 2009]
    (a) zero temperature coefficient of resistance
    (b) positive temperature coefficient of resistance
    (c) negative temperature coefficient of resistance
    (d) none of these
12. Temperature coefficient of resistance for intrinsic semiconductor is [WBUT 2009]
    (a) positive (b) negative
    (c) infinite (d) does not depend on temperature

13. Forbidden energy pap of silicon at 0 K is [WBUT 2010]
    (a) 0.78 eV (b) 1.2 eV
    (c) 1.5 eV (d) 0.3 eV
14. The major part of current flowing in an intrinsic semiconductor is due to the drift of [WBUT 2010]
    (a) conduction–band electrons
    (b) conduction–band holes
    (c) valence–band electrons
    (d) valence–band electrons
15. If a resistor has the colour code yellow–violet–gold, the value of the resistor is [WBUT 2010]
    (a) 47 (b) 0.47
    (c) 470 (d) 4.7
16. Barrier potential of a Si diode is [WBUT 2013]
    (a) 0.3 V (b) 0.7 V
    (c) 0.4 V (d) 0 V
17. If a resistor has the colour code brown – black – red, the value of the resistor equals [WBUT 2010, 2011]
    (a) 1000 (b) 10 k
    (c) 110 (d) 100
18. The sharing of electrons occurs between neighbouring atoms for
    (a) covalent bonds (b) ionic bonds
    (c) metallic bonds (d) van der Waals bonds
19. The atomic number of an atom is equal to
    (a) number of protons (b) atomic mass
    (c) number of neutrons (d) atomic weight
20. The Fermi level in a metal is about
    (a) 5 eV (b) 0.5 eV
    (c) 1 eV (d) 10 eV
21. In an *N*-type semiconductor, the position of Fermi level lies
    (a) above the centre of the energy gap
    (b) below the centre of the energy gap
    (c) in the middle of the energy gap
    (d) anywhere in the energy gap
22. In a *P*-type semiconductor, the position of Fermi level lies [WBUT 2014]
    (a) above the centre of the energy gap
    (b) below the centre of the energy gap
    (c) in the middle of the energy gap
    (d) anywhere in the energy gap

23. In an intrinsic semiconductor the Fermi level lies
    (a) above the centre of the energy gap
    (b) below the centre of the energy gap
    (c) in the middle of the energy gap
    (d) anywhere in the energy gap
24. The electron hole pairs are generated by
    (a) ionisation (b) thermal agitation
    (c) recombination (d) doping
25. The donor impurities
    (a) generate electrons (b) generate holes and electrons
    (c) generate holes (d) none of the above
26. The acceptor impurities
    (a) generate electrons
    (b) generate holes
    (c) generate holes and electrons
    (d) none of the above
27. The current density of a currentcarrying conductor due to electric field is
    (a) $J = \sigma^2 E$ (b) $J = \sigma E^2$
    (c) $J = \sigma E$ (d) $J = \sigma / E$
28. At 0 K an intrinsic semiconductor behaves as a/an
    (a) conductor (b) insulator
    (c) semiconductor (d) any of the above
29. The band gap energy of germanium at 0 K is
    (a) 0.66 eV (b) 0.5 eV
    (c) 0 eV (d) 1.2 eV
30. Fermi Dirac distribution function at 0 K is
    (a) step (b) exponential
    (c) parabolic (d) linear
31. The energy distribution of electrons obey
    (a) Bose–Einstein statistics
    (b) classical statistics
    (c) Maxwell–Boltzmann statistics
    (d) Fermi–Dirac statics
32. The temperature coefficient of resistance of silicon is
    (a) negative (b) positive
    (c) zero (d) infinity
33. The band gap energy of silicon at 300K is
    (a) 0.67 eV (b) 0.12 eV
    (c) 1.12 eV (d) 3.12 eV

34. The process of adding impurities to an intrinsic semiconductor is called
(a) annealing (b) etching
(c) doping (d) alloying
35. Which of the following trivalent elements is mostly used for P- type doping of silicon?
(a) Boron (b) Indium
(c) Silicon (d) Germanium
36. Which of the following elements does not have four valence electrons?
(a) Boron (b) Carbon
(c) Silicon (d) Germanium
37. The unit of diffusivity in the SI system of units is
(a) $Vm^{-1}$ (b) $m^2V^{-1}s^{-1}$
(c) $m^2s^{-1}$ (d) $Vs^{-1}$
38. Equation of continuity in a semiconductor signifies
(a) conservation of momentum of mobile charge carriers
(b) conservation of energy of mobile charge carriers
(c) conservation of charge of mobile charge carriers
(d) conservation of charge of ionized donors and acceptors
39. The band gap energy of a semiconductor
(a) decreases with increase in temperature
(b) decreases with decrease in temperature
(c) increases with increase in temperature
(d) increases with decrease in temperature
40. Which of the following elements is doped in silicon to make it *P*-type?
(a) Zinc (b) Boron
(c) Arsenic (d) Phosphorus
41. The energy of an electron in an orbit of radius $r$ and atomic number $Z$ is proportional to
(a) $Z^2$ (b) $Z^2r$
(c) $r^2$ (d) $Zr$
42. The most commonly used semiconductor material in manufacturing of electronic devices is
(a) boron (b) germanium
(c) gallium arsenide (d) silicon
43. Diffusibility and mobility are related by
(a) Einstein's equation (b) Gauss's law
(c) Continuity equation (d) Poisson's equation
44. The electron and hole concentrations in an intrinsic semiconductor are $n_i$ and $p_i$ respectively. When doped with a $p$-type material if these change to $n$ and $p$ respectively then
(a) $np_i = n_ip$ (b) $np = n_ip_i$
(c) $n + n_i = p + p_i$ (d) $n + p = n_i + p_i$

45. The hole concentration in p-type silicon is $2.25 \times 10^{15}$ cm$^{-3}$. if intrinsic carrire concentration is $1.5\ \mu\ 10^{10}$ cm$^{-3}$, the electron concentraction is
    (a) $10^{10}$ cm$^{-3}$ (b) $10^5$ cm$^{-3}$
    (c) $1.5 \times 10^{25}$ cm$^{-3}$ (d) 0 cm$^{-3}$
46. An electric field of 100 mV/m is applied to a copper wire of conductivity $5.7 \times 10^7$ S/m. The current density is
    (a) $5.7 \times 10^7$ A/m$^2$ (b) $877 \times 10^{-8}$ A/m$^2$
    (c) $57 \times 10^5$ A/m$^2$ (b) $114 \times 10^7$ A/m$^2$
47. An electric field of 50 mV/m is applied to a copper wire of conductivity 5.7 $10^7$ S/m and electron mobility of 0.0035 m$^2$/V-s. The charge density of free electrons per m$^3$ is
    (a) $0.0125 \times 10^{26}$ (b) $3.56 \times 10^{26}$
    (c) $1.6 \times 10^{19}$ (d) $1017.86 \times 10^{26}$
48. If The principal quantum number is $n$, the maximum number of electrons in a shell is
    (a) $8n^2$ (b) $4n^2$
    (c) $2n^2$ (d) $n^2$
49. If $N$ be the number of energy states then maximum number of electrons that can occupy these energy states is
    (a) $2N$ (b) $N$
    (c) $N/2$ (d) $4N$
50. If $n_i = 1.5 \times 10^{16}$ m$^{-3}$, $\mu_n = 0.5\ m^2$/Vs, $\mu_p = 0.045$ m$^2$/Vs then the resistivity of intrinsic silicon semiconductor at 300K is
    (a) $0.468 \times 10^{-3}$ Ωm (b) $2.137 \times 10^{-3}$ Ω m
    (c) $2.137 \times 10^3$ Ωm (d) 468 Ω m
51. An n-type semiconductor is [WBUT 2012]
    (a) negatively charged (b) positively charged
    (c) neutral

## ANSWERS

| | | | | |
|---|---|---|---|---|
| 1. (c) | 2. (d) | 3. (d) | 4. (a) | 5. (a) |
| 6. (a) | 7. (b) | 8. (c) | 9. (c) | 10. (a) |
| 11. (c) | 12. (b) | 13. (a) | 14. (a) | 15. (d) |
| 16. (b) | 17. (a) | 18. (a) | 19. (a) | 20. (d) |
| 21. (a) | 22. (b) | 23. (c) | 24. (b) | 25. (a) |
| 26. (b) | 27. (c) | 28. (b) | 29. (a) | 30. (a) |
| 31. (d) | 32. (a) | 33. (c) | 34. (c) | 35. (d) |
| 36. (a) | 37. (c) | 38. (c) | 39. (a) | 40. (b) |
| 41. (a) | 42. (d) | 43. (a) | 44. (c) | 45. (b) |
| 46. (c) | 47. (d) | 48. (c) | 49. (a) | 50. (c) |
| 51. (a) | | | | |

## UNIVERSITY QUESTIONS WITH ANSWERS

1. What is meant by cut-in voltage in a *pn* junction diode? [WBUT 2006]
   *Answer:* The cut-in voltage in a *pn* junction diode is the minimum amount of forward voltage which is required by a *pn* junction diode to start conduction. When the applied forward voltage is greater than the cut-in voltage, the barrier becomes thinner and current starts to increase rapidly through the junction as depicted in Fig. 1.

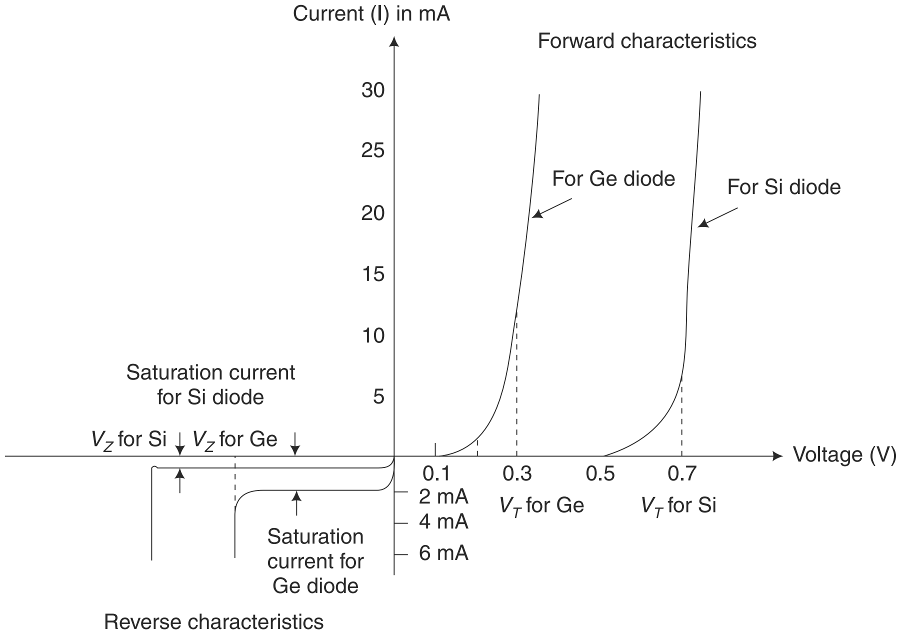



**Fig. 1** For Ge, the cut-in voltage is 0.3 volt and for Si, it is 0.7 volt.

2. (a) Why is there a reverse saturation current in a *pn* junction? Does it vary with reverse bias and temperature? [WBUT 2004]

   *Answer:* The reverse saturation current in a *pn* junction is generated due to the movement of minority carriers. The minority carrier of *p* region is electrons and holes are the minority carrier of the *n* region. The minority carriers cross the junction and reach the opposite side. Therefore, small reverse current flows through the *pn* junction and this current known as a reverse saturation current.

   This reverse saturation current depends on junction temperature. The reverse saturation current increases with temperature and this current almost doubles for every $10^{o}$C increase in temperature.

(b) The reverse saturation current at 300 K of a *pn* junction diode (Ge) is 5 $\mu$A. Find the voltage to be applied across the junction to obtain a forward current of 50 mA. [WBUT 2004]

*Answer:* Given: Reverse saturation current $I_o = 5 \times 10^{-6}$ A and forward current = 50 mA

The forward biased voltage $V = 0.2$ V

The current flowing through the diode is

$$I = I_o \left( \frac{V}{e^{\eta V_T - 1}} \right)$$

Or, $$50 \times 10^{-3} = 5 \times 10^{-6} \left( \frac{V}{e^{\eta V_T - 1}} \right)$$

Or, $$\frac{V}{e^{\eta V_T}} = \frac{50 \times 10^{-3}}{5 \times 10^{-6}} + 1 = 10001$$

After taking log on both sides of the above equation, we get

The volt equivalent of temperature is

$$V = \eta V_T \log_e (10001)$$

$$V_T = \frac{T}{11600} = \frac{300}{11600} \text{ V} = 0.026 \text{ V}$$

After substituting the value of $V_T = 0.026$ V and $\eta = 1$, we get

$$\text{V} = \eta V_T \log_e (10001)$$
$$= 1 \times 0.026 \times \log_e (10001) = 0.02394 \text{ V}$$

The applied voltage across the junction is = 0.02394 V

3. (a) What is the typical value of threshold voltage of a Ge diode? [WBUT 2005]

*Answer:* 0.3 V.

(b) Which type of carrier is responsible for reverse saturation current? [WBUT 2005]

*Answer:* Minority carrier.

(c) Why must an electric field exist in a graded semiconductor? [WBUT 2005]

*Answer:* When doping of a semiconductor is non-uniform or graded at steady state situation and there is no excitation, there will be no steady movement of charge though the carrier moves randomly due to thermal agitation.

Therefore, the total hole current is zero. For this, there must be a hole drift current which is equal and opposite of the diffusion current. But conduction current requires an electric field. As a result of non-uniform doping, an electric field is generated within the semiconductor.

4. (a) What is your concept about an ideal diode? How does it differ from an actual one? [WBUT 2007, 2009]

*Answer:*

*Ideal Diode:* It is a two-terminal solid-state device and permits only unidirectional conduction. If offers zero resistance when forward biased and infinite resistance when reverse biased.

No diode can act as an ideal diode. An actual diode neither can offer zero resistance when forward biased nor infinite resistance when reverse biased.

5. How does the depletion layer width change with doping concentration of a *pn* junction diode? Draw the ideal diode characteristic curve. [WBUT 2008]

*Answer:* When $N_A$ is the acceptor doping concentration and $N_D$ is the donor doping concentration and the contact potential $V_o$ remains constant, the depletion width is $W$.

In uniformly doping abrupt *pn* junction, the width of the depletion region is

$$W = \left[\frac{2\varepsilon V_o}{q}\,\frac{N_A + N_D}{N_A N_D}\right]^{1/2}$$

*Ideal Diode Characteristic:* Refer Section 1.29.

6. Differentiate between avalanche multiplication and Zener breakdown. [WBUT 2003]

*Answer:*

| *Avalanche multiplication* | *Zener breakdown* |
|---|---|
| Avalanche multiplication takes place in lightly doped *pn* junctions. | Zener breakdown takes place in highly doped *pn* junctions. |
| In avalanche multiplication, tunneling of electron is negligible. | In Zener breakdown, tunneling of electrons occurs from *p* side conduction band constituting a reverse current from *n* to *p*. |
| When the electric field in the transition region is large, an electron entering from *p* side may be accelerated to high enough kinetic energy to cause an ionizing collision with the lattice. This results in carrier multiplication just like a chain reaction and generates numerous electron hole pair. | Zener breakdown occurs only if the junction is reverse biased |

7. Distinguish between Zener diode and ordinary diode. Explain the working principle of Zener diode. [WBUT 2007]

*Answer:* Refer Article 1.29.

Zener diode is like an ordinary P-N junction diode except that it is properly doped so as to have a sharp reverse breakdown voltage and designed for operation in the' breakdown region in reverse-bias condition. A zener diode is always connected in reverse bias condition. When the reverse-bias voltage is gradually increased, a point is reached when the sharp junction breakdown occurs and revese current increases abruptly.

8. (a) Explain why semiconductors act as insulators at 0 K. [WBUT 2003]

   *Answer:* Refer Article 1.12.

   (b) What happens when an extrinsic semiconductor is heated? [WBUT 2003]

   *Answer:* If an extrinsic semiconductor is heated, the applied thermal energy is absorbed by the semiconductor and helps the electrons to reach the conduction band. Consequently, the number of free electrons increases which increases the conductivity of the material. Due to continuous increase in temperature, a situation may be developed when the thermally generated electron concentration will be very high and the concentration of holes and electrons in the semiconductor will become nearly equal. Therefore, when the temperature increases continuously, the extrinsic semiconductor behaves like an intrinsic one.

   (c) State the mass–action law. [WBUT 2003, 2005]

   *Answer*: Refer Article 1.21.

9. (a) Explain clearly the differences in band structure of a metal, an insulator and a semiconductor. [WBUT 2003, 2013]

   *Answer*: Refer Article 1.11.

   (b) What is the position of the Fermi level in an intrinsic semiconductor? How does the position change when (i) donors, and (ii) acceptors are added to a semiconductor? [WBUT 2003, 2014]

   *Answer*: Refer Article 1.6.

   The position of Fermi level in an intrinsic semiconductor lies in the middle of the energy gap $(E_g)$ as the concentrations of free electrons and holes are equal.

   In donor type (N-type) semiconductor, the position of Fermi level lies near the conduction band. Similarly, in acceptor type (P-type) semiconductor, the position of Fermi level lies near the valence band.

   (c) At 300 K, the intrinsic carrier concentration of Si is $1.5 \times 10^{16}$ m$^{-3}$. If the electron and hole nobilities are 0.13 and 0.05 m/Vs respectively, determine the intrinsic resistivity of Si at 300 K [WBUT 2003, 2014]

   *Answer:*

   Given: = $1.5 \times 10^{16}$ per m$^3$, m$^2$/V-s, and m$^2$/V-s

   For the intrinsic semiconductor, $n = p = n_i$

   The conductivity of an intrinsic semiconductor is

   $$\sigma_i = n_i \,.\, q\,(\mu_n + \mu_p) \qquad \text{as } q = 1.6 \times 10^{-19}$$

$= 1.5 \times 10^{16} \times 1.6 \times 10^{-19} \times (0.13 + 0.05)$ (ohm-metre)$^{-1}$

$= 0.432 \times 10^{-3}$ (ohm-meter)$^{-1}$

The intrinsic resistivity of Si is

$$\rho_i = \frac{1}{\sigma} = \frac{1}{0.432 \times 10^{-3}}\ \Omega\ \text{cm}$$

10. (a) What do you mean by an intrinsic semiconductor? Will it behave as an insulator at any temperature? [WBUT 2004, 2014]

*Answer:* Refer Article 1.17 and 1.23.

An intrinsic semiconductor is a pure semiconductor and its conductivity lies in between an insulator and metal or conductor. In an intrinsic semiconductor, no impurity is present. Intrinsic semiconductors behave just like an insulator at low temperature. At absolute temperature, the conduction band is empty. As there are no free electrons in the conduction band, it behaves as an insulator.

(b) Explain the mechanism of current flow in a biased N-type semiconductor. [WBUT 2004]

*Answer:* Refer Article 1.22.1.

(c) Find the concentration of donor atoms to be added to an intrinsic Ge sample to produce an N-type material of conductivity 480 S/m. The electron mobility in N-type Ge is 0.38 m$^2$/v.s. [WBUT 2004]

*Answer:* For N-type material, conductivity is

$$\sigma = nq\mu_n$$

where $q$ = charge of an electron

$n$ = concentration of electron

$\mu_n$ = mobility of electron

*Given*: $\sigma = 480$ s/m, $\mu_n = 0.38$ m$^2$/v.s and $q = 1.6 \times 10^{-19}$ C

After substituting the values of $\sigma$, $\mu_n$ and $q$ we get

$480 = n \times 1.6 \times 10^{-19} \times 0.38$

$$n = \frac{480}{1.6 \times 10^{-19} \times 0.38} = 789.47 \times 1019\ \text{m}^{-3}$$

The concentration of donor atoms

$N_D = n = 789.47 \times 10^{19}$ m$^{-3}$

11. How many electrons are available in the valence shell of Ge and Si? [WBUT 2005]

*Answer:* 4 electrons are available.

12. (a) What do you mean by bandgap of a semiconductor? [WBUT 2005]

(b) How can you determine it ?

*Answer:* The bandgap of a semiconductor is the difference of energy between the lower end of the conduction band and the upper end of the valence band. The minimum energy needed for an electron to reach the conduction band from the valence band is the bandgap energy.

The bandgap of a semiconductor can be determined from the following relation:

$$E_G = \frac{1.24}{\lambda}\ \text{eV}$$

where, $E_G$ is the bandgap and $\lambda$ is the wavelength.

13. State mass-action law and explain if it is applicable in a graded semiconductor also. [WBUT 2005]

*Answer:* Refer Article 1.21.

In graded junction there is a built-in potential between two sections and a contact potential is developed. The value of the contact potential is

$$V_0 = V_T \ \text{In}\ \frac{N_A N_D}{n_1^2}$$

14. (a) What are the majority and minority carriers for a phosphorous-doped germanium semiconductor? [WBUT 2006]

*Answer:* Majority carrier is electron.

Minority carrier is hole.

(b) Why do extrinsic conductions behave as good conductors? [WBUT 2006]

*Answer:* In an extrinsic semiconductor, there are sufficient numbers of free carriers. These free carriers are either electrons or holes. These carriers conduct current through it.

15. (a) The intrinsic carrier concentration of a semiconductor at 300 k is $2.5 \times 10^{19}$ per metre cube, and the motilities of electrons and holes are 0.38 $m^2$/V-s and 0.18 $m^2$/V-s. For the two different types of carries, calculate the conductivity due to holes and electrons separately. [WBUT 2006]

*Answer:*

Given: $n_i = 25 \times 10^{19}$ per $m^3$, $\mu_n = 0.38$ $m^2$/V-s, and $\mu_p = 0.18$ $m^2$/V-s

The conductivity due to electrons is

as

$$\sigma_i = n_i \,.\, q\mu_n \qquad \text{as } q = 1.6 \times 10^{-19}$$

$$= 25 \times 10^{19} \times 1.6 \times 10^{-19} \times 0.38\ \text{(ohm-metre)}^{-1}$$

$$= 15.2\ \text{(ohm-metre)}^{-1}$$

The conductivity due to holes is

$$\sigma_i = n_i \,.\, q\mu_p \qquad \text{as } q = 1.6 \times 10^{-19}$$

$$= 25 \times 10^{19} \times 1.6 \times 10^{-19} \times 0.18\ \text{(ohm-metre)}^{-1}$$

$$= 7.2\ \text{(ohm-metre)}^{-1}$$

16. (a) Explain with appropriate diagrams why a semiconductor acts as an insulator at about 0 K and why its conductivity increases with in creasing temperature. [WBUT 2007, 2009, 2010, 2011]

*Answer:* Refer Article 1.12.

(b) If a donor type of impurity is added to the extent of one atom in $10^7$ Ge atoms, calculate the resistivity and conductivity of the N-type material so formed. What is its percentage increase in conductivity compared to the intrinsic Ge at 300 K? Given at 300 K, atoms/m$^3$ of

Ge = $4.4 \times 10^{28}$, $n_i = 2.5 \times 10^{19}$/m$^3$, 0.38 m$^2$/v-s and 0.18 m$^2$/v-s.

[WBUT 2007, 2010, 2011]

*Answer:*

Given: $n_i = 2.5 \times 10^{19}$ per m$^3$, $\mu_n = 0.38$ m$^2$/V-s, and $\mu_p = 0.18$ m$^2$/V-s

For the intrinsic semiconductor, $n = p = n_i$

The conductivity of an intrinsic semiconductor is

$$\sigma_i = n_i \,.\, q\,(\mu_n + \mu_p) \text{ as } q = 1.6 \times 10^{-19}$$
$$= 2.5 \times 10^{16} \times 1.6 \times 10^{-19} \times (0.38 + 0.18) \text{ (ohm-meter)}^{-1}$$
$$= 2.24 \text{ (ohm-meter)}^{-1}$$

The intrinsic resistivity of Ge is

$$\rho_i = \frac{1}{\sigma} = \frac{1}{2.24} = 0.446\ \Omega \text{ cm}$$

Density of donor impurity is

$$N_D = \frac{4.4 \times 10^{28}}{10^7} = 4.4 \times 10^{21}/\text{cm}^3$$

The density of charge carriers $n_i = 2.5 \times 10^{19}$/m$^3$

The minority carrier (hole) density in Ge semiconductor with donor impurity $N_D = 4.4 \times 10^{21}$/m$^3$ is

$$p = \frac{n_1^2}{N_D} = \frac{(2.5 \times 10^{19})^2}{4.4 \times 10^{21}}/\text{m}^3$$

As $n >> p$, the conductivity of doped Ge semiconductor is

$$\sigma = qn\,\mu_n \quad \text{as } n = N_D = 4.4 \times 10^{21}/\text{m}^3$$
$$= (1.6 \times 10^{-19}) \times (4.4 \times 10^{21}) \times 0.38 = 2.6752 \times 10^2 \text{ S/m}$$

The resistivity $\rho = \dfrac{1}{\sigma} = \dfrac{1}{2.6752 \times 10^2}$ ohm-metre

$$= 0.3738 \times 10^{-2} \text{ ohm-meter}$$

The percentage increase in conductivity compared to the intrinsic Ge at 300 K is

$$\frac{2.6752 \times 10^2 - 2.24}{2.24}\% = 11842.86\%$$

17. Explain drift and diffusion of charge carriers in semiconductors. Derive an expression for the electron current due to drift and diffusion.

[WBUT 2008, 2014]

*Answer:* Refer Articles 1.18 and 1.19.

18. What is the importance of forbidden energy gap in material science ? What are the forbidden energies of Si and Ge'?

    *Answer:* An energy gap exists between the conduction band and valence band and this energy gap is called forbidden energy gap. When atomic spacing is decreased, the forbidden energy gap decreases and tends to zero with further reduction in spacing (say '$r_3$') as show in the Fig. 2. The two energy bands will overlap when inter atomic spacing is small enough.

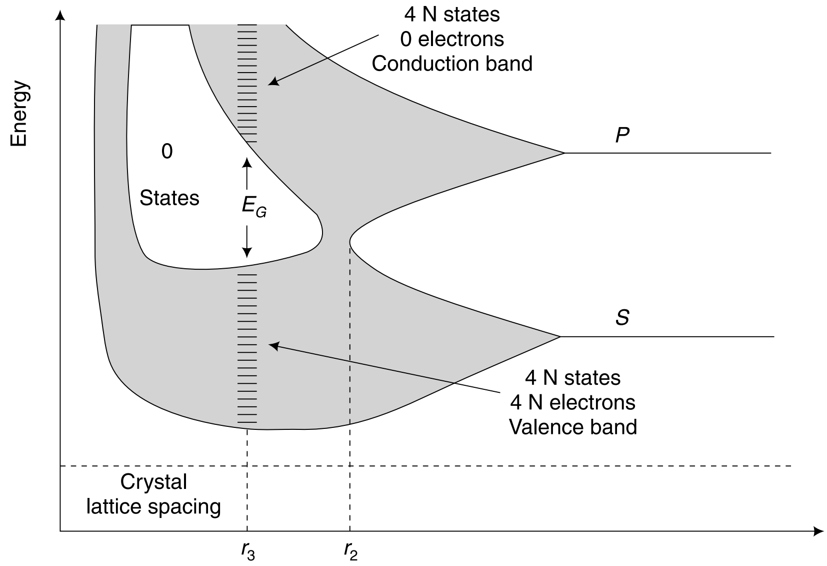



**Fig. 2** ***Energy levels in crystal as a function of inter-atomic spacing***

When the interatomic spacing is further reduced, the interaction between the atoms are extremely large and band structure is shown in Fig. 1. The forbidden energies of Si and Ge are 1.08 eV and 0.66 eV respectively.

19. Define Fermi-level in a semiconductor [WBUT 2009]

    *Answer:* Refer Article 1.6

20. Write short notes on

    (a) Diffusion current in PN junction. [WBUT 2009, 2014]

    (b) Continuity equation for holes [WBUT 2009]

    *Answer:* (a) Refer Articles 1.27 and 1.28.

    (b) Refer Article 1.21.

21. Derive the expressions for electrical conductivity of [WBUT 2009]

    (a) an intrinsic semiconductor

    (b) an N-type semiconductor

    (c) a P-type semiconductor

    *Answer:* Refer Article 1.19.

22. (a) Explain the term *drift* related to semiconductors. [WBUT 2009]

*Answer:* Refer Article 1.18.

(b) What is electrical conductivity? Derive the expression for electrical conductivity of a semiconductor. [WBUT 2009]

*Answer:* Refer Articles 1.18 and 1.19.

(c) Show the total electron current density is equal to the sum of drift and diffusion current density. [WBUT 2009]

*Answer:* Refer Article 1.18.

(d) Find

(i) conductivity, and

(ii) resistance of a bar of pure silicon of 1 cm length and 1 mm$^2$ cross-sectional area at 300 K. Given $\mu_n$ = 0.13 m$^2$/Vs, $\mu_p$ = 0.05 m$^2$/Vs, $n_i = 1.5 \times 10^{16}$/m$^3$

[WBUT 2009]

Given: $n_i = 1.5 \times 10^{16}$ per m$^3$, $\mu_n$ = 0.13 m$^2$/V-s, and $\mu_p$ = 0.05 m$^2$/V-s

For the intrinsic semiconductor, $n = p = n_i$

The conductivity of an intrinsic semiconductor is

$$\sigma_i = n_i \cdot q\,(\mu_n + \mu_p) \qquad \text{as } q = 1.6 \times 10^{-19}$$

$$= 1.5 \times 10^{16} \times 1.6 \times 10^{-19} \times (0.13 + 0.05) \text{ (ohm-metre)}^{-1}$$

$$= 0.432 \times 10^{-3} \text{ (ohm-metre)}^{-1}$$

The intrinsic resistivity of Ge is

$$\rho_i = \frac{1}{\sigma} = \frac{1}{0.432 \times 10^{-3}} = 2.3148 \times 10^3\ \Omega\text{m}$$

The resistance of the bar is equal to

$$R = \rho \frac{l}{A}$$

As l = 1 cm = $1 \times 10^{-2}$ m and $A$ = 1 mm$^2$ = $1 \times 10^{-6}$ m$^2$, the value of the

resistance is $= 2.3148 \times 10^3 \times \dfrac{1 \times 10^{-2}}{1 \times 10^{-6}} = 2.3148 \times 10^7\ \Omega$

23. Differentiate between Avalanche and Zener breakdowns.

[WBUT 2011, 2012, 2013, 2014]

***Solution:***

Zener breakdown occurs in a heavily doped p-n junction whereas avalanche breakdown occurs in lightly doped p-n junctions. In zener breakdown, the breakdown voltage is lower than that in avalanche breakdown. The temperature coefficient of breakdown voltage is negative in zener breakdown whereas it is positive in avalanche breakdown. Zener breakdown is caused by tunneling or field emission and avalanche breakdown is caused by impact ionization and avalanche multiplication.

24. (a) Explain with appropriate diagram why a semiconductor acts as an insulator at about 0 K and why its conductivity increases with increasing temperature. [WBUT 2011]

*Answer:* (a) Refer Article 1.12.

(b) If a donor type of impurity is added to the extent of one atom in 10 million Ge atoms, calculate the resistivity and conductivity of the N-type material so formed. What is the percentage of increase in the conductivity compared to the intrinsic Ge at 399 K. Given at 300 K, atoms/m$^3$ of Ge = $4.4 \times 10^{28}$, Ni = $2.5 \times 10^{19}$, $\mu = 0.38$ m$^s$/V-s, $\mu = 0.18$ m$^2$/V-s.

25. Write short note on Varactor diode. [WBUT 2011 2012, 2014]

*Anwer:* Refer Article 1.34.

26. (a) The given figure shows an electronic voltage regulator. The Zener diode may be assumed to require a minimum current of 25 mA for satisfactory operation. Find the value of $R$ required for satisfactory voltage regulation of the circuit. Assume $V_Z = 10$ V. [WBUT 2008]

*Answer:*

Given: $I_Z = 25$ mA and $V_Z = 10$ V

The current flow through load is

$$I_L = \frac{V_Z}{R_L} = \frac{10}{100} = 100 \text{ mA}$$

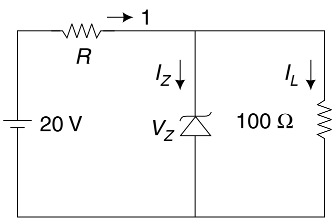



**Fig. 3**

Here, $I = I_Z + I_L$

Or, $I = 25$ mA + 100 mA = 125 mA

We can write the KVL equation

$20 = I_R + V_Z$

$$\text{Or, } R = \frac{20 - V_Z}{I} = \frac{20 - 10}{125 \times 10^{-3}} \text{ A} = 80\ \Omega$$

27. Write short notes on Zener diode used as a regulated dc power supply [WBUT 2009]

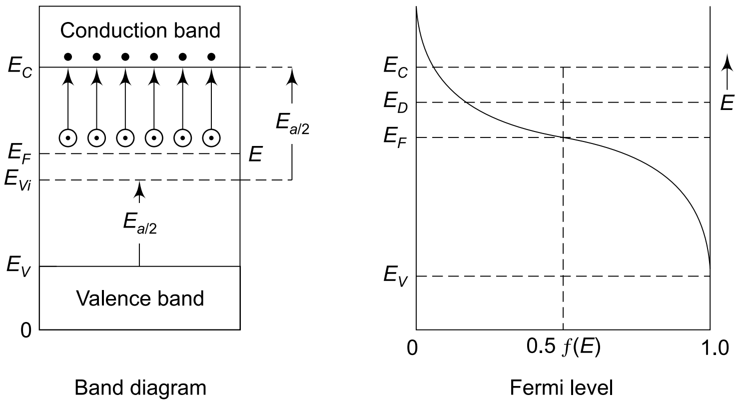
*Answer:* Refer Article 1.29.

28. What is the role of doping of impurities in pure silicon or germanium? Draw roughly the position of Fermi level for extrinsic semiconductor and explain.

*Answer:* Refer Article 1.22.

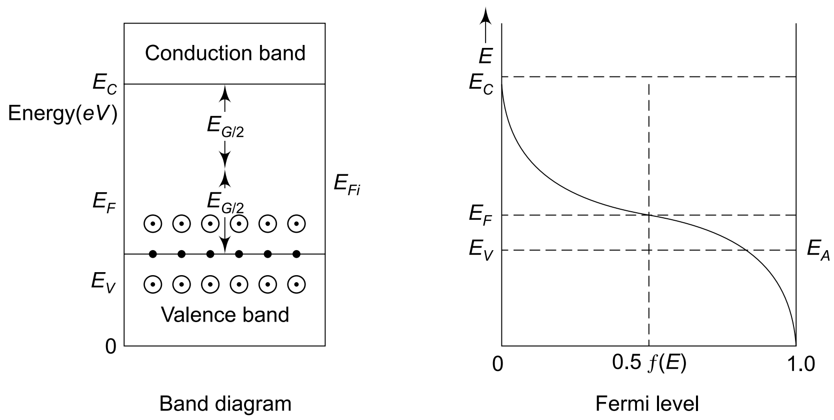
Fermi level is the measure of probability of occupancy of the allowed energy states. In *n*-type semiconductors, the number of free electrons in the conduction band is more than the number of holes in the valence band. Hence, the Fermi level must move closer to the conduction band to indicate that many energy states in that band are filled by the donor electrons and fewer holes exist in the valence band

In a *p*-type semiconductor, the number of holes in the valence band exceeds the number of free electrons in the conduction band. Hence, Fermi level must move from the centre of the forbidden gap closer to the valence band for a *p*-type material.




***n-type***

**Fig. 4**




***p-type***

**Fig. 5**

29. At 300 K. the intrinsic carrier concentration of silicon is $1.5 \times 10^{16}$ m$^{-3}$. If the electron and hole mobilities are 0.13 and 0.05 m$^2$V$^{-1}$s$^{-1}$ calculate the intrinsic resistivity of Si at 300 K. [WBUT 2012]
*Answer:* Conductivity $\delta_i = en_i(\mu_n + \mu_p) = 1.6 \times 10^{-19} \times 1.5 \times 10^{16}\,(0.13 + 0.05)$
$= 0.432 \times 10^{-3}$ s/m

$\therefore$ resistivity $= \dfrac{1}{\delta_i} = 2314.8\ \Omega$

30. What do you mean by intrinsic semiconductor? Explain drift and diffusion current for a semiconductor. [WBUT 2012, 2014]
*Answer:* Refer Articles 1.17 and 1.18.
31. Write a short note on each: [WBUT 2012, 2013, 2014]
(a) Fermi level (b) Junction capacitance
*Answer:* (a) Refer Section 1.6.
(b) Refer Article 1.31.

32. With the help of energy band diagram differentiate among conductor, semi conductor and insulator [WBUT 2013]

    *Answer*: Refer Article 1.7.

33. What is reverse saturation current? [WBUT 2013]

    *Answer:* Refer Article 1.29.

34. Determine the resistivity of germanium (a) in intrinsic condition of 300 K (b) with donor impurity of 1 in $10^7$ (c) with acceptor impurity of 1 in $10^8$. Given that for germanium at 300 K, $n_i = 2.5 \times 10^{13}$ cm$^{-3}$, $\mu_n = 3800$ cm$^2$/V-s, $\mu_p = 1800$ cm$^2$/V-s and number of germanium atoms/cm$^3$ = $4.4 \times 10^{22}$. [WBUT 2014]

    *Answer:* $n_i = 2.5 \times 10^{13}$/cm$^3$ $\mu_n = 3800$ cm$^2$/V-s $\mu_p = 1800$ cm$^2$/V-s

    (a) For intrinsic germanium semiconductor

$$\begin{aligned}\text{Conductivity} &= n_i q(\mu_n + \mu_p)\\ &= 2.5 \times 10^{13} \times 1.6 \times 10^{-19}\,(3800 + 1800)\\ &= 0.0224\ (\Omega\text{cm})^{-1}\end{aligned}$$

$$\therefore\quad \text{Resistivity} = \frac{1}{0.0224}\ \Omega\text{m} = 44.64\ \Omega\text{cm}$$

    (b) Donor impurity $N_D = \dfrac{4.4 \times 10^{22}}{10^7}/\text{cm}^3 = 4.4 \times 10^{15}/\text{cm}^3$

    The minority carrier density

$$p = \frac{n_i^2}{N_D} = \frac{(2.5 \times 10^{13})^2}{4.4 \times 10^{15}}/\text{cm}^3 = 1.42 \times 10^{11}/\text{cm}^3$$

    As $n >> p$ $(n = N_D = 4.4 \times 10^{15})$

    Conductivity of doped Ge semiconductor

$$\begin{aligned}\delta &= qn\mu_n\\ &= 1.6 \times 10^{-19} \times 4.4 \times 10^{15} \times 3800\\ &= 2.675\ (\Omega\text{cm})^{-1}\end{aligned}$$

$$\therefore\quad \text{Resistivity} = \frac{1}{2.675}\ \Omega\text{cm} = 0.3738\ \Omega\text{cm}$$

    (c) Number of donors $N_D = 4.4 \times 10^{15}$/cm$^3$

    Number of acceptors $N_A = \dfrac{4.4 \times 10^{22}}{10^8}/\text{cm}^3 = 4.4 \times 10^{14}/\text{cm}^3$

    The difference between donors & acceptors

$$= N_D - N_A = 4.4\,(10^{15} - 10^{14})/\text{cm}^3 = 39.6 \times 10^{14}/\text{cm}$$

    The number of free electrons $n = 39.6 \times 10^{14}$/cm$^3$

    The number of holes is

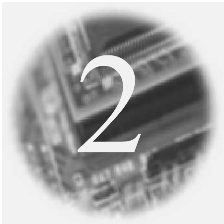
$$p = \frac{n_i^2}{N_D - N_A} = \frac{(2.5 \times 10^{13})^2}{39.6 \times 10^{14}} = 0.1578 \times 10^{12}/\text{cm}$$

$\therefore$ Conductivity $= q(n\mu_n + p\mu_p)$

$= 1.6 \times 10^{-19} \times \{39.6 \times 10^{14} \times 3800 + 0.1578 \times 10^{12} \times 1800\}$ (Ωcm)

$= 1.6 \times 10^{-19} \times 10^{12} \{39.6 \times 3800 \times 100 + 0.1578 \times 1800\} = 2.14.$ $(\Omega\text{cm})^{-1}$

$\therefore$ Resistivity $= \dfrac{1}{2.41}$ Ωcm = 0.4149 Ωcm



# 2 SEMICONDUCTOR DIODES AND THEIR ANALYSIS

## 2.1 INTRODUCTION

In the last chapter we have studied how a *pn* junction is formed. It has also been discussed that a *pn* junction conducts current easily when forward biased and practically no current flows through it when it is reverse biased. This unidirectional conduction characteristics of *pn* junction is similar to that of a vacuum diode. Therefore, a *pn* junction is called a *semiconductor* diode. Because of the peculiar property of a *pn* junction to conduct in one direction only, it is used for rectifications (i.e., conversion of ac voltage to dc voltage).

In this chapter, we will study the characteristics and applications of semiconductor diode.

## 2.2 AC TO DC CONVERSION

A dc supply is needed for many applications, viz. electroplating, electronic circuits, etc. A rectifier converts ac to dc and may be a single-phase circuit or a poly-phase circuit. When power requirement is low (less than 1 kW) a single-phase rectifier is used. For high power requirements poly-phase rectifiers are more suitable.

Generally, a complete rectifier circuit consists of a transformer, diodes, filter and a voltage regulator, connected in cascade. The functions of these devices are given below.

***Transformer*** It adjusts the ac voltage at the input of the rectifier so that appropriate dc output voltage can be obtained at the output of the rectifier.

***Rectifier*** It converts ac sinusoidal voltage to a pulsating dc voltage.

***Filter*** It smoothens the waveform by eliminating the ac components from the rectifier output.

***Regulator*** It maintains a constant dc voltage level at the output independent of load conditions or variations in the magnitude of ac supply.

## 2.3 DIODES IN A RECTIFIER CIRCUIT

A rectifier circuit generally uses semiconductor diodes (*p-n* junction). A diode has a low resistance in the forward direction and a very high resistance in the reverse direction. In all analysis, the resistance in the reverse direction is assumed to be infinite. Figure 2.1 shows the symbol for a diode. It conducts in the direction of the arrowhead.

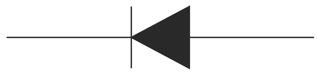


**Fig. 2.1** ***Diode symbol***

## 2.4 SPECIFICATION

A diode is said to be an ideal diode when it acts like a perfect conductor (with zero voltage across it) when forward biased and like a perfect insulator (with zero current through it) when it is reversed biased.

The *V-I* characteristics of an ideal diode are shown in Fig. 2.2(a). In a circuit an ideal diode acts like a switch. When the diode is forward biased, it is just like a closed switch, as shown in Fig. 2.2(b). Whereas, if it is reverse-biased, it acts like an open switch as shown in Fig. 2.2(c).

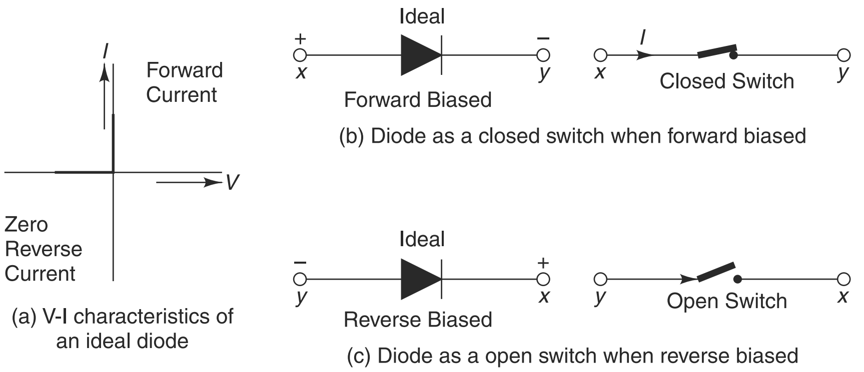



**Fig. 2.2**

In actual practice, no diode is an ideal diode i.e. neither it acts as a perfect conductor when forward-biased nor it acts as an insulator when it is reverse-biased. In other words, an actual diode offers a very small resistance (not zero) when forward biased and is called a *forward resistance*, whereas it offers a very high resistance (not infinite) when reverse-biased and is called a *reverse resistance*.

***1. Forward Resistance*** Under forward-biased condition, the opposition offered by the diode to the forward current is known as *forward resistance*. The forward current flowing through the diode may be constant (dc) or changing (ac) accordingly, the forward resistance may be (*i*) static or dc forward resistance, and (ii) dynamic or ac forward resistance.

(i) *Static or dc forward resistance*: The opposition offered by a diode to the direct current flow in forward bias condition is known as its dc forward resistance. It is measured by taking the ratio of dc voltage across the diode to the dc current flowing through it. Refer to the forward characteristics of a diode shown in Fig. 2.3. It is clear that for operating point *P*, the forward voltage is OC and the corresponding forward current is *OB*. Therefore, the static or dc forward resistance of the diode,

$$R_f = \frac{OC}{OB}$$

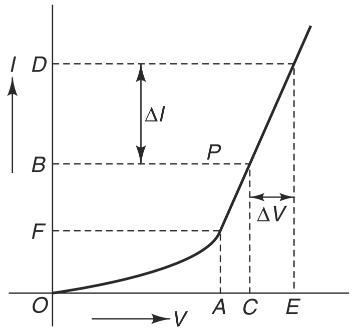



**Fig. 2.3** ***Forward characteristic of a diode***

(ii) *Dynamic or ac forward resistance*: The opposition offered by a diode to the changing current flow in forward bias condition is known as its ac forward resistance. It is measured by the ratio of change in voltage across diode to the resulting change in current through it. For instance, refer to the forward characteristics of a diode shown in Fig. 2.3. For an operating point *P*, the ac forward resistance is determined by varying the forward voltage away from the operating point and measuring the corresponding forward current.

Then dynamic or ac forward resistance, $r_f = \frac{CE}{BD} = \frac{\Delta V}{\Delta l}$

The value of forward resistance of a crystal diode is very small, ranging from 1 to 25 Ω.

***2. Reverse Resistance ($R_R$)*** Under reverse biasing, the opposition offered by the diode to the reverse current is known as reverse resistance. Ideally the reverse resistance of a diode is considered to be infinite. However in actual practice, the reverse resistance is not infinite because diode conducts a small leakage current (due to minority carriers) when reverse biased. The value of reverse resistance is very large as compared to forward resistance. The ratio of reverse to forward resistance is 100000: 1 for silicon diodes whereas it is 40000 : 1 for germanium diodes.

The important specifications of a rectifier circuit are dc output voltages, regulation, average and peak current in each diode, peak inverse voltage of each diode and ripple factor. The following analysis gives the expression for the output dc voltage from which we can define the other specification as follows:

(a) Regulation $= \frac{V_{\text{dco}} - V_{\text{dc}}}{V_{\text{dc}}} \times 100$ (2.1)

where $V_{\text{dco}}$ = dc output voltage at no load
$V_{\text{dc}}$ = dc output voltage at full load.

(b) Peak inverse voltage is the peak voltage which appears across the diode when it is not conducting. In other words, it is the maximum voltage that the diode can with stand under reverse bias condition.

(c) Ripple factor $(\gamma) = \frac{\text{r.m.s. value of ac components of voltage (or current)}}{\text{average value of voltage (or current)}}$ (2.2)

All the components of load current except $I_{dc}$ are ac components. Therefore, ripple factor is

$$\gamma = \frac{(I^2 - I_{dc}^2)^{0.5}}{I_{dc}} = \left(\frac{I^2}{I_{dc}^2} - 1\right)^{0.5} \tag{2.3}$$

## 2.5 ANALYSIS OF A HALF-WAVE RECTIFIER

Figure 2.4 shows a half-wave rectifier circuit in series with a resistance $R_L$. If the applied voltage $e = E_m$ sin ωt, then

$$i = \frac{E_m \sin \omega t}{R_f + R_L} = I_m \sin \omega t \text{ (for } 0 \le \omega t \le 0)$$

$$= 0 \qquad (\text{for } \pi \le \omega t \le 2\pi)$$

where $R_f$ = resistance of diode in forward direction

Also, $$I_m = \frac{E_m}{R_f + R_L} \tag{2.4}$$

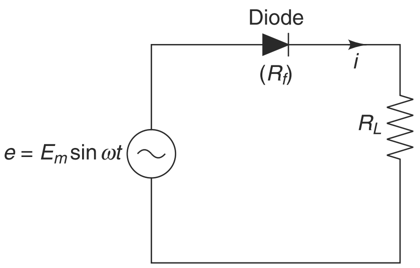



**Fig. 2.4** *Half-wave rectifier circuit*

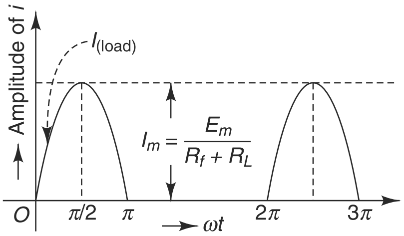



**Fig. 2.5** *Current waveform in half-wave rectifier circuit*

Figure 2.5 shows the load current waveform. $I_{dc}$ (i.e. the dc component of current) is the average value of $i$ over one full cycle of the wave shape.

$$I_{dc} = \frac{1}{2\pi}\int_0^{2\pi} i.d(\omega t) = \frac{1}{2\pi}\int_0^{\pi} I_m \sin(\omega t) + \frac{1}{2\pi}\int_{\pi}^{2\pi} (0)d(\omega t)$$

∴ $$I_{dc} = \frac{I_m}{\pi} = \frac{E_m}{\pi(R_f + R_L)} \text{ A} \tag{2.5}$$

$I$, the rms or effective value of current is given as

$$I = \left[\frac{1}{2\pi}\int_0^{2\pi} i^2.d(\omega t)\right]^{0.5} = \left[\frac{I_m^2}{2\pi}\int_0^{\pi} \text{sim}^2(\omega t)d(\omega t)\right]^{0.5}$$

or, $$I = \frac{I_m}{2} = \frac{1}{2}\frac{E_m}{(R_f + R_L)} \tag{2.6}$$

A dc ammeter connected in a rectifier circuit will read the average value, i.e., $I_{dc}$ whereas an ac ammeter in this circuit will read the rms value, i.e., $I$.

The voltage across the load resistance $R_L$ is ($iR_L$). Therefore the peak ($V_m$), average ($V_{dc}$) and rms ($V$) values of output voltage can be written directly from the above values of $I_m$, $I_{dc}$ and $I$. Thus

$$V_m = \frac{E_m R_L}{R_f + R_L} (= I_m R_L) \tag{2.7a}$$

$$V_{dc} = \frac{E_m R_L}{\pi (R_f + R_L)} (= I_{dc} R_L) \tag{2.7b}$$

$$V = \frac{E_m R_L}{2 (R_f + R_L)} (= I R_L) \tag{2.7c}$$

It is seen from Eq. 2.7b that the dc output voltage is not constant but depends on load resistance $R_L$.

PIV for half-wave rectifier is $V_m$

Also we can write, $$V_{dc} = \frac{E_m R_L}{\pi (R_f + R_L)} = \left( \frac{E_m}{\pi} - I_{dc} R_f \right) \tag{2.8}$$

At no load, the dc output voltage is ($E_m/\pi$), At load, $V_{dc}$ decreases linearly with increase of $I_{dc}$. If $R_f$ is small, regulation is good but if $R_f$ is considerable, regulation is poor(i.e., high).

$P_{dc}$, i.e. dc power delivered to the load is

$$P_{dc} = (I_{dc})^2 R_L = \frac{E_m^2 R_L}{\pi^2 (R_f + R_L)^2} \tag{2.9}$$

$P_{in}$, i.e. total power input to the circuit is

$$P_{in} = I^2 (R_f + R_L) = \frac{E_m^2}{4(R_f + R_L)} \tag{2.10}$$

the ratio ($P_{dc}/P_{in}$) is the rectifier efficiency ($\eta$).

$$\eta = \frac{P_{dc}}{P_{in}} = \left( \frac{2}{\pi} \right)^2 \frac{R_L}{R_f + R_L} = \frac{0.406}{1 + (R_f / R_L)} \tag{2.11}$$

When $R_L \gg R_f$, $\eta$ reaches its theoretical maximum value of 40.6%. The efficiency is low due to the following two reasons.

1. Only a part of the power delivered to load is dc power, the remainder is associated with the ac component of load current and is dissipated as heat. The factor $(2/\pi)^2$ or 0.406 in Eq. (2.11) accounts for this loss.
2. A part of the power is lost as heat in the diode. The factor $1/(1 + R_f/R_L)$ accounts for this loss.

Also, $$\frac{I}{I_{dc}} = \frac{\pi}{2}$$

$$\text{Ripple factor} = \sqrt{\left( \frac{I^2}{I_{dc}^2} - 1 \right)} = \left[ \left( \frac{\pi}{2} \right)^2 - 1 \right]^{0.5} = 1.21$$

The voltage across the diode, during the positive half-cycle is ($I_m R_f \sin \omega t$). During the negative half-cycle, the voltage across the diode is equal to transformer secondary voltage ($E_m \sin \omega t$). The peak inverse voltage is $E_m$ or ($\pi V_{dc}$).

The average or dc voltage across the diode is obtained as

$$V_{dc} = \frac{1}{2\pi}\left[\int_0^{\pi} I_m R_f \sin \omega t \, d(\omega t) + \int_{\pi}^{2\pi} E_m \sin \omega t \, d(\omega t)\right]$$

$$= \frac{1}{\pi}\left[(I_m R_f - V_m) = \frac{1}{\pi}\left[I_m R_f - I_m (R_f + R_L)\right]\right] \qquad (2.12)$$

$$= (-I_m R_L / \pi) = (-I_{dc} R_L)$$

It is thus seen that the dc voltage across the diode and dc voltage across load are equal in magnitude and opposite in sign. This has to be so since the sum of dc voltages across the circuit must be equal to zero (Kirchhoff's voltage law).

**2.1** In a half wave rectifier circuit, semiconductor diode feeds a load resistance of 500 ohms. The forward resistance of the diode is 100 ohms. While the resistance in the reverse direction is assumed to be infinite. It is fed from a supply having rms value of 100 V. Find: (a) the dc current through load; (b) the reading of a moving coil voltmeter connected across the load; (c) the reading of a moving coil voltmeter connected across the diode; (d) the dc power delivered to the load; and (e) the total power dissipated in the load.

***Solutions***

(a) $I_{dc} = \dfrac{E_m}{\pi (R_f + R_L} = \dfrac{100\sqrt{2}}{\pi (100 + 500)} = 0.075$ A or 75 mA

(b) The moving coil voltmeter across load will read $V_{dc}$. Here,

$V_{dc} = (I_{dc} R_L) \approx 37.53$

(c) The dc voltage across the diode is

$- I_{dc} R_L = -37.53$

(d) $P_{dc} = (I_{dc})^2 R_L = 2.81$ W

(e) $I = \dfrac{0.5 E_m}{R_f + R_L} = \dfrac{0.5 \times 100 \times \sqrt{2}}{100 + 500} = 0.118 = 118$ mA

Total power = $I^2 R_L = (0.118^2)(500) = 6.96$ W

**2.2** A half-wave rectifier circuit has a rms supply of 230 V and it uses a semiconductor diode having an internal resistance of 50 ohms. At full load, the dc current is 100 mA. Find: (a) the dc voltage at no load; (b) the dc voltage at full load; (c) the percentage voltage regulation; (d) the load resistance under full load condition; (e) the efficiency at full load; (f) the power at full load; (g) the load resistance, dc load current, efficiency and power when maximum power is delivered to load.

***Solution***

From Eq. (2.8) we have

$$V_{dc} = \left(\frac{E_m}{\pi} - I_{dc}\, R_f\right)$$

(a) At no load, $I_{dc} = 0$

$$V_{dco} = \frac{E_m}{\pi} = \frac{230 \times \sqrt{2}}{\pi} = 103.57 \text{ V}.$$

(b) At full load, $I_{dc} = 100 \times 10^{-3}$ A = 0.1 A
$V_{dc} = 103.57 - (0.1 \times 50) = 98.57$ V

(c) Regulation = [(103.57 – 98.57)/98.57] × 100 = 50.7%

(d) $R_L = V_{dc}/I_{dc} = 98.57/100 \times 10^{-3} = 985.7$ ohms

(e) $\eta = \dfrac{0.406}{1 + R_f / R_L} = 0.406/(1 + 50/985.7) = 0.386 = 38.6\%$

(f) $P_{dc} = I_{dc}^2\, R_L = (100 \times 10^{-3})^2 \times 985.7 = 9.857$ W

(g) From maximum power transfer theorem, the power is maximum when $R_L = 50$ ohm. Then,

$$I_{dc} = 230\sqrt{2}/[\pi(50 + 50)] = 1.036$$

Also, $\eta = 0.406/(1+ 50/50) = 0.203$ or 20.3%

Maximum dc power = $I_{dc}^2 \times R_L = 53.66$ W.

## 2.6 ANALYSIS OF FULL-WAVE RECTIFIER (CENTRE TAPPED TRANSFORMER CIRCUIT)

Figure 2.6 shows a full-wave rectifier circuit. This circuit comprises of two half-wave circuits connected in such a way so that one diode conducts during first half of the power cycle and the other diode conducts during the second half of the power cycle. During the first half of every power cycle, $e_{1N}$ is positive and $e_{2N}$ is negative. Therefore diode $D_1$ conducts and current flows through $R_L$. This current $i_1$ is obtained as

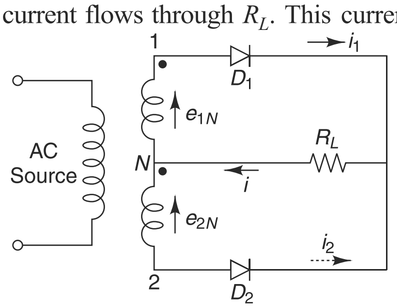



**Fig. 2.6** ***Full wave rectifier circuit with centre tapped transformer $i_1$ flows when terminal 1 is +ve; $i_2$ flows when terminal 2 is +ve***

$$i_1 = \frac{E_m \sin \omega t}{R_f + R_L} = I_m \sin \omega t \; (when\; 0 \le \omega t \le \pi)$$

$$i_1 = 0 \qquad (\text{when } \pi \le \omega \le 2\pi)$$

During the second half of each power cycle, $e_{2N}$ is negative and $e_{1N}$ is positive. Therefore diode $D_2$ conducts and current $i_2$ flows through $R_L$. This current is obtained as

$$i_2 = 0 \text{ when } (0 \le \omega t \le \pi)$$

$$i_2 = \frac{E_m \sin \omega t}{R_f + R_L} = I_m \sin \omega t \text{ when } (\pi \le \omega t \le 2\pi)$$

The current $i$ through the load is the sum of $i_1$ and $i_2$ and is unidirectional in the whole cycle. Figure 2.7 shows the profile of currents $i_1$, $i_2$ and $i$. The dc current is

$$I_{dc} = \frac{1}{2\pi}\left[\int_0^x I_m \sin(\omega t) d(\omega t) + \int_\pi^{2\pi} I_m \sin(\omega t) d(\omega t)\right]$$

or,
$$I_{dc} = \frac{2I_m}{\pi} = \frac{2E_m}{\pi(R_f + R_L)} \tag{2.13}$$

The rms current is given by

$$I = \left[\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)\right]^{0.5} = \left[\frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2(\omega t) d(\omega t)\right]^{0.5} = \frac{I_m}{\sqrt{2}} \tag{2.14}$$

It is seen that both $I_{dc}$ and $I$ in this circuit are twice of those in the halfwave circuit.

The voltage across load is ($iR_L$). Also,

$$V_m = \frac{E_m R_L}{R_f + R_L} \tag{2.15a}$$

$$V_{dc} = \frac{2E_m R_L}{\pi(R_f + R_L)} \tag{2.15b}$$

$$V = \frac{E_m R_L}{\sqrt{2}(R_f + R_L)} \tag{2.15c}$$

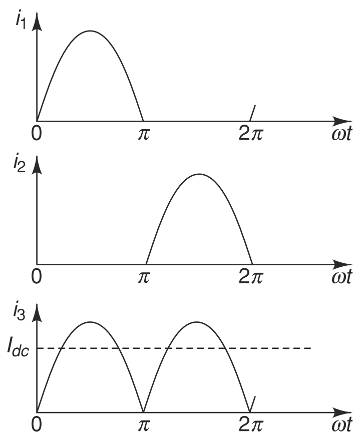



**Fig. 2.7** ***Diode currents and load current in full-wave rectifier circuit***

The dc power delivered to the load is obtained as

$$P_{dc} = I_{dc}^2 R_L = \frac{4E_m^2 R_L}{\pi^2 (R_f + R_L)^2} \tag{2.16}$$

Total power input ($P_{in}$) is given as

$$P_{in} = I^2 (R_f + R_L) = \frac{E_m^2}{2(R_f + R_L)} \tag{2.17}$$

The efficiency of the full wave circuit, is obtained as

$$\eta = \frac{P_{dc}}{P_{in}} = \frac{0.812}{1 + R_f / R_L} \tag{2.18}$$

The ratio ($I/I_{dc}$) is

$$\frac{I}{I_{dc}} = \frac{\pi}{2\sqrt{2}}$$

The ripple factor is thus given by

$$\gamma = \left(\frac{I^2}{I_{dc}^2} - 1\right) = \left(\frac{\pi^2}{8} - 1\right)^{0.5} = 0.483 \tag{2.19}$$

The output voltage at no load is obtained as

$$V_{dco} = \frac{2E_m}{\pi} \tag{2.20}$$

And the output voltage ($V_{dc}$) can be obtained as

$$V_{dc} = V_{dco} - I_{dc}\,R_f = \frac{2E_m}{\pi} - I_{dc}\,R_f \tag{2.21}$$

To find the peak inverse voltage apply K.V.L. to the outside loop of Fig. 2.6 when $D_1$ is conducting and $D_2$ is not conducting. Neglecting the voltage drop across $D_1$, the voltage across $D_2$ is $2E_m$. Thus peak inverse voltage in this full-wave rectifier circuit is $2E_m$ where $E_m$ is the peak value of ac voltage from the mid-point of the transformer secondary to either terminal.

**2.3** A full-wave rectifier circuit with a centre tap transformer uses semiconductor diodes having a forward resistance of 100 ohm and infinite backward resistance. The load resistance is 200 ohms. The rms value of the transformer secondary voltage from midpoint to each terminal is 200 V. Find: (a) the dc load current; (b) the rms value of load current; (c) the dc power output; (d) the power input; (e) efficiency; and (f) voltage regulation.

***Solution***

(a) $I_{dc} = \dfrac{2E_m}{\pi(R_f + R_L)} = \dfrac{2 \times 200 \times \sqrt{2}}{\pi(100 + 200)} = 0.6$ A

(b) $I = \dfrac{I_m}{\sqrt{2}} = \dfrac{E_m}{\sqrt{2}(R_f + R_L)} = \dfrac{200 \times \sqrt{2}}{\sqrt{2}(100 + 200)} = 0.67$ A

(c) $P_{dc} = I_{dc}^2\,R_L = (0.6)^2 \times (200) = 72$W

(d) $P_i = I^2\,(R_f + R_L) = (0.67)^2 \times (100 + 200) = 134.67$ W

(e) $\eta = \dfrac{0.812}{1 + 100/200} = 0.541 = 54.1\%$

(f) At no load, $V_{dco} = \dfrac{2E_m}{\pi} = \dfrac{2 \times 200 \times \sqrt{2}}{\pi} = 180.13$V

$V_{dc} = V_{dco} - I_{dc}\,R_f = 180.13 - (0.6 \times 100) = 120.13$ V

Regulation $= \dfrac{180.13 - 120.13}{120.13} \simeq 50\%$

**2.4** A full-wave rectifier circuit is required to give a dc output voltage of 100 V. Neglect resistance of the diode. Find: (a) transformer secondary voltage; (b) transformation ratio if transformer is fed from 230 V mains; (c) dc load current if $R_L$

is 500 ohm; (d) peak current through diode; (e) peak inverse voltage across each diode; (f) efficiency of rectifier; (g) voltage regulation.

***Solution***

(a) Peak value of transformer secondary voltage
$= 2E_m = V_{dco} \times \pi = 100 \times 3.14 = 314\text{V}$ $[\because V_{dco} = (2E_m/\pi)]$

R.M.S. value of secondary voltage $= \dfrac{314}{\sqrt{2}} = 222.07$ V

(b) Transformation ratio $= \dfrac{230}{222.07} = 1.04$

(c) $I_{dc} = \dfrac{2E_m}{\pi R_L} = \dfrac{314}{\pi \times 500} = 0.20$ A

(d) Peak current through diode $= \dfrac{E_m}{R_L} = \dfrac{0.5 \times 314}{500} = 0.314$A

(e) Peak inverse voltage $= 2E_m = 314$ V

(f) $\eta = \dfrac{0.812}{1 + R_f / R_L} = 0.812\,\eta = [\therefore R_f = 0] \therefore \eta = 81.2\%$

(g) Since $R_f = 0$, the voltage drop in the diode is zero

$\therefore \quad V_{dc} = V_{dco} - I_{dc} R_f = V_{dco} = 100$ V

$\therefore \quad \dfrac{V_{dc} - V_{dc}}{V_{dc}} = 0$ (i.e., voltage regulation is zero).

## 2.7 BRIDGE RECTIFIER CIRCUIT

Figure 2.8 shows a full-wave rectifier known as bridge rectifier circuit. In the positive half-cycle, diodes $D_1$ and $D_2$ conduct while diodes $D_3$ and $D_4$ act as open circuits. In the negative half-cycle diodes $D_3$ and $D_4$ conduct while diodes $D_1$ and $D_2$ act as open circuits. The direction of current in the load $R_L$ is the same in the both the half-cycles and has the same waveshape as that in the full-wave rectifier circuit with centre tapped transformer. It is seen that two diodes each having forward resistance $R_f$ are in the circuit in each half-cycle. Therefore,

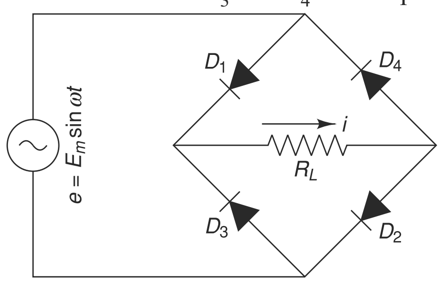


**Fig. 2.8** ***Bridge rectifier circuit***

$$i \text{ (current through } R_L) = \frac{E_m \sin \omega t}{2R_f + R_L} \tag{2.22}$$

$$I_{dc} \text{ (average dc current through } R_L) = \frac{1}{2\pi}\int_0^{2x} \frac{E_m \sin \omega t}{2R_f + R_L} = \frac{2E_m}{\pi(2R_f + R_L)} \tag{2.23}$$

$$I(\text{RMS value of load current}) = \left[\frac{1}{2\pi}\int_0^{2\pi} i^2\, d(\omega t)\right]^{0.5} = \frac{E_m}{\sqrt{2}\,(2R_f + R_L)} \quad (2.24)$$

Also, $$P_{dc} = I_{dc}^2\, R_L = \frac{4E_m^2}{\pi^2 (2R_f + R_L)^2} \quad (2.25)$$

and $$P_{in} = I^2\,(2R_f + R_L) = \frac{E_m^2}{2(2R_f + R_L)} = \frac{E_m^2 \times 0.5}{2(2R_f + R_L)} \quad (2.26)$$

And $$\eta = \frac{P_{dc}}{P_{in}} = \frac{0.812}{1 + 2R_f / R_L} \quad (2.27)$$

Hence, $$\gamma = \left[\frac{I^2}{I_{dc}^2} - 1\right]^{0.5} = 0.483 \quad (2.28)$$

Also, $$V_{dco} = \text{Output voltage at no load} = \frac{2E_m}{\pi}$$

$$V_{dc}\ (\text{Output voltage at no load}) = V_{dco} - I_{dc}\,(2\,R_f) \quad (2.29)$$

$$\text{Peak inverse voltage} = E_m = \frac{\pi V_{dco}}{2} \quad (2.30)$$

***Advantages of a Bridge Rectifier Circuit*** The advantages of a bridge rectifier circuit over the full-wave centre tapped transformer rectifier circuit are

(i) It does not require a centre-tapped transformer
(ii) The peak inverse voltage is half of that in a full-wave circuit

***Disadvantages of a Bridge Circuit***

(i) It requires 4 diodes as compared to two in the other circuit
(ii) Since two diodes are always in the circuit, the voltage drop is more, efficiency and regulation are poorer as compared to those in the full-wave centre tapped circuit.

***Applications***

As the PIV rating of diode in bridge rectifier is half than that in a full wave rectifier the bridge rectifier is suitable for high voltage applications. For low voltage applications full wave rectifier is more efficient. The bridge circuit can be used when a large dc power is required.

**2.5** A bridge rectifier uses four diodes having forward resistance each of 100 ohms. The load resistance is 200 ohms. It is fed from a 250 V ac supply. Find: (a) dc load current; (b) rms load current; (c) dc power output; (d) power input; (e) efficiency; (f) voltage regulation.

***Solution***

(a) $$I_{dc} = \frac{2E_m}{\pi\,(2R_f + R_L)} = \frac{2 \times 250\sqrt{2}}{\pi\,(2 \times 100 + 200)} = 0.56\ \text{A}$$

(b) $$I_{dc} = \frac{E_m}{\sqrt{2}\,(2R_f + R_L)} = \frac{250 \times \sqrt{2}}{\sqrt{2}\,(200 + 200)} = 0.625\ \text{A}$$

(c) $P_{dc} = I_{dc}^2 R_L = (0.56)^2(200) = 62.72$ W

(d) $P_i = I^2 (2R_f + R_L) = (0.625)(400) = 250$ W

(e) $\eta = \dfrac{P_{dc}}{P_i} = \dfrac{0.812}{1 + 2R_f / R_L} = 0.406 = 40.6\%$

(f) $V_{dco} = \dfrac{2E_m}{\pi} = 2 \times 250 \times \dfrac{\sqrt{2}}{\pi} = 225.16\,\text{V}$

$$V_{dc} = V_{dco} - I_{dc}(2R_f) = 225.16 - 0.56 \times 200 = 113.16 \text{ V}$$

$$\text{Voltage Regulation} = \frac{225.16 - 113.16}{113.16} \times 100 = 98.97\%$$

## 2.8 RECTIFIER FILTERS

The load current in a rectifier circuit contains both dc and ac components. The waveshape can be improved by using a filter.

***(a) Inductance Filter*** An inductor presents zero impedance to dc but a high impedance to ac. Thus an inductor (choke) blocks the ac component of current. If an inductor is connected in series with the load resistance, it can smoothen the current waveshape. Figure 2.9 shows a full-wave rectifier circuit with an inductance filter. Generally, iron core inductors are used for this purpose.

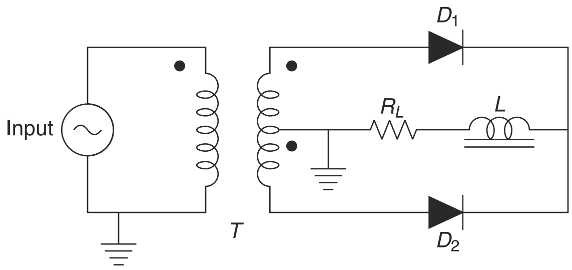



**Fig. 2.9** ***Inductance filter***

***(b) Capacitance Filter*** Figure 2.10(a) shows a capacitance connected across $R_L$ in a half-wave bridge circuit. The capacitor presents infinite impedance to dc but a low impedance to ac. When the diode is conducting, the capacitor is charging. During the time when the diode is not conducting, the capacitor discharges through $R_L$. Therefore the output current and voltage do not have the wide fluctuations which are present in the circuit without the capacitor. Figure 2.10(b) shows the ac input voltage V, current $i$ in the diode and voltage across $R_L$. The diode conducts only when voltage $e_d$ is positive. The voltage $e_d$ is

$$e_d = e - v_0$$

Thus the diode conducts only during the interval $\theta_1 \le \omega t \le \theta_2$ as shown in Fig. 2.10(b). When the diode is not conducting, the outpul voltage $V_0$ decays exponentially. If time constant is large, the voltage decays only by a small amount during the period when the diode is not conducting. For a specified $I_{dc}$, the diode current becomes more peaked and the period of conduction decreases as $C$ is made larger. The use of a capacitance filter may impose serious duty conditions on the diode because even if $I$ is below its current rating, the peak current may be excessive.

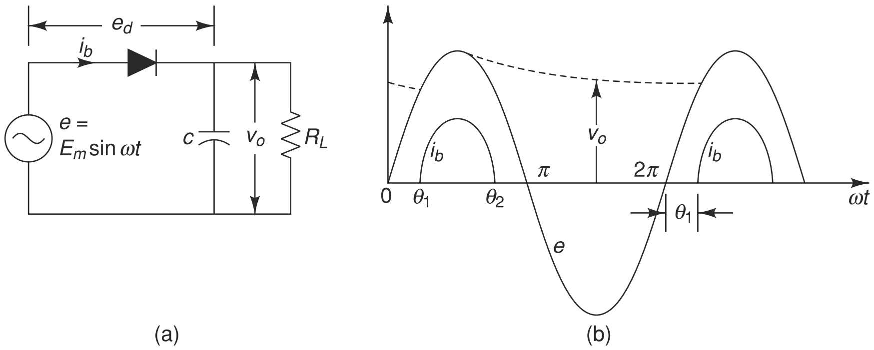



**Fig. 2.10** ***Half-wave rectifier with capacitance filter (a) circuit (b) current and voltage waveforms***

***(c) L-Section Filter*** (Inductance-Capacitance Filter) Figure 2.11 shows a full-wave circuit with an $L$-section (or inductance-capacitance filter). The shunt capacitance presents infinite impedance to dc and low impedance to ac whereas the series inductance presents zero impedance to dc and high impedance to ac. Therefore, the dc component of load current tends to flow through $R_L$ and the ac component tends to flow through $C$. The combined effect of inductance and capacitance results in a load current which is approximately dc with only very small harmonic components. The filtering can be made better by using two or more $L$-sections in cascade.

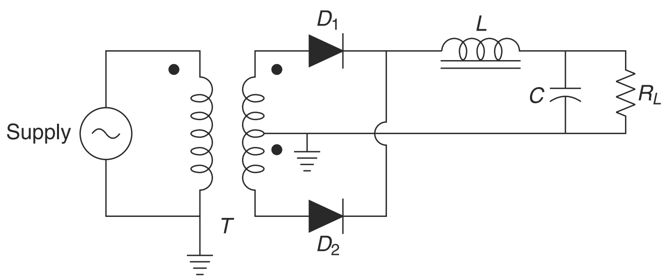



**Fig. 2.11** ***L-section filter***

***(d) Capacitance Input Filter*** ($\pi$-Section Filter) A very smooth output can be obtained by using a filter having two shunt capacitances $C_1$ and $C_2$ and one series inductance $L$ as shown in Fig. 2.12. This filter is equivalent to two filters in cascade, one consisting of $C_1$ and the second consisting of $L$ and $C_2$.

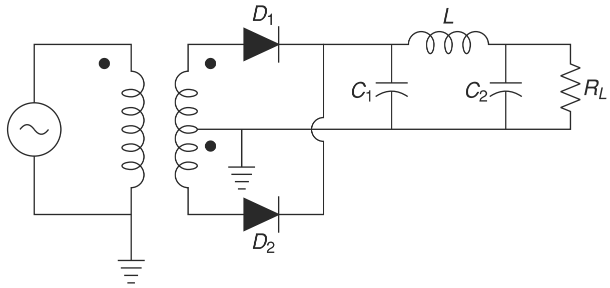



**Fig. 2.12** ***π*-section filter**

## 2.9 VOLTAGE REGULATOR

A rectifier, along with a suitable filter, provides output voltage which is very nearly dc. However a change in ac input voltage or a change in load resistance $R_L$ causes a change in output voltage. Many systems require a constant dc voltage. A voltage regulator circuit used in conjunction with rectifier and filter can provide a constant dc voltage.

A semiconductor diode has a very high resistance in the reverse direction. As the voltage applied to it in the reverse direction is increased, it breaks down at a voltage known as the zener breakdown voltage. After zener breakdown, the voltage across the diode remains constant and current through it can be limited only by the external circuit. Some practical *p-n* junctions are manufactured to operate in the zener breakdown region and are known as zener diodes. The zener diode is not damaged by reverse current if its temperature rise is kept within limits. The maximum and minimum currents for such diodes are specified. Zener diodes are available in fairly large voltage ratings. They can also be connected in series to obtain still higher ratings.

Figure 2.13 shows a simple voltage regulator circuit showing a zener diode. The load is in parallel with the zener diode. Since the voltage across the zener diode remains constant, the voltage across the load also remains constant. If $R_L$ increases, the load current decreases. The zener current $I_2$ increases so that the current through the load and voltage across it remain constant. A similar action occurs when load resistance decreases or supply voltage changes. Thus, as load current or supply voltage changes, the zener diode current accommodates itself to these changes to maintain a nearly constant dc output voltage. The diode regulates till the circuit operation requires the diode current to fall to its minimum value. The minimum diode current is usually kept at about 10% of maximum load current. The resistance $R$ is so selected as to keep the output voltage $V_a$ constant for the specified range of variations in input voltage and load resistance $R_L$. The resistance $R$ is given by

$$R = \frac{V_{\min} - V_0}{I_{L\max}}$$

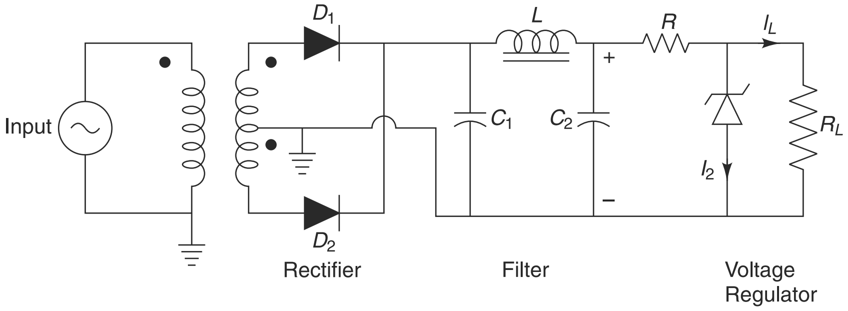



**Fig. 2.13** *Voltage regulator circuit*

## 2.10 COMMONLY USED DIODES

There are a large number of semiconductor diodes of different ratings which are used in electronic circuits as per the requirement. These diodes are of different shapes, sizes and colours. Only a few of them are shown in Fig. 2.14.

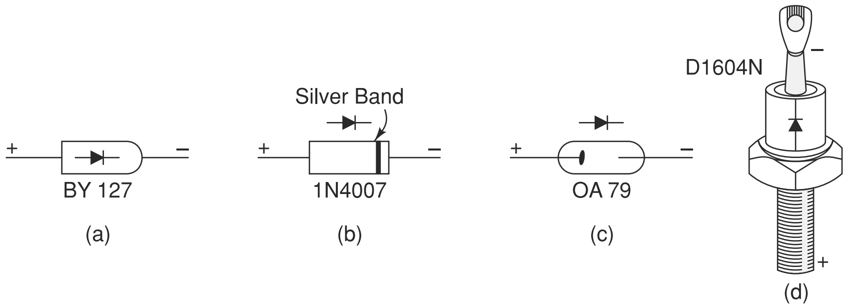



**Fig. 2.14** *Different diodes*

Figure 2.14(a) shows a BY 127 diode which can carry 1 A forward current with PIV of 1000 V safely. Its body is green in colour and the direction in which it can conduct is marked by the symbol on its body as shown in Fig. 2.14(a). In the same series, there are also other diodes like BY 118, etc.

Figure 2.14(b) shows the shape of a IN 4007 diode. It can carry 1 A forward current with PIV of 1000 V safely. Its body is black in colour. On one side, a silver colour band is printed which shows the negative end (cathode) of the diode. In the same series, the other diodes are IN 4001, IN 4002, IN 4003, IN 4004 etc. Another diode of the same series is IN 5406 which can carry 6 A current with PIV of 200 V.

Figure 2.14(c) shows the shape of an OA 79 diode. Its body is of transparent glass. Red mark on its body denotes the positive (arrow) terminal. The other diodes of the same series are OA 80, OA 85, etc.

Figure 2.14(d) shows the shape of a power diode D 1604N. It has metallic body and can handle large power. It can carry 16A current with PIV of 400 V safely, Its polarity is marked in Fig. 2.14(d). Another power diode is 10 KLR 12 which can carry 10A current at PIV of 1200 V.

Similarly there are many other diodes of different ratings. The rating of the diode can be determined from the manufacturer's data book.

## 2.11 CHECKING OF DIODE TERMINALS

If the symbol or mark on the body of a diode showing its polarity is missing or vanished off, then the terminals' polarity can be determined with the help of an ohm-meter (multimeter). The polarity of the terminals of the battery contained within an ohmmeter appears at the leads of the ohmmeter. In Fig. 2.15, lead *P* is positive and *Q* is negative.

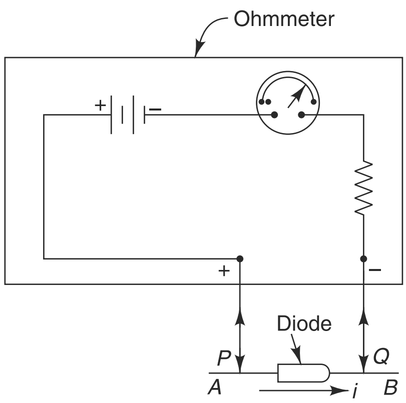


**Fig. 2.15** ***Checking diode terminals***

To check the terminals of the diode, it is connected across the leads *P* and *Q* as shown in Fig. 2.15. If diode conducts and meter gives deflection, then terminal *A* of the diode is +ve (anode) and *B* is –ve (cathode). However, if diode does not conduct and there is no deflection in the meter, the terminals of the diode are opposite as mentioned earlier. If the diode conducts even if the terminals are reversed, it means that the diode is shorted.

## 2.12 DIODE CLIPPING CIRCUIT

Clipping circuits are used to transmit the desired part of a signal lying above or below the reference value. The basic components required for a clipping circuit are an ideal diode and a resistor. To fix the clipping level to the desired amount, a dc battery must be included. Different levels of clipping can be obtained by varying the amount of voltage of the battery and interchanging the positions of diode and resistor. The diode clippers may be positive or negative depending upon the positive or negative region of the input signal being clipped.

In a positive clipper circuit, the positive half cycle of the signals are removed. The circuit for positive clipper both for series and shunt positive clipper are shown in Fig 2.16. In series clipper circuit, the diode is in series with the load whereas in shunt clipper circuit the diode is parallel to the load. For Fig. 2.16 (a), the diode is reverse biased during the positive half cycle of the input signal and hence output voltage is 0. The diode acts as an open switch in this case. During the negative half cycle of the input, the diode is forward biased and hence output voltage is available only in the negative half cycle. In Fig. 2.16 (b), the diode is forward biased during the positive half cycle of input signal and heavy current flows through diode. This results in 0 voltages across the diode or the load resistance $R_L$ connected in parallel to the diode. The diode here acts as a closed switch. Hence no output voltage is available in the positive half cycle. In the negative half cycle as the diode acts as reverse bias, full voltage appears across $R_L$.

In the negative clipper circuit, the diode in Fig. 2.17 are connected with reverse polarity and the output voltage is available only during the positive cycle of the input voltage signal.

In a practical diode, the breakdown voltage will exist and the output voltages for positive and negative clipping circuits are shown in Fig. 2.18.

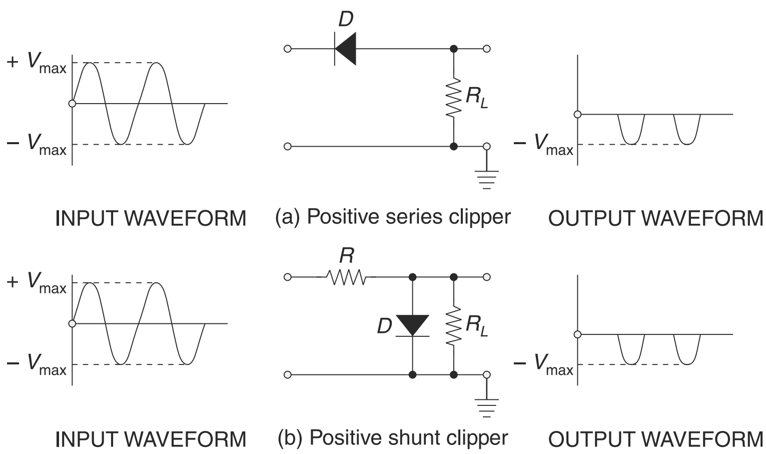



**Fig. 2.16** ***Positive series clipper and positive shunt clipper***

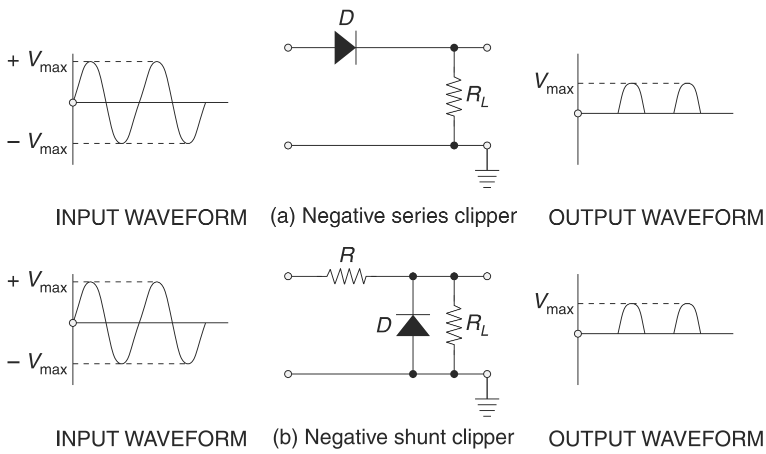



**Fig. 2.17** ***Negative series clipper and negative shunt clipper***

Biased clipper circuits are used when a small portion of either positive are negative half cycle is to be removed. The circuit diagram of biased negative clipper is shown in Fig. 2.19. During the positive half cycle of the input signal the diode is reverse biased and full input voltage appears across $R_L$. During the negative half cycle when the signal becomes more than the battery voltage, the diode is forward biased and it starts conducting. Hence full voltage appears across $R_L$ till input signal is less than the battery voltage. The output voltage stays at –$V$ till the input voltage gain decreases and becomes less than battery voltage. In a positive clipper circuit, both the battery and diode connections are reversed and desired clipping is obtained in the positive half cycle of the output waveform.

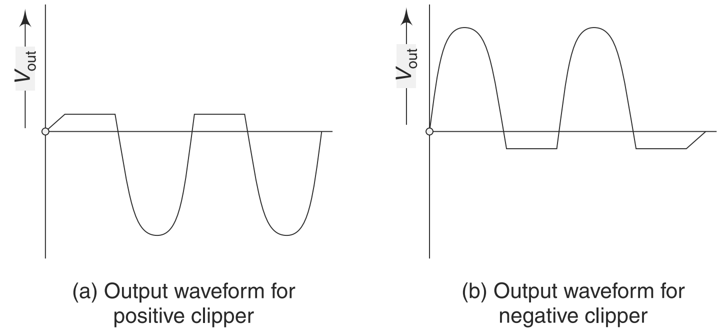



**Fig. 2.18** ***Positive clipper and negative clipper***

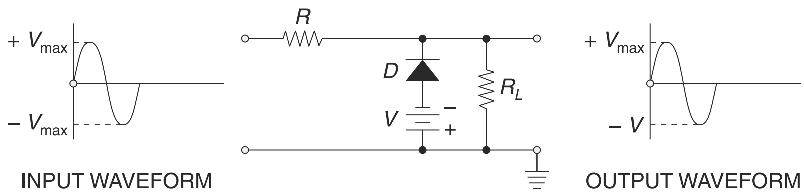



**Fig. 2.19** ***Biased negative clipper***

**2.6** A diode is connected to a 50 V (rms) source with forward resistance of 60 Ω. It supplies power to a load resistance of 2000 Ω. Determine the (i) dc and ac load current (ii) dc voltage across the diode (iii) dc output power (iv) conversion efficiency and regulation.

***Solution***

(i) The peak load current $(I_m) = \dfrac{50\sqrt{2}}{60 + 2000} = 0.034$ A

The dc load current $(I_{dc}) = \dfrac{I_m}{\pi} = \dfrac{0.034}{3.14} = 0.0108$ A

The ac load current $(I_{ac}) = \sqrt{I_{rms}^2 - I_{dc}^2}$

As diode is a half wave rectifier

$$\therefore \quad I_{\text{rms}} = \frac{I_m}{2} = \frac{0.034}{2} = 0.017 \text{ A}$$

$$\therefore \quad I_{\text{ac}} = \sqrt{(0.017)^2 - (0.0108)^2} = 0.0131 \text{ A}.$$

(ii) The dc voltage across the diode is $I_{\text{dc}} R_f = 0.0108 \times 60 = 0.648$ V.

(iii) The dc output power $P_{\text{dc}} = I_{\text{dc}}^2 R_L = (0.0108)^2 \times 2000 = 0.23328$ W.

(iv) Conversion efficiency $(\eta) = \left(\frac{2}{\pi}\right)^2 \times \frac{1}{1+\frac{R_f}{R_2}} \left(\frac{2}{\pi}\right)^2 \times \frac{1}{1+\frac{60}{2000}}$

$$= 0.3935 \text{ or } 39.35\%.$$

(v) Voltage regulation is $\frac{V_{nl} - V_{RL}}{V_{RL}} \times 100$

$$= \frac{I_{dc}\, R_f}{I_{dc}\, R_L} \times 100 = \frac{R_f}{R_L} \times 100 = \frac{60}{2000} 100 = 3\%.$$

**2.7** The dc voltage and peak to peak ripple voltage across the load resistance of an inductor filter connected to a bridge rectifier is 50V and 2V respectively. Determine the ripple factor and the percentage ripple.

***Solution***

The peak ripple voltage $V_P = \frac{2}{2} = 1$ V

The rms ripple voltage $= \frac{1}{\sqrt{2}} = 0.707$

$V_{\text{dc}} = 50$ V

The ripple factor $= \frac{0.707}{50} = 0.014$

$\therefore$ % ripple is $0.014 \times 100 = 1.4\%$.

**2.8** A full wave bridge rectifier feeds a load resistance of 3000 Ω from a 50 V (rms) supply. The forward resistance of each diode is 40 Ω. Determine (i) the dc load voltage (ii) percentage regulation and (iii) ripple voltage at the output.

***Solution***

Peak load current $I_m = \frac{V_m}{R_2 + 2R_f} = \frac{\sqrt{2} \times 50}{3000 + 2 \times 40} = 0.023$ A..

$\therefore$ dc load current $I_{\text{dc}} = \frac{2I_m}{\pi} = \frac{2 \times 0.023}{\pi} = 0.0146$ A

(i) The dc load voltage $V_{dc} = I_{dc} \times R_L = 0.0146 \times 3000 = 43.8$ V.

(ii) Percentage regulation is $\dfrac{2R_f}{R_L} 100 = \dfrac{2 \times 40}{3000} \times 100 = 2.67\%$

(iii) The ripple voltage at the output is $\sqrt{I_{rms}^2 - I_{dc}^2} \times R_L = \sqrt{\left(\dfrac{I_m}{\sqrt{2}}\right)^2 - I_{dc}^2} \times R_L$

$$= \sqrt{\left(\frac{0.023}{\sqrt{2}}\right)^2 - (0.0146)^2} \times 3000 = 21.49.$$

**2.9** The input voltage to the circuit shown in Fig. 2.20(a) is the square wave shown in Fig. 2.20(b). The forward and reverse resistances of the diode is zero and 3 $\mu\Omega$ respectively. Sketch the output voltage waveform if the cutin voltage of the diode $V_r$ = O.

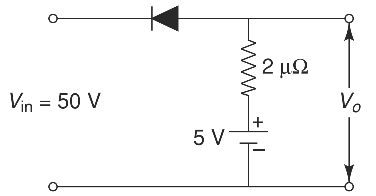



**Fig. 2.20(a)**

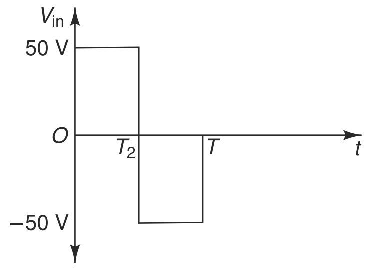



**Fig. 2.20(b)**

***Solution***

In the positive half cycle the diode is reversed biased. The current $i_b$ flowing through the diode

$$i_b = \frac{50 - 5}{3 + 2}\ \mu\text{A} = \frac{45}{5}\ \mu A = 9\ \mu A$$

The output voltage

$V_0 = 5 + 2 \times 10^6 \times 9 \times 10^{-6}$

$= 5 + 18 = 23$ V

In the negative half cycle the diode is forward biased.

As $R_F = 0$,

$\therefore$ $V_0 = -50$ V

The voltage waveform is shown in Fig. 2.20(c).

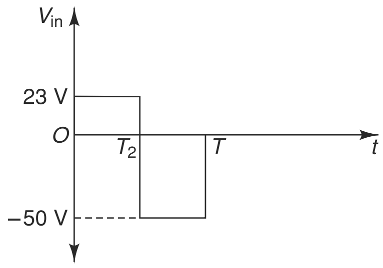



**Fig. 2.20(c)**

**2.10** Design an *L* type *LC* filter for full wave rectification which gives dc output voltage of 20 V when the load current is 200 mA. Assume ripple factor to be 0.05.

Assume the ripple factor is given by the expression, $\gamma = \dfrac{\sqrt{2}}{12\omega^2\ LC}$ and L = 0.106 H.

***Solution***

$$\because \text{ Ripple factor } \gamma = \frac{\sqrt{2}}{12\omega^2\ LC} = 0.05$$

$$\text{Here,} \quad LC = \frac{\sqrt{2}}{0.05 \times 12 \times (314)^2} = 2.39 \times 10^{-5} \qquad [\omega = 2\pi f = 314]$$

$$\text{Load resistance; } R_L = \frac{V_{\text{dc}}}{I_{\text{dc}}} = \frac{20}{200 \times 10^{-3}} = 100\ \Omega$$

$$\text{When } L_C \text{ is 0.106 H, } C = \frac{2.39 \times 10^{-5}}{0.106} = 225\ \mu\text{F}$$

**2.11** The primary to secondary turns ratio of a centre tapped transformer is 20 : 1. Its primary is fed from a 230 V ac source and the secondary is connected to a full wave rectifier, the forward resistance of whose each element is 150 Ω. The rectifier supplies current to a 2000 Ω load resistance. Calculate (i) the dc current in the load and in each diode (ii) the dc power output (iii) ripple voltage across load resistance and (iv) regulation and efficiency of rectification.

***Solution***

Primary voltage of transformer is 230 V

$$\text{Secondary voltage } \frac{230}{20} = 11.5 \text{ V}$$

The secondary voltage from the centre tap is $\dfrac{11.5}{2} = 5.75$ V.

The peak value of secondary voltage is $5.75 \times \sqrt{2} = 8.132$ V.

(i) The dc current in the load is $I_{dc} = \dfrac{2I_m}{\pi}$

$$\text{where } I_m = \frac{V_m}{R_f + R_L} = \frac{8.132}{150 + 2000} = 3.78 \times 10^{-3} \text{ A}$$

$$\therefore \quad I_{\text{dc}} = \frac{2 \times 3.78 \times 10^{-3}}{\pi} = 2.41 \text{ mA}$$

The dc current in each diode is then 2.41 mA

(ii) The dc power output

$$I_{\text{dc}}^2 R_L = \left(2.41 \times 10^{-3}\right)^2 \times 2000 = 11616.2 \times 10^{-6} \text{ W} = 0.0116 \text{ W}$$

(iii) The ripple voltage across the load resistance

$$= \sqrt{I_{\text{rms}}^2 - I_{\text{dc}}^2} \times R_L = \sqrt{\left(\frac{3.78 \times 10^{-3}}{\sqrt{2}}\right)^2 - \left(1.205 \times 10^{-3}\right)^2} \times 2000$$

$$= \sqrt{7.1442 - 1.45} \times 10^{-3} \times 2000$$

$$= 4.772 \text{ V}$$

(iv) $\text{Regulation} = \frac{R_f}{R_L} \times 100\% = \frac{150}{2000} \times 100\% = 7.5\%$

Efficiency of rectification $\eta = \frac{81.2}{1 + R_f / R_L}\ \% = \frac{81.2}{1 + \frac{150}{2000}}\ \% = 75\%$

## 2.13 DIODE CIRCUIT ANALYSIS USING LOAD LINE

Let us consider that a diode is connected to a source voltage $v$ through a load resistance $R_L$. If current $i_a$ flows in the circuit under forward-biased condition then

$$v = v_a + i_a R_L$$

where $v_a$ is the voltage drop across the diode. The variation of $i_a$ with $v_a$ is known as static characteristic, and the variation of $i_a$ with the supply voltage $v$ is known as the dynamic characteristic.

Fig. are 2.21 shows the plot of $i_a$ against $v_a$ which is a straight line represented by $AB$. The intercept of the straight line on the voltage axis is $v$ and on the current axis is $v/R_L$. This line is called the load line which has a slope of $-1/R_L$. The load line and the static curve intersect each other at the point $P$. The coordinates of point $P$ are $v_P$ and $i_P$.

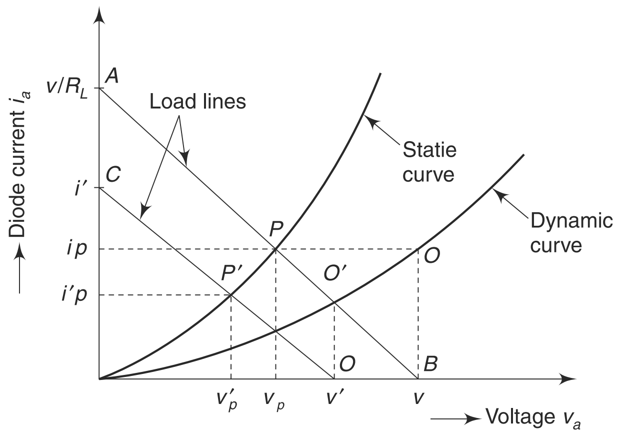



**Fig. 2.21** ***Load line and dynamic curve of diode circuit***

Now if the source voltage $v$ is varied, the diode current will change. For a new value of source voltage, the load line shifts parallel to the previous load line *AB*. The new load line is *CD* and the intercepts on the voltage and current axes are $v^/$ and $i^/$ respectively. The new load line intersects the static curve at $P^/$, and the coordinates of $P^/$ are $v^/_P$ and $i^/_P$. the point $O$ is obtained by drawing two perpendicular lines from $i_P$ and $v$, one perpendicular to the current axis and the other perpendicular to the voltage axis and $O$ is the point of intersection of the two lines. Similarly point $O^/$ is obtained by drawing two perpendicular lines from $i^/_P$ and $v^/$. More such points can be obtained by the variation of the supply voltage. The curve passing through the origin and the points $O$, $O^/$,…..is known asdynamic characteristic of the diode for the given load resistance. For a given supply voltage, the diode current can be directly determined from dynamic characteristics.

## 2.14 LINEAR PIECEWISE MODEL OF DIODE

In the linear piecewise model, the diode is represented by a hypothetical linear circuit which shows the same characteristics as that of the nonlinear device diode. In this model, the original nonlinear *V-I* characteristics of the diode can be segmented into different linear parts. The points where the linear parts meet are known as break points.

Let us consider that the threshold voltage of the diode under consideration is $V_T$ and the breakdown voltage is $V_B$. the forward and reverse diode resistance and the resistance of the diode after breakdown are $R_f$, $R_r$ and $R_Z$ respectively. Thus, the nonlinear characteristic of the diode can be represented by three straight lines: one with slope $1/R_f$ indicating the forward-bias condition, one with slope $1/R_r$ indicating the reverse bias condition and the third with slope $1/R_Z$ which represents the operation of the diode under breakdown condition. Fig. 2.22 shows the piecewise linear characteristic of a diode and Figure. 2.23 shows the circuit model of a practical diode using piecewise linear approximation when supply voltage is $V$.

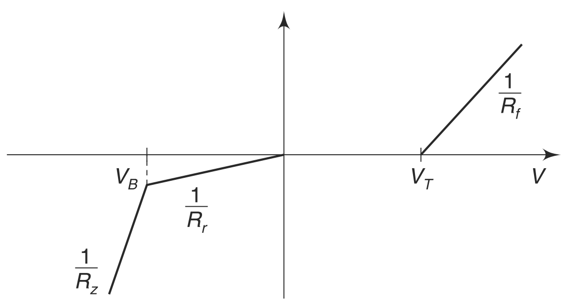



**Fig. 2.22** ***Piecewise linear V-I characteristic of diode***

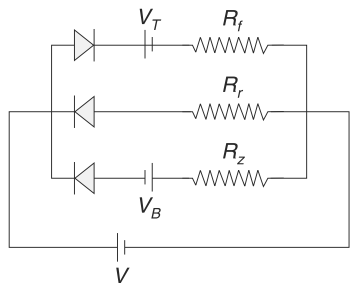



**Fig. 2.23** ***Diode-circuit model according to linear piecewise approximation***

## EXERCISES

1. What is a semiconductor diode? Draw its symbol.
2. Explain the operation of a semiconductor diode in forward bias and reverse bias modes.
3. Draw and explain the *V–I* characteristics of a semiconductor diode.
4. What are the various resistances of a diode?
5. Explain the terms: Forward voltage, forward current, breakdown voltage and PIV.
6. Explain the operation of a semiconductor diode as a halfwave rectifier. Draw the waveforms of output to voltage and current.
7. Explain the operation of a full wave rectifier with centre tapped transformer. Draw the waveforms of dc outputs.
8. Draw the circuit of a bridge rectifier and explain the operation with the waveforms.
9. Why and when the bridge rectifier circuit is preferred over centre tapped transformer circuit?
10. Why are filter circuits used in rectifier circuits? State the types of filter circuits.
11. A half-wave rectifier has ac input power of 140 W and dc output power of 60 W. What is the rectification efficiency? (Ans. 42.8%)

$$\left[ H\,\text{int} : \eta = \frac{\text{dc output power}}{\text{ac input power}} \times 100 = 42.8\% \right]$$

12. An ac supply of 230 V is applied to a half-wave rectifier circuit through a transformer of twins ratio 10 : 1. If the diode resistance is 10 Ω and the load resistance is 800 ohm, find
    (i) $I_{dc}$, $I_{rms}$
    (ii) efficiency of rectification
    (iii) dc output voltage
    (iv) PIV
    (Ans. $I_{dc}$ = 12.79 mA; $I_{rms}$ = 20.08 mA, $\eta$ = 40%
    $E_{dc}$ = 10.24 V; PIV = 32.53 V

*Hint*: $E_2 = \frac{N_2}{N_1} \times E_1 = \frac{1}{10} \times 230 = 23\text{ V}$

$E_m = \sqrt{2}\ E_2 = 32.53\text{ V}.$

$I_m = \left[E_m / (R_f + R_L)\right] = \frac{32.53}{10+800} = 40.16\text{ mA}$

$I_{dc} = \frac{I_m}{\pi} = 12.79\text{ mA}$

$I_{rms} = \frac{I_m}{2} = 20.08\text{ mA}.$

$P_{rc} = I_{rms}^2\,(r_f + r_L) = (20.08)^2 \times 10^{-6} \times 810\ 325.3\text{ mW}$

$\eta = \frac{130}{325.3} \times 100 = 40\%$

$E_{dc} = R_L \times I_{dc} = 10.24\text{ V}.$

PIV = $E_m$ = 32.53 V.

13. A centre tapped transformer rectifier has two diodes each of 20 Ω. The transformer secondary voltage from centre to each and of the secondary is 100 V. Assuming the load resistence to be 800 Ω, find $I_{dc}$ and $I_{rms}$.
[Ans. 110.3 mA; 122.8 mA]

$$\left[Hint : I_m = \frac{E_m}{R_f + R_L} = \frac{100\sqrt{2}}{20+800} \times 10^3 = 173.17\text{ mA}\right.$$

$$I_{dc} = \frac{2I_m}{\pi} = 110.3\text{ mA}$$

$$\left. I_{rms} = \frac{I_m}{\sqrt{2}} = 122.8\text{ mA}\right]$$

14. The turns ratio of the transformer used in a bridge rectifier is 12 : 1. The primary is connected to 230 V 50 Hz ac supply. Find the dc voltage across the load and PIV of each diode. If the same dc voltage is to be obtained by a centre tapped transformer recitifer what will be the PIV?

[Ans. 17.21 V; 27.021 V; 54.04 V]

$$\left[Hint : E_{pri(max)}\ \sqrt{2} \times 230 = 342.3\text{ V}.\right.$$

$$E_{(sec)(max)} = \frac{324.3}{12} = 27.02\text{ V}$$

$$\left. E_{dc} = \frac{2E_{sec(max)}}{\pi} = 17.21\text{ V}\right]$$

PIV (bridge) = $E_{sec(max)}$ = 27.02 V

PIV (centre tapped) = 27.02 × 2 = 54.04]

## MULTIPLE CHOICE QUESTIONS

1. The cut-in voltage of a Si *pn* diode is about [WBUT 2008]
   (a) 0.6 V (b) 0.3 V
   (c) 0.6 mV (d) 0.3 mV
2. The diffusion capacitance of forward-biased *pn* junction diode varies [WBUT 2009]
   (a) linearly with current
   (b) inversely with current
   (c) as the square of the current
   (d) as the square root of the current
3. When the reverse voltage across a *pn* junction is gradually decreased, the depletion region inside the diode [WBUT 2009]
   (a) does not change in width
   (b) initially increases up to a certain width and then decreases
   (c) continuously increases in width
   (d) continuously decreases in width
4. The capacitance of a varactor diode can be changed by its [WBUT 2010]
   (a) doping level (b) temperature
   (c) forward bias (d) reverse bias
5. In reverse-biased condition, junction capacitance of step-graded *pn* junction diode varies proportionally as [WBUT 2010]
   (a) $V^{-1/2}$ (b) $V^{-1/3}$
   (c) $V^{-1/4}$ (d) none of these
6. For a full-wave rectifier [WBUT 2011]
   (a) one centre-tapped transformer is required
   (b) two centre-tapped transformers are required
   (c) more than two centre-tapped transformers are required
   (d) centre-tapped transformer is not required
7. When a *pn* junction is forward biased,
   (a) drift current increases
   (b) drift current decreases
   (c) diffusion current increases
   (d) diffusion current decreases
8. When a *pn* junction is reverse biased,
   (a) drift current increases
   (b) drift current decreases
   (c) diffusion current increases
   (d) diffusion current decreases

9. A forward biased *pn* junction has a resistance of the order of
   (a) kilo Ω (b) mega Ω
   (c) Ω (d) milli Ω
10. A reverse biased *pn* junction has a resistance of the order of
   (a) kilo Ω b) mega Ω
   (c) Ω (d) milli Ω
11. The breakdown occurs in *pn* junction when the junction is
   (a) damaged (b) heated
   (c) forward biased (d) reverse biased
12. The leakage current across a *pn* junction is due to
   (a) majority mobile carrier
   (b) minority mobile carrier
   (c) both majority and minority mobile carriers
   (d) immobile charges
13. Zener breakdown is due to
   (a) avalanche phenomenon
   (b) tunneling phenomenon
   (c) both avalanche and tunneling phenomenon
   (d) burn out failure
14. The Zener voltage is of the order of
   (a) 100 V (b) 0.6 V
   (c) 60 V (d) 6 V
15. Tunneling through a *pn* junction occurs when
   (a) barrier potential is very low
   (b) peak field at the junction is low
   (c) doping concentration is low
   (d) depletion layer is extremely thin
16. Diffusion capacitance of a diode arises in
   (a) reverse bias
   (b) forward bias
   (c) both reverse and forward bias
   (d) unbiased condition
17. The *pn* junction device used in a tuned circuit is the
   (a) varistor (b) thermistor
   (c) zener (d) varactor

18. When a *pn* junction diode is reverse biased it acts like
    (a) on switch (b) off switch
    (c) low resistance (d) inductor
19. A zener diode is used as a
    (a) rectifier (b) amplifier
    (c) multiplier (d) voltage regulator
20. A *pn* junction dynamic resistance is
    (a) directly proportional to applied voltage
    (b) directly proportional to current
    (c) inversely proportional applied voltage
    (d) none of the above
21. The reverse saturation current of a *pn* junction diode is
    (a) decreased with temperature
    (b) inversely proportional with temperature
    (c) increased with temperature
    (d) independent of temperature
22. A diffusion current flows through a *pn* junction just after its formation due to
    (a) majority carriers (b) minority carriers
    (c) forward bias (d) reverse bias
23. The knee voltage of an Si diode is
    (a) 0.8 V (b) 0.3 V
    (c) 0.5 V (d) 0.7 V
24. Light emitting diode is made by
    (a) Si (b) Ge
    (c) GaAs (d) combination of Si and Ge
25. The transition capacitance of a reverse-biased junction having uniform doping on the *p* type and *n* type region varies with the applied voltage ($V$) as
    (a) $V^{-1/3}$ (b) $V^{-1/2}$
    (c) $V^{-1/4}$ (d) $V$
26. The typical operating current of a light emitting diode is approximately
    (a) 5 to 20 mA (b) 20 to100 mA
    (c) 1 to 5 mA (d) 100 to 500 mA
27. The zone in a semiconductor diode where no free charge carrier exists is known as
    (a) anode region (b) cathode region
    (c) depletion region (d) none of the above

28. In its pure state, silicon has the properties of
    (a) semiconductor (b) insulator
    (c) conductor (d) none
29. A varistor is a
    (a) current-dependent resistor
    (b) current-dependent diode
    (c) voltage-dependent diode
    (d) voltage-dependent resistor
30. Which type of transformer is required to create 180° input to a rectifier?
    (a) step-down secondary (b) step-up secondary
    (c) centre-tapped secondary (d) split winding primary
31. The dc current through each diode in a bridge rectifier equals
    (a) load current (b) twice the dc load current
    (c) half the dc load current (d) one fourth of the dc load current
32. If a 169.7 V half-wave peak has an average voltage of 54 V, what is the average of two full wave peaks?
    (a) 108 V (b) 119.9 V
    (c) 339.4 V (d) 115.7 V
33. In a power supply diagram, which block indicates a smooth dc output?
    (a) Filter (b) Regulator
    (c) Transformer (d) Rectifier
34. Since diodes are destroyed by excessive currents, circuits must have
    (a) higher current sources (b) higher voltage sources
    (c) more dopants (d) current limiting resistors
35. A diode in which you can change the reverse bias and thus vary capacitance is called a
    (a) tunnel diode (b) zener diode
    (c) varactor diode (d) switching diode
36. The peak inverse voltage across a nonconducting diode in a bridge rectifier equals approximately
    (a) twice the peak secondary voltage
    (b) half the peak secondary voltage
    (c) four times the peak value of the secondary voltage
    (d) peak value of secondary voltage
37. When checking a diode, low resistance readings both ways indicate the diode is
    (a) satisfactory (b) faulty
    (c) open (d) short

38. What is the percent regulation if $V_{nl}$ = 20 V and $V_{fl}$ = 19.8 V?
    (a) 0.1% (b) 5%
    (c) 0% (d) 1%
39. The area at the junction of *p*-type and *n*-type materials that has lost its majority carriers is called the
    (a) depletion region (b) *n*-region
    (c) *p*-region (d) barrier potential
40. Ripple factor of a full-wave rectifier is [WBUT 2008]
    (a) 1 (b) 0
    (c) 0.483 (d) 10
41. The dc voltage and peak-to-peak ripple voltage across the load resistance of an inductor filter connected to a bridge rectifier is 70 V and 2 V respectively. What is the ripple factor?
    (a) 0.09 (b) 0.5
    (c) 1 (d) 0.0101
42. The slope of the load line is given by
    (a) $-1/R_L$ (b) $-R_L$
    (c) $R_L$ (d) $1/R_L$
43. The ripple factor ã is equal to
    (a) $V_{dc}/V_{ac}$ (b) $V_{rms}/V_{ac}$
    (c) $V_{dc}/V_{rms}$ (d) $V_{rms}/V_{dc}$
44. The ripple factor of a half-wave rectifier is [WBUT 2009, 2012]
    (a) 0.482 (b) 0.41
    (c) 1.21 (d) 1.11
45. The intersection of the load line and the diode characteristics gives
    (a) *Q*-point (b) load resistance
    (c) load voltage (d) source voltage
46. The applied voltage to a diode is $v = 50 \sin \omega t$. The average or dc value of voltage is
    (a) 25 V (b) 50 V
    (c) 10 V (d) 15.9 V
47. The number of diodes in a three-phase bridge rectifier is
    (a) 2 (b) 3
    (c) 6 (d) 4
48. A clipping circuit is used to
    (a) restore dc level to the signal
    (b) remove a certain portion of the input voltage signal

(c) clamping the input voltage

(d) regulate the output voltage with change in the input voltage signal

49. A clamping circuit consists of

(a) a resistor and a capacitor

(b) a resistor, a capacitor and a diode

(c) a resistor and a diode

(d) a diode and a capacitor

50. Which of the following elements is not an essential element of a dc power supply?

(a) Voltage amplifier (b) Voltage regulator

(c) Filter (d) Rectifier

51. If the line frequency is 60 Hz, the output frequency of a bridge rectifier is [WBUT 2010]

(a) 30 Hz (b) 60 Hz

(c) 120 Hz (d) 240 Hz

52. Which one is used as a reference voltage source? [WBUT 2008]

(a) Junction diode (b) Zener diode

(c) Transistor (d) Op-amp

53. A full-wave rectifier has twice the efficiency of a half-wave rectifier because [WBUT 2008]

(a) it makes use of a transformer

(b) its ripple factor is much less

(c) it utilizes both half cycles of the input

(d) its output frequency is double the line frequency

54. Compared to avalanche breakdown, a Zener diode has [WBUT 2009]

(a) less doping concentration

(b) less barrier field intensity

(c) higher barrier field intensity

(d) higher depletion width

55. The ripple factor of a power supply is a measure of

(a) its filter efficiency (b) its voltage regulation

(c) diode rating (d) purity of power output

56. The maximum efficiency of a full-wave rectifier can be [WBUT 2009, 2010]

(a) 37.2% (b) 40.6%

(c) 53.9% d) 81.2%

57. Without a dc source, a clipper acts like a [WBUT 2010]
    (a) rectifier (b) clamper
    c) chopper (d) demodulator
58. In an unbiased negative parallel clipper using an ideal *pn* junction diode, when the diode is forward biased, the output is [WBUT 2008]
    (a) positive half cycle of the input
    (b) negative half cycle of the input
    (c) zero
    (d) breakdown voltage of the diode
59. If the filtered load current is 10 mA, which of the following has a diode current of 10 mA? [WBUT 2008]
    (a) Half-wave rectifier (b) Full wave rectifier
    (c) Bridge rectifier (d) Impossible to say
60. If the load resistance decreases in a Zener regulator, the series current [WBUT 2008]
    (a) decreases
    (b) stays the same
    (c) increases
    (d) equals the source voltage divided by the series resistance

## ANSWERS

| | | | | |
|---|---|---|---|---|
| 1. (a) | 2. (a) | 3. (d) | 4. (d) | 5. (a) |
| 6. (a) | 7. (c) | 8. (d) | 9. (c) | 10. (b) |
| 11. (d) | 12. (b) | 13. (b) | 14. (d) | 15. (d) |
| 16. (b) | 17. (d) | 18. (b) | 19. (d) | 20. (a) |
| 21. (c) | 22. (b) | 23. (d) | 24. (c) | 25. (b) |
| 26. (a) | 27. (c) | 28. (b) | 29. (d) | 30. (c) |
| 31. (a) | 32. (a) | 33. (b) | 34. (d) | 35. (c) |
| 36. (d) | 37. (b) | 38. (d) | 39. (a) | 40. (c) |
| 41. (d) | 42. (a) | 43. (d) | 44. (c) | 45. (a) |
| 46. (d) | 47. (c) | 48. (b) | 49. (d) | 50. (a) |
| 51. (c) | 52. (b) | 53. (c) | 54. (d) | 55. (d) |
| 56. (d) | 57. (a) | 58. (c) | 59. (a) | 60. (b) |

## UNIVERSITY QUESTIONS WITH ANSWERS

1. Can the contact potential developed across a *pn* junction be measured? [WBUT 2003]

   *Answer:* There is no current flow through a short circuited *pn* junction since there is no external source of energy. When we assume that a current flows through a *P-N* junction, the metal wire should be heated. As there is no external energy source, a simultaneous cooling of the *P-N* junction will take place. But this is not possible under thermal equilibrium. Therefore, we can say that no current flows in a short-circuited *P-N* junction

Since the sum of voltages around the closed loop must be zero, the junction potential must be exactly compensated by the metal to semiconductor contact potentials at the ohmic contacts. When we connect a voltmeter across the junction, it detects zero reading. As a result, it is not possible to measure contact potential directly with a voltmeter though we can calculate it when the different carrier concentrations are known.

2. A half-wave rectifier used a diode with an equivalent forward resistance of 0.3 ohm. If the input ac voltage is 10 V(rms) and the load resistance of 2 ohms, Calculate $I_{dc}$ and $I_{rms}$. [WBUT 2002]

*Answer:* The rms input voltage is $V_{rms}$ = 10 V, the forward resistance of diode is $r_f$ = 0.3 ohm and the load resistance is $R_L$ = 2 ohms.

The maximum voltage $V_m = \sqrt{2}\ V_{rms}\ \sqrt{2}\ \times 10\text{ V} = 14.142\text{ V}$

The maximum current is

$$I_m = \frac{V_m}{r_f + R_L} = \frac{14.142}{0.3+2}\text{ A} = 6.148\text{ A}$$

The dc current is $I_{dc} = \frac{I_m}{\pi} = \frac{6.148}{3.142} = 1.956\text{ A}$

The rms current is $I_{rms} = \frac{I_m}{2} = \frac{6.148}{2} = 3.074\text{ A}$

3. (a) A silicon diode having internal resistance $R_f$= 30 is used for half wave rectification. [WBUT 2003, 2014]

The input ac voltage is $V_i$ = 6 sin volt and load resistance is 500.

Find (i) *dc* output voltage, (ii) *ac* input power, and (iii) efficiency of the rectifier.

Given: The maximum supply voltage = 6 V, $R_f$ = 30 Ω and $R_L$ = 500 Ω

(i) The dc load current is

$$I_{dc} = \frac{I_m}{\pi} = \frac{1}{\pi}\frac{V_m}{R_f + R_L} = \frac{1}{\pi} \times \frac{6}{30+500} = 3.6\text{ mA}$$

The dc output voltage is

$$I_{dc} R_L = 3.6 \times 10^{-3} \times 500\text{ V} = 1.8\text{ V}$$

Peak value of the current is

$$I_m = \frac{V_m}{R_f + R_L} = \frac{6}{30+500} = 11.3\text{ mA}$$

The rms current is

$$I_{\text{rms}} = \frac{I_m}{2} = \frac{11.3}{2} \text{ mA} = 5.65 \text{ mA}$$

(ii) The ac input power is

$P_{\text{ac}} = I^2_{\text{rms}} (R_f + R_L)$

$$= (5.65 \times 10^{-3})^2 (30 + 500) = 16.92 \text{ mW}$$

The dc power delivered to the load is

$$P_{\text{dc}} = I^2_{\text{dc}} R_L = (3.6 \times 10^{-3})^2 \times 500 = 6.48 \text{ mW}$$

(iii) The efficiency is

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{I^2_{dc} R_L}{I^2_{rms} (R_f + R_L)} = \frac{6.48}{16.92} \times 100\% = 38.29\%$$

(b) Draw the circuit of a bridge rectifier and compare its performance with that of a full-wave rectifier using centre tapped transformer.

[WBUT 2003, 2010]

*Answer:* Refer Fig. 2.8.

**Table** *Comparison of centre-tapped full-wave and full-wave bridge rectifier*

| *Parameter* | *Centre-tapped full-wave rectifier* | *Full-wave bridge rectifier* | *Half wave rectifier* |
|---|---|---|---|
| Number of diodes | 2 | 4 | 1 |
| Peak inverse voltage | $2V_m$ | $V_m$ | $V_m$ |
| Average of dc output voltage | $0.636V_m$ | $0.636V_m$ | $0.318\ V_m$ |
| Ripple factor | 0.482 | 0.482 | 1.21 |
| Ripple frequency | $2f$ | $2f$ | $2f$ |
| Percentage efficiency | 81.2 | 81.2 | 40.6 |
| Transformer utilization factor | 0.693 | 0.812 | 0.287 |

4. Each of the two diodes in a full-wave rectifier circuit has a forward resistance of 50. The dc voltage drop across a load resistance of 1.2 kΩ is 30 V. Find the primary to total secondary turn's ratio of the center tapped transformer, primary being fed from 220 $V_{\text{rms}}$. [WBUT 2004]

*Answer:*

Assume that the primary to secondary terms ratio is $n : 1$

Given, $R_f = 50\ \Omega$, $R_L = 1.2\ \text{k}\Omega$, rms primary voltage is 220 V

Therefore, rms secondary voltage is equal to $\dfrac{220}{n}$

The voltage across half-secondary winding between centre tapping and one end of the secondary

$$= \frac{220}{2n}$$

The maximum voltage across half-second winding is $V_m = \dfrac{220\sqrt{2}}{2n}$

Peak value of current is

$$I_m = \frac{V_m}{R_f + R_L} = \frac{V_m}{50 + 1200} = \frac{220\sqrt{2}}{2n \times 1250} \text{ A}$$

The dc load current is

$$I_{dc} = \frac{2I_m}{\pi} = \frac{2}{\pi}\frac{V_m}{R_f + R_L} = \frac{2}{\pi} \times \frac{220\sqrt{2}}{2n \times 1250} \text{ A}$$

The dc output voltage is

$$I_{dc} R_L = 2\pi \times \frac{220\sqrt{2}}{2n \times 1250} \times 1200 = 30$$

Therefore, the primary to total secondary turn's ratio of the centre tapped transformer is

$$n = \frac{2}{\pi} \times \frac{220\sqrt{2}}{2 \times 1250} \times 1200 \times \frac{1}{30} = 3.17$$

5. A centre-tap (CT) transformer has 230 V primary winding rated at 12-0-12 volts. This transformer is used in the FW rectifier circuit with a load resistance of 100. What are the dc output voltages, dc load current and the rms voltage developed across the diode? Assume the diodes and the transformer to be ideal. [WBUT 2007]

*Answer:*

The rms voltage between centre-tap and one end of secondary winding is $V = 12$ V

The peak value of the supply voltage is $V_m = \sqrt{2}\, V = \sqrt{2} \times 12 = 16.97$ V

$R_L = 100$

The dc load current is

$$I_{dc} = \frac{2I_m}{\pi} = \frac{2}{\pi}\frac{V_m}{R_L} = \frac{2}{\pi} \times \frac{16.97}{100} = 0.108 \text{ mA}$$

The peak voltage across the diode is

$$2V_m = 2\sqrt{2}\, V = 2\sqrt{2} \times 12 = 33.94 \text{ V}$$

The rms voltage across diode is

$$\frac{2V_m}{2} = V_m = 16.97 \text{ V}$$

6. (a) Explain the operation of a full-wave bridge rectifier with centre-tapped transformer the help of a circuit diagram. Also draw the dc output waveform. [WBUT 2013]

*Answer:* Refer Article 2.6.

(b) Obtain a mathematical expression for the efficiency of a full-wave rectifier and show that its ripple factor is 0.482. [WBUT 2007, 2008]

*Answer:* Refer Article 2.6:

7. (a) What is built-in potential across a *P-N* junction? [WBUT 2006]

   *Answer:* At the surrounding of the junction, there is some uncovered charge. Due these uncovered charges, there is a built-in potential across the *P-N* junction.

   (b) What is the role of minority carrier in a *P-N* junction diode? [WBUT 2006]

   *Answer:* Due to presence of minority carrier in a *P-N* junction diode, a reverse saturation current flows through the diode.

   (c) What is understood by peak inverse voltage ratting of a junction diode? [WBUT 2006]

   *Answer:* The peak inverse voltage ratting of a junction diode is the maximum amount of voltage applied across the diode in reverse bias condition without destroying it.

8. Compare two types of full-wave recitifier: [WBUT 2011]

   *Answers:* Refer Article 2.6 and 2.7

   (a) Centre-tapped transformer

   (b) Bridge type

9. Write short notes on any two of the following:

   *Anwers:* Refer Article 2.12 [WBUT 2008, 2010, 2011, 2013]

   (a) Clipper circuit

   (b) Ripple factor

   *Answers:* ***Ripple factor*** Ripple factor is one of the factors determining the performance of a rectifier circuit. It is a measure of the fluctuating component present in the signal. Ripple factor is defined as the ratio of the rms value of the alternating component of load voltage or current to the average value of the load voltage or current.

   The rms value of the alternating component of the load voltage is

$$V_{rms}^{/} = \left\{ \frac{1}{2\pi} \int_0^{2\pi} (V_L - V_{dc})^2 \, d(\omega t) \right\}^{1/2}$$

   where $V_L$ is the total load voltage and $V_{dc}$ is the average or dc load voltage. Therefore,

$$V_{rms}^{/} = \left\{ \frac{1}{2\pi} \int_0^{2\pi} (V_L^2 - V_{dc}^2 - 2V_L V_{dc}) \, d(\omega t) \right\}^{1/2}$$

   If $V_{\text{rms}}$ is the rms or the effective value of the total load voltage then

$$V_{rms}^2 = \frac{1}{2\pi} \int_0^{2\pi} V_L^2 \, d(\omega t)$$

Now as $\frac{1}{2\pi}\int_0^{2\pi} V_L\, d(\omega t) = V_{dc}$

Therefore, $V_{rms}^2 = \{V_{rms}^2 + V_{dc}^2 - 2V_{dc}^2\}^{1/2} = (V_{rms}^2 - V_{dc}^2)^{1/2}$

Hence, ripple factor $\frac{\sqrt{V_{rms}^2 - V_{dc}^2}}{V_{dc}}$

10. Each of the two diodes in a full-wave rectifier circuit has a forward resistance of 50 Ω. The dc voltage drop across a load resistance of 1.2 kΩ is 30 V. [WBUT 2007]
Find the primary to total secondary turns ratio of the centre-tap transformer, primary being fed from 220 $V_{\text{rms}}$
*Answer:*

The forward resistance of each diode is $R_f$ = 50 ohms and the load resistance is $R_L$ = 1.2 ohms.

Then maximum voltage across each half of the secondary winding is

$$V_m = \frac{\sqrt{2}\,V_2}{2}\ \frac{\sqrt{2}\times 44}{2}\ \text{V} = 31.11\ \text{V}$$

The peak value of current is

$$I_m = \frac{V_m}{R_f + R_L}$$

The average value of load current is

$$I_{\text{dc}} = \frac{2I_m}{\pi} = \frac{1}{\pi}\,\frac{2V_m}{R_f + R_L}$$

The voltage drop across load is

$$V_{\text{dc}} = I_{\text{dc}}\,R_L = \frac{1}{\pi}\,\frac{2V_m}{R_f + R_L} \times R_L$$

or, $$30 = \frac{2V_m}{50 + 1200} \times 1.2$$

The $$V_m = \frac{30 \times \pi \times 1250}{2 \times 1200} = 49.06\ \text{V}$$

The total secondary voltage is $2V_m$ = 98.12 V

The turn ratio is $$n = \frac{\text{primary voltage}}{\text{secondary voltage}} = \frac{220\sqrt{2}}{98.12} = 3.17$$

11. (a) Draw the circuit diagram of clipper and explain the working principle of it. [WBUT 2008]

*Answer:* Refer Article 2.12.

(b) Explain the working principle of a bridge rectifier with diagram. [WBUT 2008]

*Answer:* Refer Article 2.7.

(c) Evaluate the ripple factor and efficiency of a full-wave rectifier. [WBUT 2008]

*Answer:* Refer Article 2.6.

12. Calculate the maximum conversion efficiency of a half-wave rectifier. [WBUT 2009]

*Answer:* Refer Article 2.5.

13. What are the advantages and disadvantages of bridge rectifier using two diodes? [WBUT 2009]

Refer Article 2.7.

14. (a) With regard to full-wave rectification, explain the working of a bridge rectifier and compare its PIV with other rectifiers. Give two advantages and disadvantages of the bridge rectifier. [WBUT 2009]

*Answer:* Refer Article 2.6 and 2.7.

(b) Find out the expression for efficiency, form factor and ripple factor for a half-wave rectifier. [WBUT 2009]

*Answer:* Refer Article 2.5.

(c) A full-wave rectifier uses a diode, the forward resistance of each element being 100 ohms. The rectifier supplies current to a load of 1000 ohms. The primary to secondary turns ratio of the centre-tapped transformer is 10:1. The transformer primary is fed from a supply of 240 $V_{rms}$. [WBUT 2009]

Find:

(i) dc load current
(ii) direct current in each diode
(iii) the ripple voltage
(iv) the efficiency of rectification

*Answer:*

Given: Turn ratio is $\frac{N_1}{N_2} = 10$, the primary voltage of transformer is $V_1 = 240\ V_{rms}$

The secondary voltage of transformer is

$$V_2 = \frac{N_2}{N1} V_1 = \frac{1}{10} \times 240 = 24\ V_{rms}$$

Then maximum voltage across each half of the secondary winding is

$$V_m = \frac{\sqrt{2}\,V_2}{2} = \frac{\sqrt{2} \times 24}{2}\ \text{V} = 16.968\ \text{V}$$

The forward resistance of each diode is $R_f$ = 100 ohms and the load resistance is $R_L$ = 1000 ohms.
The peak value of current is

$$I_m = \frac{V_m}{R_f + R_L} = \frac{16.968}{100 + 1000} = 15.425 \text{ mA}$$

The average value of load current is

$$I_{\text{dc}} = \frac{2I_m}{\sqrt{2}} = \frac{2 \times 15.425 \times 10^{-3}}{\pi} = 9.825 \text{ mA}$$

The direct current in each diode is $\frac{I_{dc}}{2} = \frac{9.825}{2} = 4.9125$ mA

The rms value of load current is

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} = \frac{15.425}{\sqrt{2}} \text{ mA} = 10.908 \text{ mA}$$

The rms value of the ripple current is

$$I_{\text{ripple, rms}} = \left[I_{\text{rms}}^2 - I_{dc}^2\right]^{1/2} = [(10.908)^2 - (9.825)^2]^{1/2} \text{ mA}$$
$$= 4.738 \text{ mA}$$

The ripple factor can be given by

$$\gamma = \frac{\left[I_{\text{rms}}^2 - I_{dc}^2\right]^{1/2}}{I_{dc}} = \frac{4.738}{9.825} = 0.4822$$

The ripple voltage is $V_{\text{ripple, rms}} = \gamma V_{\text{dc}} = 0.4822 \times 0.636 V_m$

$0.422 \times 0.636 \times 16.968 \text{ V} = 5.203 \text{ V}$

The dc power is equal to

$$P_{\text{dc}} = I^2_{\text{dc}} R_L = (9.825 \times 10^{-3})^2 \times 1000 \text{ watts} = 96.53 \text{ mW}$$

The ac power input to the rectifier is

$$P_{\text{ac}} = I^2_{\text{rms}} (R_f + R_L)$$
$$= (10.908 \times 10^{-3})^2 \times (100 + 1000) \text{ watts} = 130.88 \text{ mW}$$

The efficiency of rectifier is equal to

$$\eta = \frac{P_{ac}}{P_{dc}} = \frac{96.53 \text{ mW}}{130.88 \text{ mW}} \times 100\% = 73.75\%$$

15. (a) Explain how a Zener diode can act as a voltage regulator. 4

*Answer:* Refer Article 2.9. [WBUT 2014]

(b) Write a short note on clipper circuit. 3

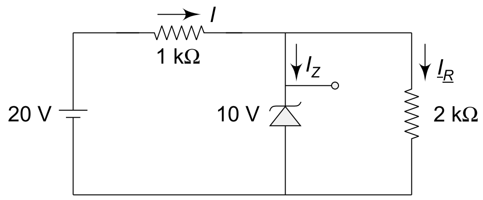
*Answer:* Refer Article 2.12.

16. Calculate the current $I$, $I_R$ and $I_Z$ for the following circuit. [WBUT 2012]

*Answer:* $I = \frac{20 - 10}{1000} = 10$ mA

$$I_R = \frac{10}{2 \times 10^3} = 5 \text{ mA}$$

$$I_Z = I - I_R = 5 \text{ mA}$$




17. Write a short note on damper circuit. [WBUT 2012]
*Answer:* A clamper circuit is an electronic circuit that fixes the positive or the negative peak excursions of a signal to a fixed value by shifting its dc value. This circuit moves the whole signal up or down so as to place the peaks at the reference level. The most common clamper circuit is the diode clamp which consists of a diode. It conducts electric current in only one direction and prevents the signal exceeding the reference value. It also consists of a capacitor which provides a dc offset from the stored charge. The capacitor forms a time constant with the resistor load which determines the range of frequencies over which the clamper will be effective.
Clamping circuits are also known as dc voltage restorers. Clampers can be constructed in both positive and negative polarities. When unbiased, positive clamping circuits will fix the voltage lower limit and negative clamping circuits will fix the voltage upper to 0 volts. Clamp circuits are classified by their operation; negative or positive, and biased or unbiased. A positive clamp circuit outputs a purely positive waveform from an input signal; it offsets the input signal so that all of the waveform is greater than 0V. A negative clamp is the opposite of this—this clamp outputs a purely negative waveform from an input signal.
18. Explain the operation of a full-wave rectifier with centre-tapped transform and draw the dc output waveform.
*Answer:* Refer article 2.6
19. The ripple factor for a half wave rectifier is [WBUT 2014]
(a) 0.482 (b) 0.41
(c) 1.21 (d) 1.11
*Answer* (c)
20. Explain the operation of a half wave rectifier with the help of circuit diagram. Optain mathematical experssion for the efficiency of the halfwave rectifier and show that its ripple factor is 1.21. [WBUT 2014]
*Answer:* Refer Article 2.5
21. A full wave bridge rectifier fed from 15 $V_{rms}$ ac source and is connected across a 100 Ω load calculate

(a) PIV

(b) Rms current drawn from the supply

(c) Average dc current across the load [WBUT 2013]

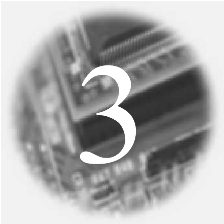
*Solution*

(a) PIV = $\sqrt{2} \times 15$ V = 21.21 V

(b) $I_m = \dfrac{V_m}{R_L} = \dfrac{21.21}{100} = 0.2121$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{0.2121}{\sqrt{2}} A = 0.15 \text{ A}$$

(c) $I_{dc} = 2\dfrac{I_m}{\pi} = \dfrac{2 \times 0.2121}{\pi} A = 0.135 \text{ A}$




# INTRODUCTION TO TRANSISTORS

## 3.1 PREAMBLE

*Transistor* was invented by J. Barden and W.H. Brattain in 1948 in USA. Because of its innumerable merits viz. longer life, high efficiency, light weight, low power loss, it replaced almost all vaccum tube triodes. It is also known as *Bipolar Junction Transistor* (BJT).

## 3.2 TRANSISTOR AND ITS CONFIGURATION

A transistor is a semiconductor device obtained by sandwitching a *p* or *n* type semiconductor material between a pair of *n* type or *p* type semiconductor materials respectively

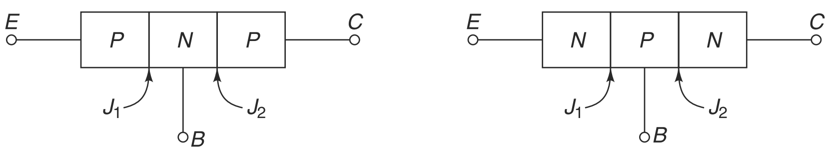
Accordingly we have 2 types of transistors.

(i) *p-n-p* transistor

This is obtained by sandwiching an *n*-type material between two *p*-type materials [sec Fig. 3.l(a)]

(ii) *n-p-n* transistor

This is obtained by sandwiching a *p*-type material between two *n*-type materials [see Fig. 3.l(b)]




(a) *pnp* transistor (b) *npn* transistor

**Fig. 3.1** ***Types of transistors***

It may be noted that

(a) the transistor is a 3-semiconductor layer, 3-terminal and 2-function device.

(b) a transistor is also called a bi-junction transistor as it has 2 junctions $J_1$ and $J_2$.

(c) it is a bipolar device, i.e., conduction in a transistor takes place due to the movements of charges of both polarities, viz. electrons (– ve charge) and (+ ve charge) .

(d) a transistor can be assumed to be a device obtained by keeping two diodes back to back. Refer Fig. 3.2(a) and (b) for obtaining *p-n-p* and *n-p-n* transistors.

As it will be discussed in the following pages, junction $J_1$ is forward biased and $J_2$ is reverse biased. As a result $J_1$ has a low resistance path and $J_2$, a high resistance path. The signal is fed in the low resistance path ($J_1$) and output is received from the high resistance path ($J_2$); thus the device transfers a signal "through resistances". This is the reason why they have been named 'transistors' (which *transfer* through *resistors*)

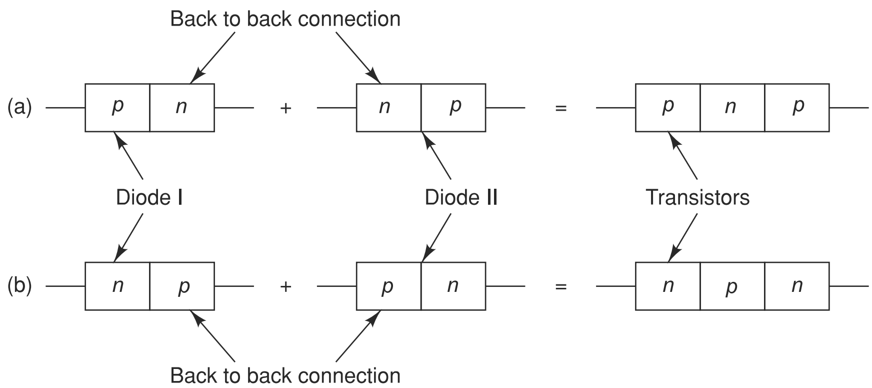



**Fig. 3.2** ***Transistors using back to back connection of diodes***

## 3.3 TERMINALS OF A TRANSISTOR

Every transistor has three terminals called *emitter, collector* and *base.*

***Emitter*** The side of the transistor that supplies a large number of "majority carriers" is called *emitter.* The emitter is always *forward biased* with respect to base so that it can supply a large number of majority carriers to its junction with the base. The biasing of emitter base junction of *npn* transistor and *pnp* transistor are shown in Fig. 3.3 and 3.4 respectively. Since emitter is to supply or inject a large amount of majority carriers into the base, it is *heavily doped* but *moderate* in size.

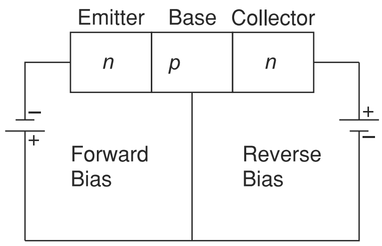



**Fig. 3.3** ***Proper biasing arrangement of npn transistor***

***Collector*** The other side of the transistor that collects the major portion of the majority carriers supplied by the emitter is called *collector.* The collector-base junction is always *reverse biased.* Its main function is to remove majority carriers (or charges) from its junction with base. The biasing of collector-base junction of *npn* transistor and *pnp* transistor is shown in Fig. 3.3 and 3.4 respectively. The collector is *moderately doped* but *larger is size* so that it can collect most of the majority carriers supplied by the emitter.

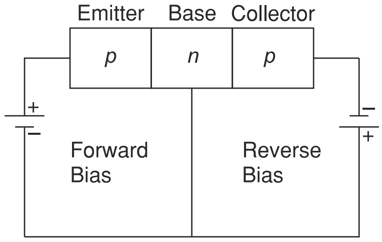



**Fig. 3.4** ***Proper biasing arrangement of pnp transistor***

***Base*** The middle section which forms two *pn* junctions between emitter and collector is called *base.* The base forms two circuits, one input circuit with emitter and the other output circuit with collector. The base-emitter junction is forward biased, providing low resistance for the emitter circuit. The base-collector junction is reversed biased, offering high resistance path to the collector circuit. The base is *lightly doped* and *very thin* so that it can pass on most of the majority carriers supplied by the emitter to the collector.

## 3.4 OPERATING PRINCIPLE OF A TRANSISTOR

The transistor action can be explained in two ways: 1. Unbiased and 2. biased.

### 3.4.1 Transistor is Unbiased

In an unbiased transistor, electrons and holes diffuse (Fig. 3.5) across the junctions (as in case of a diode) and this gives rise to depletion layers at the two junctions [Fig. 3.5(a)]. The barrier potential at this layer is about 0.7 V for a silicon transistor and 0.3 V for a germanium transistor. Also, the width of these layers are different due to the difference of the doping of the three regions. The depletion layer at the emitter base (EB) junction is small in width, whereas at the collector base (CB) junction it is wider [Fig. 3.5(b)]. It also gives an important conclusion that the emitter base junction will need a smaller bias while collector base junction will need a larger bias for the operation.

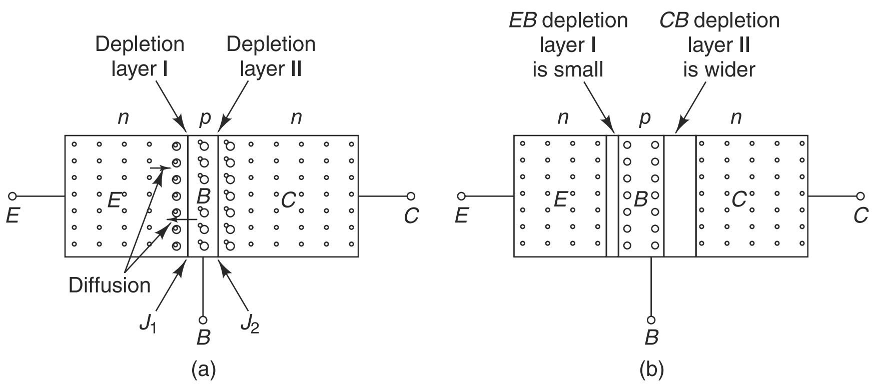



**Fig. 3.5** ***Unbiased transistor***

### 3.4.2 Transistor is Biased

We will discuss operation for the two transistors separately:

(a) An *n-p-n* transistor for operation is forward bias at its emitter base junction and reverse biased at its collector base junction.

If the biased transistor is *n-p-n* type, its emitter (*n* side) will have plenty of electrons and on biasing they diffuse into the base region which has holes and thus creating current $I_E$ (the emitter current). In the base, recombination of electrons and holes take place and valence electrons are created, which constitute 'base current' ($l_B$). But as the base is thin and also lightly doped only about 5% electrons can combine with holes and the rest 95% electrons find their way to the collector through base collector junction, and constitute collector current ($I_C$).

Here, $I_E = I_B + I_C$ (Fig. 3.6)

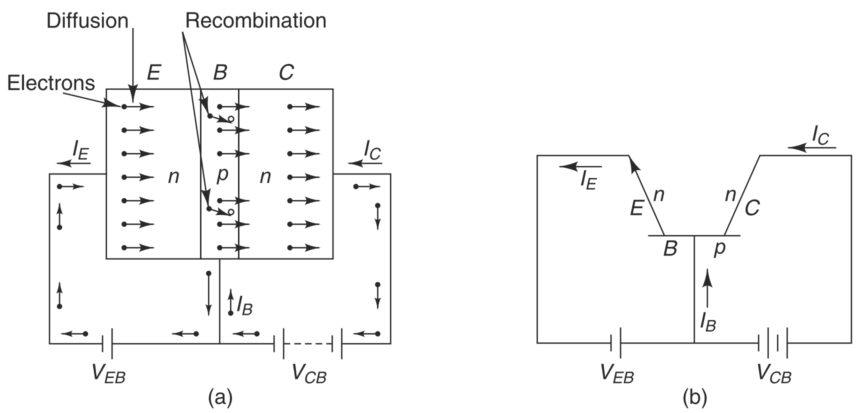



**Fig. 3.6** *Operation of npn transistor*

In this way, current conduction takes place in the transistor. Remember that the conventional direction of current is opposite to the direction of flow of electrons.

(b) A *p-n-p* transistor for operation is also forward biased at its emitter base junction and reverse biased at its collector base junction.

If the transistor is *p-n-p*, its emitter (*p* side) has plenty of holes, which will diffuse into emitter-base junction and constitute emitter current $I_E$ into the base (*n* region), which has electrons. Here recombination of electrons and holes takes place and a base current ($I_B$) is resulted. But as the base is thin (lightly doped) only a few holes can combine with electrons and the rest 95% holes find their way through CB junction and constitute collector current ($I_C$). In this case also.

$$I_E = I_B + I_C \text{ (Fig. 3.7)}$$

In this way, current conduction takes place in the transistor. Remember that inside the transistor current is due to the movement of holes, but in the outside wires it is due to electrons. Further, direction of current is opposite to the direction of flow of electrons.

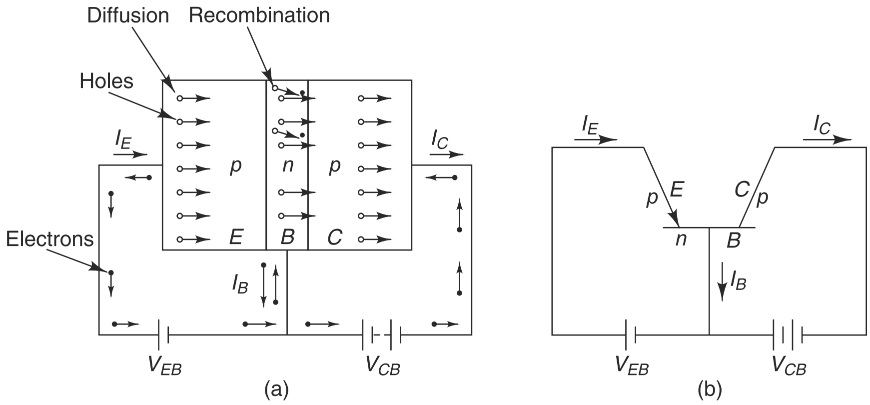



**Fig. 3.7** ***Operation of a pnp transistor***

For a transistor, whether a *p-n-p* or an *n-p-n*, the following equation holds good in all conditions:

$$I_E = I_B + I_C$$

This is called "transistor equation".

If the above equation is valid, it should be taken care of showing current direction in a transistor.

(a) In case of an *n-p-n* transistor $I_E$ comes out of it and $I_B$ and $I_C$ enter into the transistor [Fig. 3.8(a)].

(b) In case of an *p-n-p* transistor, $I_E$ enters into it and $I_B$ and $I_C$ come out of the transistor [Fig. 3.8(b)].

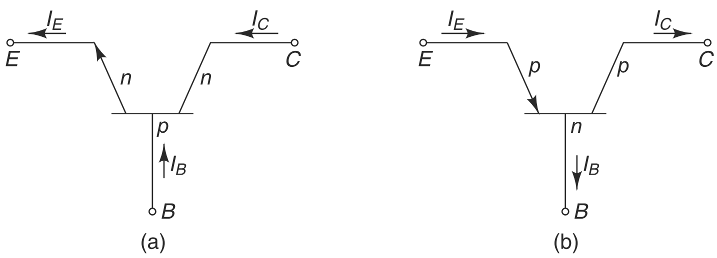



**Fig. 3.8** ***Current directions in transistor***

The most important application of a transistor is as an amplifier, i.e. it amplifies (increases) the strength of the ac signal fed at its input junction. This will be discussed in the subsequent articles.

When a transistor is used as an amplifier, the equation for the transistor shall be

$\Delta I_E = \Delta I_B + \Delta I_C$

$\Delta I_E$ = change in the emitter current

$\Delta I_B$ = corresponding change in base current

$\Delta I_C$ = corresponding change in the collector current

### 3.4.3 Current Components in a Transistor

It will be of interest to know about the various current components which flow through a transistor. When its emitter junction is forward biased and collector junction is reverse biased, Fig. 3.9 shows these components for a *p-n-p* transistor.

(i) The emitter current $I_E$ has two parts:
   (a) Hole current $I_{PE}$ constituted by holes of the emitter and flowing to base crossing the emitter junction $J_1$.
   (b) Electron current $I_{NE}$ flowing to base crossing the emitter junction. Current $I_{NE}$ constituted by electrons of the emitter and crossing $J_1$ from emitter to base. In *pnp* transistors, $I_{NE}$ is negligibly small as compared to $I_{PE}$ ($I_{PE} << I_{NE}$) and therefore the emitter current consists almost of holes $I_{PE}$.

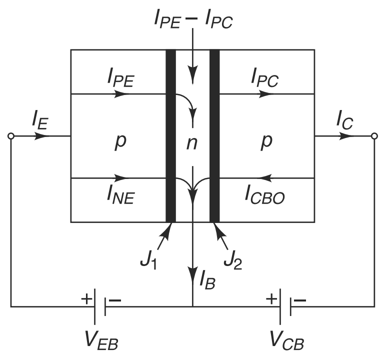



**Fig. 3.9** ***Current components in a transistor***

(ii) $I_{PC}$ represents the hole current at collector junction $J_2$ by holes coming from emitter.

(iii) ($I_{PE} – I_{PC}$) is the current resulting from recombination process going on in the base region.

(IV) ($I_{CBO}$) is the reverse current due to electrons (minority carriers) from the *P* region.

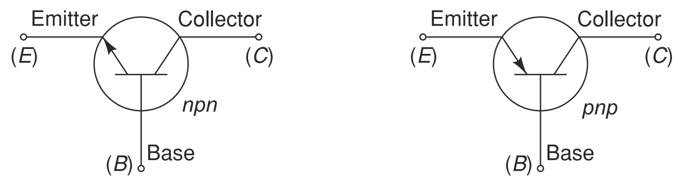
A similar sketch can be drawn for an *n-p-n* transistor with reversing direction of the various current components.

From the above discussions, we can summarize as under:

1. A transistor has three regions, namely *emitter, base* and *collector.*
2. The emitter is heavily doped so that it can inject or emit a large number of charge carriers (electrons or holes) into the base. It is moderate in size. The base is lightly doped and very thin so that it can allow most of the charge carriers to diffuse across this region and reach the collector space charge layer. The collector is intermediately doped and large in size so that it can dissipate the heat produced at the collector junction.
3. There are two junctions, one is between emitter and base and is called emitter junction, whereas, the other one is between the collector and base and is called collector junction.
4. The depletion layer at the emitter junction is quite small as compared to the depletion layer at the collector junction.
5. For perfect operation of a transistor, the emitter junction is forward biased, whereas the collector junction is reverse biased.
6. The forward biased voltage $V_{EB}$ is very small, whereas the reverse biased voltage $V_{CB}$ is very large.
7. The resistance of emitter junction is very small, whereas the resistance of collector junction is very large.

## 3.5 SYMBOL OF A TRANSISTOR

For the sake of convenience, the transistors are generally represented by schematic diagrams (i.e. symbols), the symbols for *npn* and *pnp* transistors are shown in Fig. 3.10.




(a) Symbol for *npn* transistor (b) Symbol for *pnp* transistor

**Fig. 3.10** ***Symbolical representation of a transistor***

It can be seen that emitter is shown by an arrow which indicates the direction of flow of conventional current in emitter with forward biasing applied to emitter-base junction. The only difference between *npn* and *pnp* transistor symbols is in the direction of arrow head placed on the emitter.

## 3.6 TRANSISTOR CONNECTIONS

A transistor has three terminals but we need two supplies for biasing and operation and hence four terminals are required to connect the transistor in a circuit. One terminal of the transistor is kept common and this arrangement gives us three connections/configurations for a transistor. The input is given between the common terminal and one of the two other terminals. Similarly, output is obtained between the common terminal and the left out third terminal. For satisfactory operation in all the configurations, the input side is to be forward biased and the output side is to be reverse biased. By keeping one transistor terminal common at a time, we can have three transistor configurations:

1. Common base (*CB*) configuration
2. Common emitter (*CE*) configuration
3. Common collector (*CC*) configuration

### 3.6.1 Common Base Connection

In this case, the base is made common. The emitter-base is forward biased and acts as input side, the collector base is reverse biased and acts as output (collector load $R_C$) side.

Figure 3.11 shows *CB* connections for an *n-p-n* transistor and Fig. 3.12 for a *p-n-p* transistor.

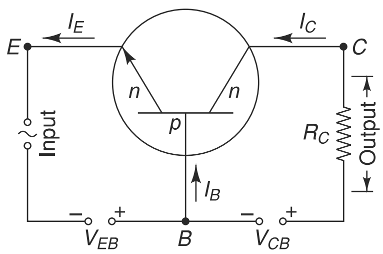



**Fig. 3.11** ***CB connection for npn transistor***

***1. Current Amplification Factor ($\alpha$)*** The ratio of output current ($I_C$) to the input current ($I_E$) in a *CB* common base connection is "current amplification factor". It is represented by $\alpha$. It is of two types:

*(i) dc alpha ($\alpha_{dc}$)* When only dc conditions prevail, i.e., the transistor biased by dc supply on both sides, but not used as an amplifier (ac signal is not applied), then the ratio of the output current to the input current is called *dc amplification factor*.

$$\alpha_{dc} = \frac{I_C}{I_E} \text{ or, } I_C = \alpha_{dc} \cdot I_E$$

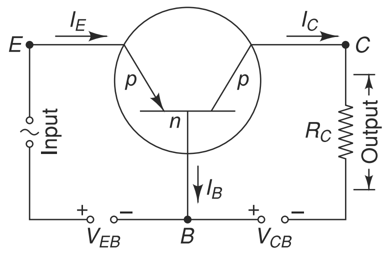



**Fig. 3.12** ***CB connection for pnp transistor***

*(ii) ac alpha ($\alpha_{ac}$)* When both the dc as well as ac conditions prevail in the circuit (i.e. the transistor is biased and signal is fed at its input side) then we consider it as ac alpha ($\alpha_{ac}$). It is defined as the ratio of change in the collector current ($\Delta I_C$) to the change in the emitter current ($\Delta I_E$)

i.e., $$\alpha_{dc} = \frac{\Delta I_C}{\Delta I_E} \text{ (at constant } V_{CB})$$

or, $\Delta I_C = \alpha_{ac} \cdot \Delta I_E$.

In *CB* configuration, the value of $\alpha_{dc}/\alpha_{ac}$ is 0.98 (less than unity). Though $\alpha$ is less than unity, this circuit will give current gain (or power gain). Here it is to be noted that value of $R_c$ is very high. Therefore, this circuit can give some current and power gain. This configuration is used either as a current buffer or as a voltage amplifier (non-inverting)

***2. Collector Current*** Here,

(i) $I_C = \alpha \cdot I_E$ (as $\alpha = I_C/I_E$); this is the part of emitter current reaching at the collector.

(ii) As the collector base junction is reverse biased small 'leakage current' will also flow due to minority carriers (i.e. due to holes) in case of *n-p-n* and due to electrons is case of *p-n-p* transistor.

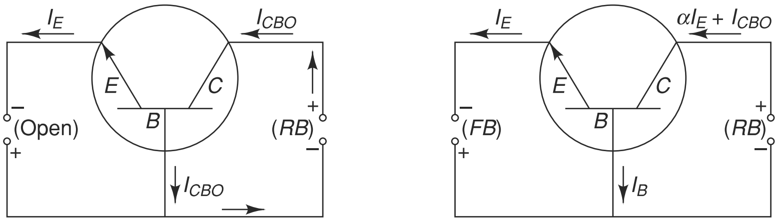
Therefore, the total collector current. $I_C = \alpha I_E$ + leakage current

In the above expression, if $I_E$ is kept equal to zero, it can be seen that in that case also, the leakage current will flow. In other words, if *EB* junction is kept open, i.e., unbiased ($I_E = 0$), the leakage current will flow from collector to base. This is called "collector to base current when emitter is open" and is abbreviated as $I_{CBO}$ or $I_{CO}$.

Thus total collector current is them given by

$$I_C = \alpha I_E + I_{CBO}$$

Figure 3.13 shows that when *EB* side is open, and *CB* side is reverse biased, $I_{CBO}$ flows from collector to base. When both the sides are given proper biasing, then $I_C = \alpha I_E + I_{CBO}$. Since the common pare mode has the lowest input resistance and highest output resistance, the *CB* configuration is used to match a low-inpendance source with a high-inpendance source.




(a): *EB* sides open (b): *EB* sides forward biased

**Fig. 3.13** ***Components of collector current***

**3.1** Determine the voltage gain and power gain of a transistor having $\alpha = 0.97$ and used in a common base amplifier. The load resistance and resistance of the emitter junction is 5000 Ω and 100 Ω respectively.

***Solution***

Voltage gain $(A_V) = \dfrac{I_C R_L}{I_E R_E}$, $R_L$ being the collector resistance and serving as load resistance $= \alpha \dfrac{R_L}{R_e}$ ($R_e$ is the emitter junction resistance and $\alpha$ (= current gain) = $(I_C/I_E)$)

Here $R_L = 5000\ \Omega$ and $R_e = 100\ \Omega$

$$\therefore \quad A_V = 0.97 \times \frac{5000}{100} = 48.5$$

and power gain $(A_P) = \alpha \times A_V = 0.97 \times 48.5 = 47.045$.

### 3.6.2 Characteristics of Common Base Connections

To determine the characteristics of a transistor in *CB* configuration, the circuit is arranged as shown in Fig. 3.14(a). The emitter to base voltage $V_{EB}$ can be varied by adjusting the potentiometer. *A* series resistor $R_s$ is inserted in the emitter circuit to limit the emitter current $I_E$ otherwise the value of $l_E$ may change to a large value even if the setting of potentiometer $R_i$ is changed slightly.

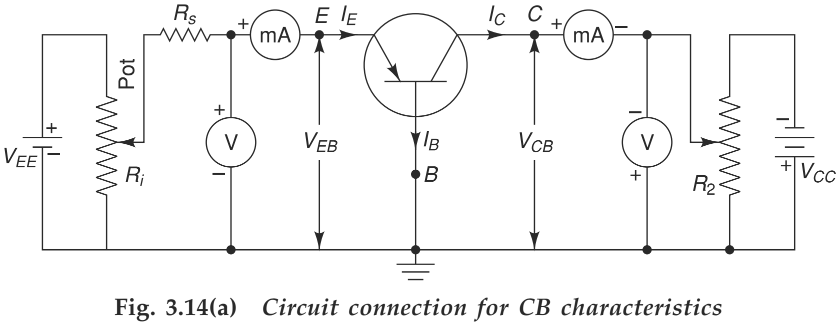



**Fig. 3.14(a)** ***Circuit connection for CB characteristics***

The collector voltage can be varied by adjusting the setting of potentiometer $R_i$. For different settings, the currents and voltages are read from the milli ammeters and voltmeters connected in the circuit.

***1. Input Characteristics*** (In *CB* configuration), the curve, plotted between emitter current $I_E$ and the emitter-base voltage $V_{EB}$ at constant collector-base voltage $V_{CB}$ is called *input characteristics*.

A number of characteristic curves can be plotted for different settings of $V_{CB}$. Figure 3.14(b) shows the input characteristics of a typical *PNP* transistor in common-base configuration. The following points may be noted from these characteristics:

(i) For a particular value of $V_{CB}$, the curve is just like a diode characteristic in the forward region. In fact, here *pn* emitter junction is forward biased.

(ii) When $V_{CB}$ is increased, value of $I_E$ increases slightly for the given value of $V_{EB}$. It also reveals that emitter current and hence collector current is almost independent of collector-base voltage $V_{CB}$.

(iii) The emitter current increases rapidly with a small increase in emitter-base voltage $V_{EB}$. It shows that input resistance is very small.

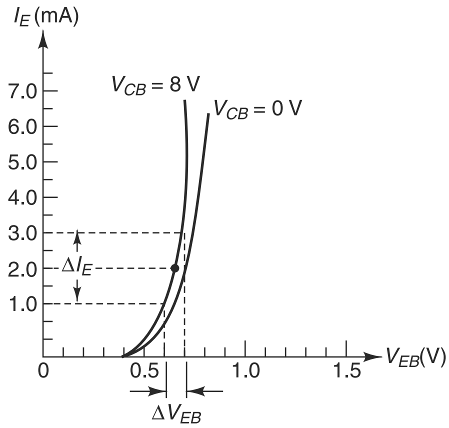



**Fig. 3.14(b)** ***Input characteristics***

*Input resistance*: The ratio of change in emitter-base voltage ($\Delta V_{EB}$) to the resulting change in emitter current ($\Delta I_E$) at constant collector-base voltage ($V_{CB}$) is known as input resistance.

$$r_i = \frac{\Delta V_{EB}}{\Delta I_E} \text{ at constant } V_{CB}$$

The value of input resistance is very low. Its value further decreases with the increase in collector-base voltage $V_{CB}$ since the curve tends to become more vertical. The typical value of input resistance varies from a few ohms to 100 ohms.

***2. Output Characteristics (In CB Configuration)*** The curve plotted between collector current $I_C$ and collector-base voltage $V_{CB}$ at constant emitter current $I_E$ is called *output characteristics*.

A number of characteristic curves can be plotted for different settings of $I_E$. Figure 3.15 shows the output characteristics of a typical *PNP* transistor in *CB* configuration. The following points may be noted from these characteristics:

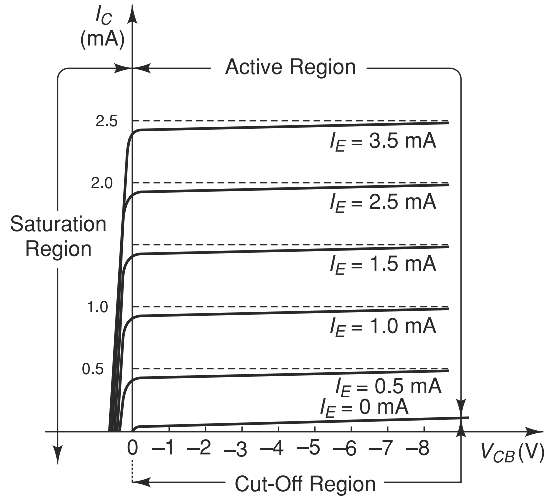



**Fig. 3.15** ***Output characteristics of pnp transistor in CB configuration***

(a) In the active region, where collector-base junction is reverse-biased, the collector current $I_C$ is almost equal to the emitter current $I_E$. The transistor is always operated in this region. In the active region, the curves are almost flat. A very large change in $V_{CB}$ produces only a tiny change in $I_C$. It means that the circuit has very high output resistance $r_0$.

(b) When $V_{CB}$ becomes positive i.e., the collector-base junction is forward biased, the collector current $I_C$ (for a given $I_E$) decreases abruptly. This is the saturated region. In this region $I_C$ does not depend much upon $I_E$.

(c) When $I_E = 0$, collector current $I_C$ is not zero although its value is very small. This is the reverse leakage current i.e. $I_{CBO}$ that flows in the collector circuit. This current is temperature dependent and its value ranges from 0.1 to 1.0 μA for silicon transistors and 2 to 5 μA for germanium transistors.

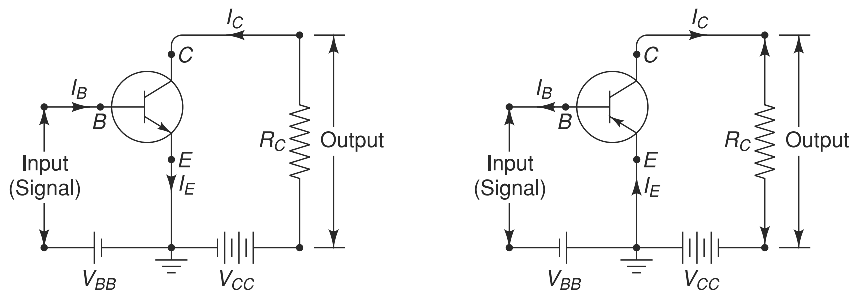
*Output resistance:* The ratio of change in collector-base voltage $\Delta V_{CB}$ to the resulting change in collector current $\Delta I_C$ at constant emitter current ($I_E$) is known as output resistance, i.e.

Output resistance, $$r_o = \frac{\Delta V_{CB}}{\Delta I_C} \text{ at constant } I_E$$

It may be seen from the output characteristics that the change in collector current is very little with the change in $V_{CB}$ as the curves are almost flat. Therefore, the output resistance of *CB* configuration is very high (of the order of several tens of kilo-ohms).

### 3.6.3 Common Emitter Connection

The common emitter circuit arrangements for *npn* transistor and *pnp* transistor are shown in Fig. 3.16(a) and 3.16(b) respectively. In this arrangement, the input is connected between base and emitter while output is taken from the collector and emitter. Thus, the emitter of the transistor is common to both input and output circuits and hence the name common emitter connection or common emitter configuration.



(a) Common emitter circuit of *npn* transistor (b) Common emitter circuit of *pnp* transistor

**Fig. 3.16** ***Common emitter connection***

***Base Current Amplification Factor*** The ratio of output current to input current in common emitter configuration is known as base current amplification factor. In this arrangement, the output current is collector current $I_C$ and the input current is base current $I_B$.

Thus, the ratio of change in collector current to the change in base current is known as *base current amplification factor.* It is generally represented by the Greek letter (beta).

i.e., $$\beta = \frac{\Delta I_C}{\Delta I_B}$$

*Relation between* $\beta$ *and* $\alpha$*:*

We know, $$\beta = \frac{\Delta I_C}{\Delta I_B} \quad \text{...(i)}$$

$$\alpha = \frac{\Delta I_C}{\Delta I_E} \quad \text{...(ii)}$$

Also, $I_E = I_C + I_B$

or, $\Delta I_E = \Delta I_C + \Delta I_B$

or, $\Delta I_B = \Delta I_E - \Delta I_C$

Substituting the value of $\Delta I_B$ in Eq. (i) we get

or, $$\beta = \frac{\Delta I_C}{\Delta I_E - \Delta I_C}$$

or, $$\beta = \frac{\Delta I_C/\Delta I_E}{\Delta I_E/\Delta I_E - \Delta I_C/\Delta I_E} = \frac{\alpha}{1-\alpha}$$

$\therefore$ $$\beta = \frac{\alpha}{1-\alpha}$$

The above relation clearly shows that as $\alpha$ approaches unity, $\beta$ approaches infinity. In other words, the current gain in common emitter configuration is very high. It is because of this reason that this circuit arrangement is generally used in almost all of transistor applications.

***Collector Current*** In *CE* configuration, the input current is $I_B$ and the output current is $I_C$. These currents are related by the equations:

$$I_E = I_C + I_B \quad \text{...(i)}$$

and $$I_C = \alpha I_E + I_{CBO} \quad \text{...(ii)}$$

or $$I_C = \alpha (I_C + I_B) + I_{CBO}$$

$$I_C(1 - \alpha) = \alpha I_B + I_{CBO}$$

or $$I_C = \frac{\alpha}{1-\alpha} I_B + \frac{1}{1-\alpha} I_{CBO} = \frac{\alpha}{1-\alpha} I_B + (\beta + 1) I_{CBO} \quad \text{...(iii)}$$

In *CE* configuration, if the base circuit is open (i.e. $I_B = 0$), as shown in Fig. 3.17, the collector current will be the current to the emitter. This current is abbreviated as $I_{CEO}$; this means collector-emitter current with base open.

Hence,

$$I_{CEO} = \frac{\alpha}{1-\alpha} I_B + I_{CEO} = \beta I_B + I_{CEO} \left[ \because \frac{1}{1-\alpha} = \beta + 1 \right]$$

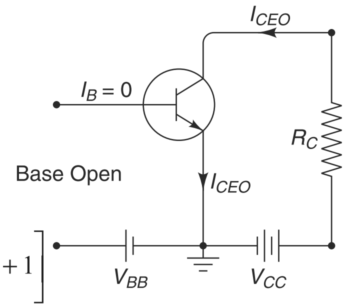



**Fig. 3.17** ***CE configuration with $I_B = 0$***

Substituting this value in exp ...(iii) we get,

$$I_C = \frac{\alpha}{1-\alpha} I_B + I_{CEO} = \beta I_B + I_{CEO}$$

**3.2** Find the value of $\beta$ for a transistor operated in *CE* configuration having $\alpha = 0.99$ and a reverse saturation current $I_{CBO} = 20$ μA. Also calculate the emitter current and the collector current if the base current is 300 μA.

***Solution***

We know $\beta = \frac{\alpha}{1-\alpha}$ Here $\beta \frac{0.99}{1-0.99} = 99$

The collector current $I_C = \beta I_B + I_{CBO} (1 + \beta)$

$= 99 \times 300 \times 10^{-6} + 20 \times 10^{-6} (1 + 99)$

$= 31.7$ μA

If $I_E$ be the emitter current then we can write $\left| \frac{I_E}{I_B} \right| = 1 + \beta$

i.e., $I_E = (1 + 99) \times 300 \times 10^{-6} = 30$ μA

**3.3** In a transistor, operating in *CE* mode, the collector current changes from 3 μA to 3.5 μA when the collector emitter voltage changes from 10 V to 15 V. If the base current remains constant at 50 μA calculate output resistance, $\beta$ and $\alpha$ at $V_{CE} = 15$ V.

***Solution***

$$\text{Output resistance} = \frac{\text{Change in collector emitter voltage}}{\text{Change in collector current}}$$

$$= = \frac{15 - 10}{(3.5 - 3) \times 10^{-3}} = \frac{5}{0.5} = 10\,\text{K}\Omega$$

$$\beta = \frac{I_C}{I_B} = \frac{3.5 \times 10^{-3}}{50 \times 10^{-6}} = 0.07 \times 10^3 = 70$$

$\because$
$$\beta = \frac{\alpha}{1 - \alpha}$$

$\therefore$
$$\alpha = \frac{\beta}{1 - \beta} = \frac{70}{70 + 1} = 0.986.$$

## 3.6.4 Characteristics of Common Emmiter Connection

For determining the characteristics of a transistor in *CE* configuration, the circuit arrangement is shown in Fig 3.18. In *CE* circuit, the supply voltage connected between base and emitter is represented by $V_{BE}$ and between collector and emitter by $V_{CE}$. The actual voltages working across the input and the output sides are represented by $V_{BE}$ and $V_{CE}$ respectively, which can be varied by potentiometers $R_1$ and $R_2$ respectively.

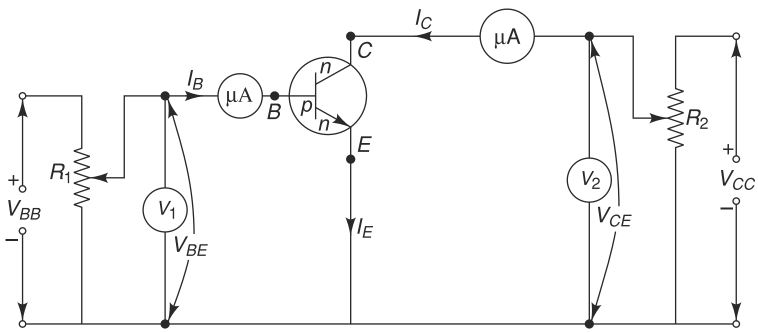



**Fig. 3.18** ***Circuit connection to determine common emmiter characteristics***

***1. Input Characteristics*** This is the characteristic for the input side and is the characteristic between base current ($I_B$) and base-emitted voltage $V_{BE}$ at constant collector emitter voltage ($V_{CE}$).

This can be obtained by varying the potentiometer and reading is taken on voltmeter ($V_1$) and micrometer (µA), keeping the value of voltmeter ($V_2$) constant. Figure 3.19 shows the input characteristic.

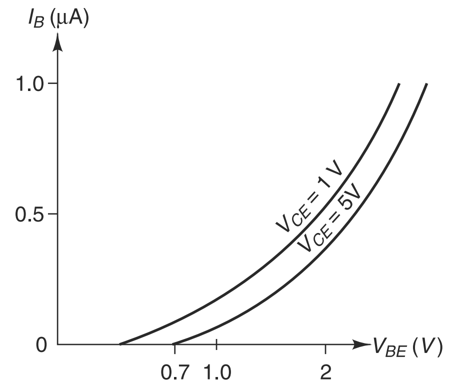



**Fig. 3.19** ***Input characteristic in CE configuration***

It may be noted here that

(a) The characteristic is similar to that obtained in the case of input characteristic in *CB* configuration.

(b) As compared to *CB* configuration, $I_B$ increases less rapidly with increase in $V_{BE}$.

*Input resistance* This is the resistance offered by the input ($V_{BE}$) junction in the path of input current. This is the ratio of the change in base emitter voltage ($\Delta V_{BE}$) to the corresponding change in the base current ($\Delta I_B$) at constant collector emitter voltage ($V_{CE}$).

Thus the input resistance $r_i = \dfrac{\Delta V_{BE}}{\Delta I_B}$ (at constant $V_{CE}$) (Fig. 3.20).

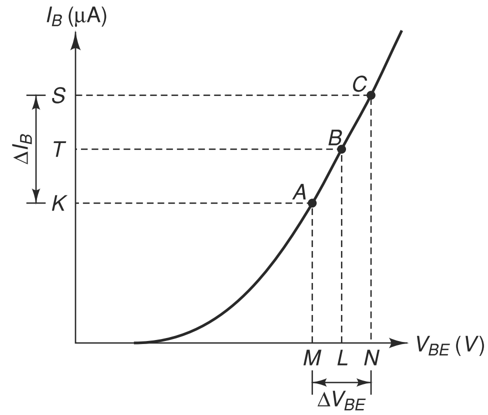



**Fig. 3.20** ***Profile of input resistance in CE connection***

**2. *Output Characteristics*** This is the curve for the output side of a *CE* transistor. In other words, this is the curve between collector current ($I_C$) and collector emitter voltage ($V_{CE}$), keeping base current ($I_B$) at constant value.

For obtaining the characteristic, readings are taken in the voltmeter ($V_2$) and milli-ammeter (mA) by varying the potentiometer $R_2$ and keeping the value of microammeter (μA) at a constant value.

The output characteristic is shown in Fig. 3.21(a). A family of curves has been drawn. This characteristic is very important as the transistor is used as an amplifier mostly in *CE* configuration.

Figure 3.21(b) shows only one curve for giving better concept. The characteristic has three regions:

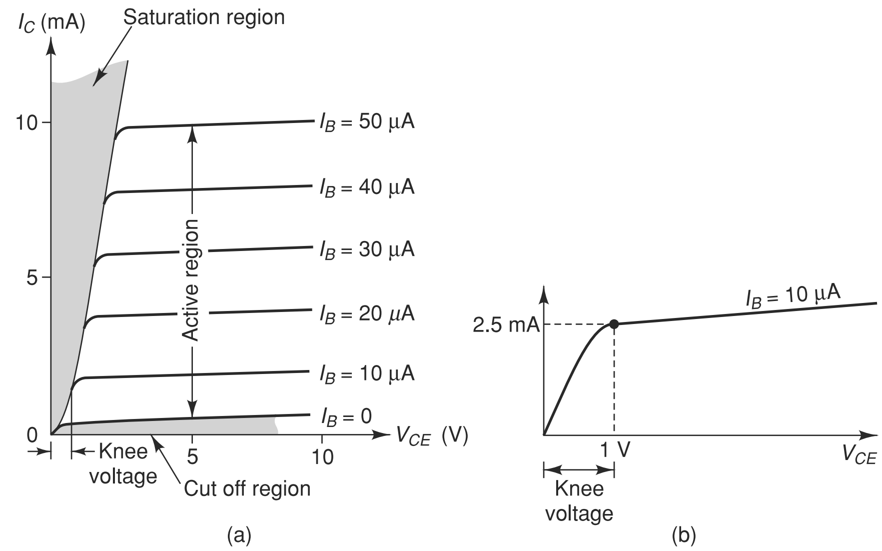



**Fig. 3.21** ***Output characteristic of CE transistor***

***Active Region*** In this region $I_C$ slightly increases with increase in $V_{CE}$. Hence $\beta$ is also increasing with the increase of $V_{CE}$.

As an amplifier, the transistor operates in active region.

***Saturation Region*** When $V_{CE}$ falls below $V_{BE}$, $I_C$ decreases rapidly. At this stage, collector base junction is also forward biased. In other words, both the junctions of the transistor become forward biased. Then $I_C$ is dependent of $I_B$, therefore the transistor is *said* to be working in saturation region. Actually a transistor at this stage acts as a closed switch. The transistor becomes 'fully ON' and carries constant and saturated current.

***Cut off Region*** When $I_B = 0$, $I_C$ is not zero but it is equal to $I_{CEO}$.

The transistor behaves as an open switch, the region is therefore called a "cut off region". The transistor is said to be "OFF" as it carries a very small and negligible current in microamperes.

When a transistor is used as an ON/OFF switch (e.g., in oscillators), it works alternately in 'Saturation' and 'cut off' regions.

***Output Resistance*** This is the resistance offered by output junction (*CE* junction in this case) in the path of the load current.

This is the ratio of the change in the collector emitter voltage $\Delta V_{CE}$ to the corresponding change in the collector current $I_C$ at constant base current ($I_B$). Output resistance

$$r_o = \Delta V_{CE} / \Delta I_C \quad \text{at constant } I_B.$$

The output resistance of a transistor in *CE* connections is about 50 K.

### 3.6.5 Common Collector Connection

In this configuration the collector of the transistor is made a common terminal. The input is applied between base and collector and the output is obtained between emitter and collector. Figure 3.22 shows an *npn* transistor and Fig. 3.23 a *pnp* transistor in *CC* configuration.

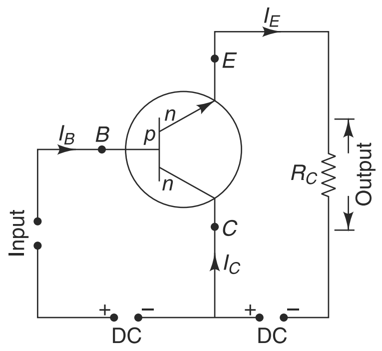



**Fig. 3.22** *CC connection of npn transistor*

***1. Current Amplification Factor (γ)*** The ratio of the output current ($I_E$) to the input current ($I_B$) is called current amplification factor ($\gamma$). The ratio of change in emitter current ($\Delta I_E$) to the change in the base current ($\Delta I_B$) is also the current amplification factor $\gamma$ in common collector circuit.

$$\therefore \qquad \gamma = \frac{\Delta I_E}{\Delta I_B}$$

This value of current amplification or gain ($\gamma$) is same as $\beta$ as that in the case of *CE* circuit ($\beta$).i.e, between 20 and 500

$$\gamma \cong \beta \cong 20 \text{ to } 500.$$

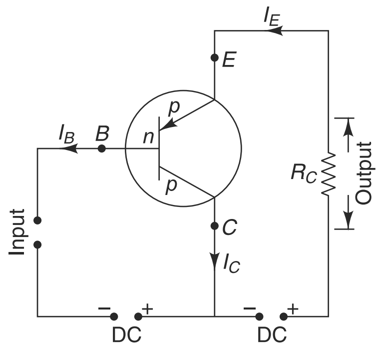



**Fig. 3.23** *CC connection of pnp transistor*

***Relation between γ and α***

We know that

$$\gamma = \frac{\Delta I_E}{\Delta I_B}$$

$$= \frac{\Delta I_E}{\Delta I_E - \Delta I_C} \; [\Delta I_E = \Delta I_B + \Delta I_C \text{ or } \Delta I_B = \Delta I_E - \Delta I_C]$$

Dividing the numerator and demumerator by $\Delta I_E$

$$\gamma = \frac{\Delta I_E / \Delta I_E}{\Delta I_E / \Delta I_E - \Delta I_C / \Delta I_E}$$

$$\gamma = \frac{1}{1 - \alpha} \left[ \frac{\Delta I_C}{\Delta I_E} = \alpha \right]$$

***2. Collector Current***

We know that

$$I_E = I_C + I_B$$

or $$I_E = (\alpha I_E + I_{CBO}) + I_B \quad [I_C = \alpha I_E + I_{CBO}]$$

$$I_E - \alpha I_E = I_B + I_{CBO}$$

$$I_E(1 - \alpha) = I_B + I_{CBO}$$

or $$I_E = \frac{1}{1-\alpha} \cdot I_B + \frac{1}{1-\alpha} I_{CBO}$$

or $$I_E = (\beta + 1)I_B + (\beta + 1)\, I_{CBO} \left[\frac{\alpha}{1-\alpha} = \beta \therefore \frac{1}{1-\alpha} = \beta + 1\right]$$

The following points may be noted:

1. The CC (common collector) circuit has a very high input resistance ($r_i$) of about 750 KΩ and output resistance ($r_0$) of about 25 ohms. The voltage gain therefore of this circuit is very less (< 1). This configuration is not used for amplification. However due to low output resistance it is used for impedance matching purposes.
2. This circuit is also called an 'emitter follower'.

[When the input (ac signal) goes through its positive half cycle, the output across the load also goes through its positive half cycle. In other words, input and output are in the same phase. In magnitude also the output is almost equal to the input. Thus we see that the output (emitter voltage) follows the input hence the name 'emitter follower'.]

Relation between $\alpha$, $\beta$ and $\gamma$ would be as follows in CC connection.

$$\gamma = \frac{1}{1-\alpha} = \frac{\alpha}{1-\alpha} + 1 = \beta + 1$$

## 3.7 DIFFERENT OPERATING CONDITIONS OF A TRANSISTOR

In the previous articles, we have seen, how a transistor works. When the emitter junction of a transistor is forward biased and the collector junction is reverse biased it is said to be operated in its active region. However, as a transistor has two junctions (which can be biased in different ways) it may work in different conditions as listed in the table below:

| *Condition* | *Emitter junction* | *Collector junction* | *Region of operation* |
|---|---|---|---|
| I FR | Forward-biased | Reverse-biased | Active |
| II FF | Forward-biased | Forward-biased | Saturation |
| III RR | Reverse-biased | Reverse-biased | Cut-off |
| IV RF | Reverse-biased | Forward-biased | Inverted |

***I. FR (Forward-Reverse Condition)*** In this condition, emitter junction is *forward* biased whereas the collector junction is reverse biased. The transistor is in active region and collector current depends upon the emitter current. Generally, transistor is operated in this region for amplification.

***II. FF (Forward-Forward Condition)*** In this condition, base emitter and collector junctions are forward biased. The transistor is in saturation and the collector current becomes independent of the base current. The transistor acts like a closed switch.

***III. RR (Reverse-Reverse Condition)*** In this condition, both the junctions are reverse biased. The emitter does not emit charge carriers into the base and no carriers are collected by the collector (except a little thermally generated minority carriers). Thus, the transistor acts like an open switch.

***IV. RF (Reverse Forward Condition)*** In this condition, the emitter junction is reverse biased where as the collector junction is forward biased. As the collector is not doped to the extent as the emitter is doped, therefore it cannot emit the majority carriers to the base. In this case very poor transistor action is achieved. Thus the emitter collector terminals of a transistor are not interchangeable.

## 3.8 TRANSISTOR AS AN AMPLIFIER

A transistor is a device which raises the strength of weak signal and thus acts as an amplifier. The basic transistor amplifier circuit is shown in Fig. 3.24. The input (weak signal) is applied across emitter-base and the output (amplified signal) is obtained across the load resistor $R_c$ connected in the collector circuit. It may be noted that a dc voltage $V_{EE}$ is applied in the input circuit in addition to the signal to achieve faithful amplification. This dc voltage $V_{EE}$ keeps the emitter-base junction under forward biased condition regardless of the polarity of the signal and is known as bias voltage.

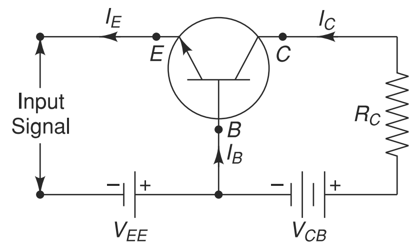



**Fig. 3.24** ***Operation of transistor as an amplifier***

When a weak signal is applied at the input, a small change in the signal voltage causes an appreciable change in emitter current (say a change 0.1 V in signal voltage causes a change of 1 mA in emitter current) as the input circuit has very low resistance. This causes almost the same change in collector current due to transistor action. The collector circuit has a load resistor $R_C$ of high value (say 10 KΩ) and when collector current flows through such a high resistance, it produces a large voltage drop ($V_O$ = 10 kΩ × 1 mA = 10 V) across it.

Thus a weak signal (0.1 V) applied at the input circuit appears in the amplified form (10 V) in the collector circuit.

## 3.9 COMPARISON BETWEEN THREE TRANSISTOR CONFIGURATIONS

It has been seen that a transistor can be connected in any one of the three configurations i.e. *CB, CE* and *CC* configuration. Its behaviour is different in different configurations. The configuration in which a transistor is to be connected depends upon the requirement, e.g., *CC* configuration has very high input resistance but a small value of voltage gain, therefore, it is best suited for impedance matching.

Hence, it is concluded that for the application of a particular configuration its characteristics must be known, and to know the characteristics of the three configurations at a glance, we study their comparison in the tabular form given below:

| *S. No.* | *Characteristics* | *Common-base configuration* | *Common-emitter configuration* | *Common-collector configuration* |
|---|---|---|---|---|
| 1. | *Input resistance* | Low | Low | Very high |
| 2. | *Output resistance* | Very high | High | Low |
| 3. | *Current gain* | Less than unity (0.98) | High (100) | High (100) |
| 4. | *Voltage gain* | Small (150) | High (500) | Less than one |
| 5. | *Leakage current* | Very small (5 uA for Ge 1 uA for Si) | Very large (500 μA for Ge 20 uA for Si) | Very large (500 μA for Ge 20 uA for Si) |
| 6. | *Applications* | For high frequency applications | For audio frequency applications | For impedance matching |

## 3.10 TRANSISTOR LOAD LINES

The dc load line on the output characteristic of a transistor is the locus of $V_{CE}$ and $I_C$. Figure 3.25(a) shows an *n-p-n* transistor in dc conditions only; Figure 3.25(b) shows its output-characteristic.

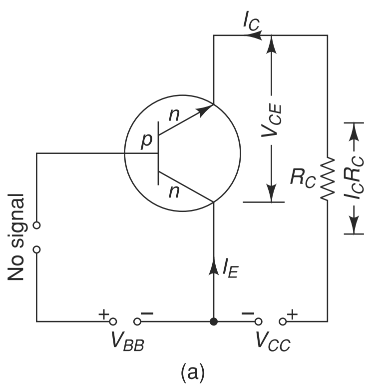



**Fig. 3.25(a)** ***npn transistor in dc condition***

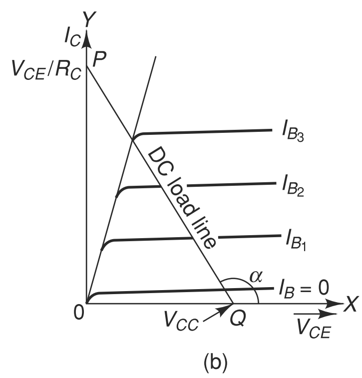



**Fig. 3.25(b)** ***Ouput characteristic and dc load line***

Applying Kirchoff's law on the collector side, we get

$$V_{CC} = V_{CE} + I_C R_C$$

This is the equation of a straight line, which can be drawn on the output characteristic of the transistor and it is called the *dc load line*.

Next we will see how to draw the dc load line *AB:*

$I_C$ will be maximum when $V_{CE} = 0$. With $V_{CE} = 0$ in the load line equation, we get $V_{CC} = 0 + I_C R_C$

or, $$I_C = \frac{V_{CC}}{R_C}$$

This gives point *P* on *Y*-Axis.

$V_{CE}$ will be maximum when $I_C = 0$; with $I_C = 0$ in the load ine equation, we get

$$V_{CE} = V_{CC}$$

This gives point *Q* on the *X*-axis. Now join *AB*, to get the dc load line.

The slope $\alpha$ of the line is $-1/R_C$ and therefore, it will be decided by the value of $R_C$ which is the dc load. When signal will be applied, we shall consider ac load which will be different from this load. This is the reason why the line is called dc load line.

Also, $$V_{CC} = V_{CE} + I_C R_C$$

or, $$\frac{V_{CC}}{R_C} = \frac{V_{CE}}{R_C} + \frac{I_C R_C}{R_C}$$

or, $$\frac{V_{CC}}{R_C} = \frac{1}{R_C}(V_{CE}) + I_C$$

or, $$I_C = -\frac{1}{R_C} V_{CE} + \frac{V_{CC}}{R_C}$$

Comparing it with the standard form of the equation for the straight line $y = mx + c$, it is seen that the slope $m = -1/R_C$.

## 3.11 OPERATING POINT (QUIESCENT, *Q*-POINT)

We can find directly the values of $I_C$ and $V_{CE}$ at any time during the transistor action. These values are known as operating point ('quiescent or *Q*-point'). This name is given as transistor is operating in quiescent (dc) conditions. Moreover, the transistor is silent as no signal has been introduced to work upon.

The *Q*-point can be defined as 'the zero signal (no signal) value of $V_{CE}$ and $I_C$ of the transistor'. In other words $V_{CE}$ and $I_C$ are coordinates of *Q*-point.

Suppose we are to find coordinates of *Q*-point when the base current $I_B = 20$ μA. The point of intersection of the load line and the base current line ($I_B = 20$ μA) be *Q*-point. Next draw perpendiculars from *Q* on *X*- and *Y*-axes. The coordinates of *Q*-point are 4 V, 2 mA.

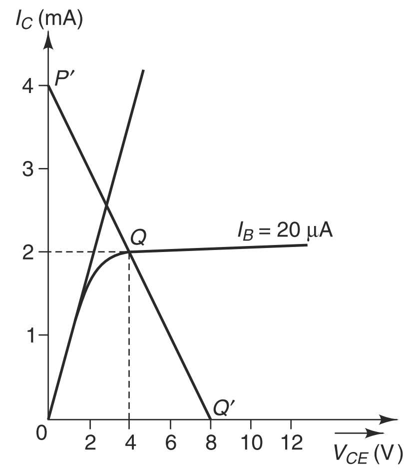



**Fig. 3.26** ***Q point of transistor (Load line = P′Q′)***

## 3.12 SIGNIFICANCE OF OPERATING POINTS ON TRANSISTOR CHARACTERISTICS

The output characteristic of a transistor along with dc load line is shown in Fig. 3.27. It has following points:

***(i) Cut off Point*** The point, where the load line intersects the $I_B = 0$ curve is called 'cut off point'. The transistor acts as an off (open) switch and a very small negligible current ($I_{CEO}$) flows. The *BE* junction does not remain forward biased and transistor action is lost. At this point $V_{CE} = V_{CC}$.

***(ii) Saturation Point*** This is the point when load line intersects the curve of saturated value of base current $I_B$ (sat). At this point, $I_B$ as well as $I_C$ are maximum, i.e., saturated. At this point the transistor acts as a short circuited (closed) switch. Here also collector base junction does not remain reverse biased and transistor action is again lost.

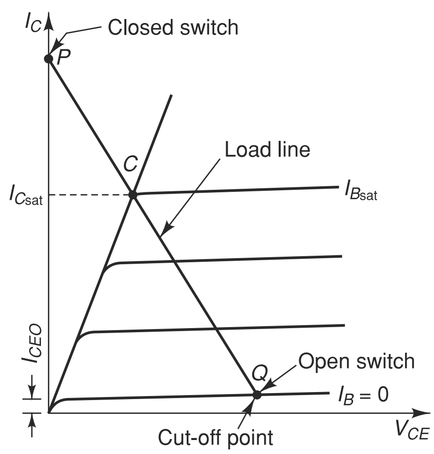



**Fig. 3.27** ***Output characteristic with dc load line***

At saturation $I_C$ (sat) = $V_{CC}/R_C$ and $I_C$ does not increase even if $I_B$ increases.

The region between OFF and saturation points is called 'active' region. The transistor remains ON, as the *BE* (base emitter) junction remains forward biased and *CE* junction reverse biased during this period. The transistor is fully ON between points *P* and *Q*.

## 3.13 TRANSISTOR RATINGS AND SPECIFICATIONS

$V_{CEO}$ is defined as the *maximum collector to emitter voltage* with base open. It is the collector emitter breakdown voltage and is a very important factor particularly in switching transistors; also used for designing switched mode power supplies.

$V_{CBO}$ is the *collector to base breakdown voltage* with emitter open. $V_{CBO}$ determines the output breakdown voltage when the transistor is connected in common base configuration. $V_{ER}$ is the maximum base emitter voltage of the transistor.

$I_{C\ (max)}$ is the peak collector current rating of the transistor (either continuous or pulsed).

$P_{D(max)}$ is the *maximum power dissipation capability* of the transistor. It is usually specified at a given ambient temperature and it should be derated at higher temperatures as per the derating curve supplied by the manufacturer. Also, in the absence of the heat sink, the dissipation capability will be much less. The actual dissipation capability can be determined from the known values of maximum operating junction temperature, ambient temperature and the thermal resistances involved.

The *turn-on time* ($t_{ON}$) is the sum of *delay time* ($t_d$) and *rise time* ($t_r$). Delay time is the time taken by the collector current to change from its OFF-state value to 10 percent of its final ON-state value after the turn-on pulse is applied at the base. The rise time ($t_r$) is the time taken for the collector current to rise from 10 percent to 90 percent of its final ON-state value.

The *turn-off time* ($t_{OFF}$) is the sum of storage time ($t_s$) which is the time for the collector current to change from its final ON-state value to 90 percent of its final ON-state value and the fall time ($t_f$) which is the time for the collector current to change from 90 percent of its final ON-state value to 10 percent of its final ON-state value.

$C_{ob}$ is the *output capacitance* of the transistor when connected in common base configuration. It is particularly useful in switching and high frequency applications.

## 3.14 THERMAL RUNAWAY

Thermal runaway is a phenomenon related to the bias stability in transistors. The change of temperature causes variation in the reverse saturation current $I_{CO}$ of the transistor. For every 10°C rise in temperature, $I_{CO}$ attains twice its original value. Accordingly, $I_C\,[= \beta I_\beta + (1 + \beta)\, I_{CO}]$ increases with the increase of temperature. This further raises the junction temperature which in turn increases $I_{CO}$ also. It is a cumulative process causing the junction temperature to exceed its rated value. This may result in burn out or failure of a transistor due to thermal instability. This phenomenon is known as thermal runaway.

**3.4** Find the base current and the collect or current of an *NPN* transistor whose current gain is 0.97 and operated in *CB* configuration. The emitter current is 5 μA and the reverse saturation current is $I_{CBO}$ = 20 μA.

***Solution***

For an *NPN* transistor,

$I_C = \alpha I_E + I_{CBO}$ where $I_C$ and $I_E$ are the collector and emitter current respectively.

Here $\quad I_E = 5$ mA, $I_{CBO} = 20$ µA and $\alpha = 0.97$

$\therefore \quad I_C = 0.97 \times 5 \times 10^{-3} + 20 \times 10^{-6} = 4.87$ mA

For *NPN* transistor, we have

$= I_E + I_C + I_B = 0$

This gives, $\quad I_B = (I_E - I_C) = 5 - 4.87 = 0.13$ mA.

**3.5** Determine the collector emitter voltage of a transistor shown in Fig. 3.28 operating in *CE* mode. Assume $\beta = 100$ and base emitter voltage is 1 V.

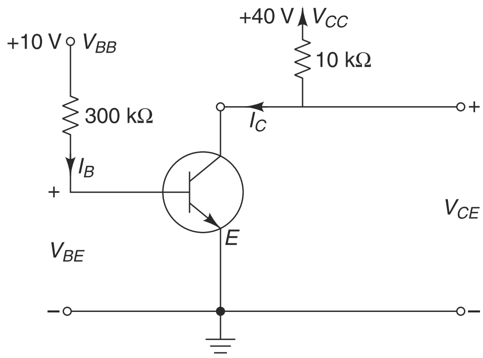



**Fig. 3.28**

***Solution***

Given, $\quad \beta = 100$

$V_{BE} = 1$ V, $V_{BB} = 10$ V; $R_B = 300$ kΩ

$$\therefore \quad I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{10 - V_{BE}}{300 \times 10^3} \frac{10 - 1}{300 \times 10^3} \text{ A} = 0.03 \text{ mA}$$

Also, $\quad I_C = \beta I_B = 100 \times 0.03$ mA $= 3$ mA

Now the collector emitter voltage can be obtained

as, $\quad V_{CE} = 40 - I_C \times 10 \times 10^3$

$= 40 - 3 \times 10^{-3} \times 10 \times 10^3 = 10$ V.

**3.6** An *NPN* transistor shown in Fig. 3.29 has $\beta = 100$ and $I_{CO} = 0.05$ mA. It is operated in *CE* configuration. If the base emitter voltage is 1 V find the base, collector and emitter current.

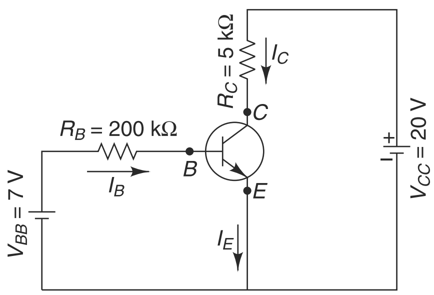



**Fig. 3.29**

***Solution***

The base current

$$I_B = \frac{V_{BB} - V_{BE}}{R_B}, \text{ where } V_{BE} \text{ is the base emitter voltage}$$

$$\therefore \qquad I_B = \frac{7-1}{200 \times 10^3} = 0.03 \text{ mA}$$

The collector current

$$I_C = \beta I_B = 100 \times 0.03 \times 10^{-3} = 3 \text{ mA}$$

Applying Kirchhoff's law in collector loop,

$$-V_{CC} + I_C R_C + V_{CB} + V_{BE} = 0$$

$$\therefore \qquad V_{CB} = 20 - 3 \times 10^{-3} \times 5 \times 10^3 - 1 = 4 \text{ V}$$

As $V_{CB}$ is positive hence the collector junction is reverse biased. The emitter current $I_E = -(I_C + I_B) = -(3 + 0.03) = -3.03$ mA

**3.7** In the circuit shown in Fig. 3.30 the voltage $V_{BE}$ and $V_{CE}$ at saturation is 1 and 0.5 respectively. Obtain the values of $I_C$ and $I_B$.

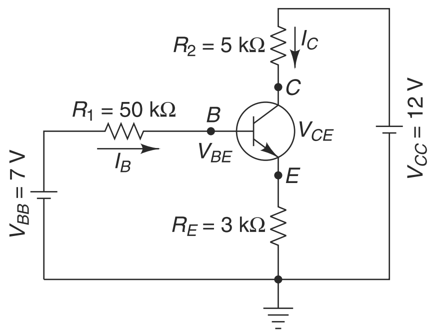



**Fig. 3.30**

***Solution***

Applying Kirchhoff's voltage law to the base circuit

$$R_1 I_B + V_{BE} + R_E(I_B + I_C) - V_{BB} = 0$$

$$\therefore \qquad 50 \times 10^3\, I_B + V_{BE} + 3 \times 10^3\, (I_B + I_C = 7 \qquad \ldots\text{(i)}$$

Applying Kirchhoff's voltage law to the collector circuit

$$R_2 I_C + V_{CE} + R_E(I_B + I_C) - V_{CC} = 0$$

or, $$5 \times 10^3\, I_C + V_{CE} + 3 \times 10^3 (I_B + I_C) = 12 \qquad \ldots\text{(ii)}$$

Solving Eq. (i) and Eq. (ii) and substituting the values of $V_{BE}$ and $V_{CE}$ at saturation

$$I_B = 0.097 \text{ mA and } I_C = 2.07 \text{ mA}$$

**3.8** What is the significance of thermal resistance of a power transistor? How do you define ($\theta_{J-C}$) and ($\theta_{C-A}$).

***Solution***

The thermal resistance is one of the parameters that indicates the power handling capability of a transistor. Quantitatively, it is given by the ratio of rise in transistor junction temperature with respect to the power dissipated. Obviously, thermal resistance is measured in °C/Watt.

There is another parameter called maximum junction temperature (more precisely, it is a transistor rating) which tells us as to what temperature of the transistor junction, the device will retain its properties. An estimate of transistor junction temperature can be made only when the thermal resistance parameter is also known in addition to the quantum of power dissipated.

One of the thermal resistances encountered with power transistors is ($\theta_{J-C}$) called thermal resistance from junction to case. If $\theta_J$ and $\theta_C$ are respectively the junction and case temperatures in °C for a power dissipated equal to $P$ watts, then

$$\theta_{J-C}\left(\frac{\theta_J - \theta_C}{P}\right)$$

The above expression can be used to obtain the value of ($\theta_J$) from known values of ($\theta_{J-C}$), ($\theta_C$) and ($P$).

Another thermal resistance normally encountered is ($\theta_{C-A}$). Mathematically,

$$\theta_{C-A}\left(\frac{\theta_C - \theta_A}{P}\right)$$

where $\theta_A$ = Ambient temperature.

So, if $\theta_{C-A}$ and $\theta_{J-C}$ are known, we can determine the junction temperature from the knowledge of ambient temperature and power dissipated.

**3.9** Derive an expression for the power gain to be provided by transistor $Q$ of Fig. 3.31.

***Solution***

Input power = Input voltage × Input current

Input voltage $= I_b R_{IN}$

Therefore, Input power $= I_b R_{IN} \times I_b$

$= I_b^2 R_{IN}$

Output power = Output Voltage × Output current

$= I_C R_C \times I_C$

$= \beta I_b R_C \times \beta I_b$

$= (\beta^2 I_b^2 \; R_C)$

Power gain $= \dfrac{\text{Output power}}{\text{Input power}}$

$$= \left[\frac{\beta^2 I_b^2 \, R_C}{I_b^2 \, R_{IN}}\right]$$

$$= \beta^2 \frac{R_C}{R_{IN}}$$

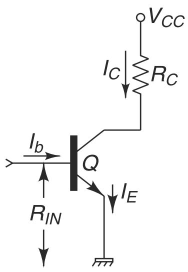



Fig. 3.31

*3.10* The input impedance, as illustrated in the previous question (Example 3.9), affects the power gain provided by the transistor. How is it then that input impedance parameter does not figure in the transistor manuals?

***Solution***

The input impedance parameter does figure in the usual list of parameters. However, it is not given as such as it comprises of a number of components. It is usually specified as a maximum base to emitter voltage ($V_{BE}$) under specified input current conditions.

## EXERCISES

1. Define a transistor. Why is a transistor a bipolar device?
2. What are the three terminals of a transistors? What are their characteristics.
3. Why is the emitter region heavily doped and the base region lightly doped in a transistor?
4. Describe the operating principle of (i) *npn* and (ii) *pnp* transistor under biased condition.
5. Discuss how a transistor can be used as current amplifier.
6. Why common base connection is rarely used for amplication purposes?
7. Draw and explain the input and output characteristics of transistor in common base configuration.
8. What is base current amplification factor referred to common emitter connection of transistor?
9. With respect to common base output characteristic of transistor, explain the active, saturation and cutoff region.
10. Define input and output resistance of transistor connected in common base configuration.
11. Explain the current amplification factors $\alpha$ and $\beta$ for common base and common emitter configuration respectively of a transistor. Obtain relation between them.
12. Draw the common emitter circuit of a junction transistor. Sketch its output characteristic and indicate the active, saturation and cutoff regions.
13. Obtain the relation between $\alpha$, $\beta$ and $\gamma$ referred to common collector configuration of transistor.
14. What are the different operating conditions of a transistor?
15. Explain how transistor load line can be obtained graphically.
16. What do you mean by Quiescent point in a transistor?
17. What is the significance of thermal resistances of a power transistor?
18. What are the specifications of a transistor?

# MULTIPLE CHOICE QUESTIONS

1. $\alpha$ and ß of a BJT are related as [WBUT 2012]

   (a) $\alpha = \dfrac{\beta + 1}{\beta}$ (b) $\beta = \dfrac{\alpha}{1 - \alpha}$

   (c) $\beta = \dfrac{\alpha}{1 + \alpha}$ (d) $\alpha = \dfrac{\beta}{\beta - 1}$

2. The load line of a transistor amplifier circuit is a graph between

   (a) $V_{CE}$ and $I_B$ (b) $V_{CE}$ and $I_C$

   (c) $V_{BE}$ and $I_C$ (d) $V_{BE}$ and $I_B$

3. The transistor operates in cut-off region if

   (a) collector junction is forward biased and the emitter junction is reverse biased.

   (b) both collector and emitter junctions are reverse biased.

   (c) both collector and emitter junctions are forward biased.

   (d) collector junction is reverse biased and the emitter junction is forward biased.

4. Early effect in a transistor is known as

   (a) zener breakdown

   (b) thermal breakdown

   (c) reduction in width of base or base narrowing

   (d) avalanche breakdown

5. When transistors are used in digital circuits, they usually operate in the

   (a) active region

   (b) breakdown region

   (c) saturation and cut-off regions

   (d) linear region

6. In CE configuration, an emitter resistor is used for

   (a) ac signal by pass (b) collector bias

   (c) higher gain (d) stabilization

7. To operate properly, a transistor‘s emitter base junction must be forwarded biased with reverse bias applied to which junction?

   (a) Collector base (b) Base emitter

   (c) Base collector (d) Collector emitter

8. The ends of a load line on a family of curves determine

   (a) the operating point (b) power curve

   (c) amplification factor (d) saturation and cut-off

9. If $V_{CC}$ = 18 V, the voltage divider resistors are 4.7 kΩ and 1500Ω, the base bias voltage is
   (a) 4.35V (b) 2.9 V
   (c) 8.7 V (d) 0.7 V
10. The CB configuration is used to provide _________ gain.
   (a) power (b) resistance
   (c) current (d) voltage
11. The *Q*-point on a load line may be used to determine
   (a) $V_B$ (b) $V_{CC}$
   (c) $I_C$ (d) $V_C$
12. The current gain for a common base configuration where $I_E$ = 4.2 mA and $I_C$ = 4 mA is
   (a) 1.05 (b) 0.95
   (c) 16.8 (d) 0.2
13. If a 3 mV signal produces 2 V output, the voltage gain is [WBUT 2009, 2012]
   (a) 1000 (b) 100
   (c) 0.004 (d) 0.001
14. Most of the electrons in the base of an *n-p-n* transistor flow
   (a) out of the base load (b) into the emitter
   (c) into the base supply (d) into the collector.
15. What is the collector current for a CE configuration with $\beta$ =100 and a base current of 50 $\mu$A?
   (a) 0.5 $\mu$A (b) 5 mA
   (c) 5 MA (d) 50 $\mu$A
16. The input/output relationship of the common collector and common base amplifiers is
   (a) 180 degrees (b) 90 degrees
   (c) 0 degrees (d) 270 degrees
17. In a transistor, the collector current is controlled by
   (a) base current (b) collector voltage
   (c) collector resistance (d) all the above
18. The CC circuit is used in a BJT for [WBUT 2011, 2012]
   (a) switching (b) impedance matching
   (c) high voltage gain (d) amplification
19. The output impedance of a transistor in CB configuration is [WBUT 2004, 2012]
   (a) low (b) zero
   (c) high (d) moderate
20. The arrow in the circuit symbol of *n-p*-n transistor is directed from
   (a) emitter to base (b) base to emitter
   (c) collector to base (d) base to collector

21. The arrow in the circuit symbol of a *p-n-p* transistor is directed from
    (a) emitter to base (b) base to emitter
    (c) collector to base (d) base to collector
22. The number of depletion layers in a transistor is
    (a) one (b) two
    (c) three (d) four
23. The base current as a percentage of emitter current in a transistor is
    (a) 2% (b) 90%
    (c) 70% (d) 40%
24. The emitter of a transistor is
    (a) moderately doped (b) lightly doped
    (c) heavily doped (d) undoped
25. The base of a transistor is
    (a) moderately doped (b) lightly doped
    (c) heavily doped (d) undoped
26. The common base current gain $\alpha$ is defined as
    (a) $\frac{I_E}{I_C}$ (b) $\frac{I_C}{I_E}$
    (c) $\frac{I_C}{I_B}$ (d) $\frac{I_B}{I_E}$
27. The common collector current gain $\gamma$ is defined as
    (a) $\frac{I_E}{I_C}$ (b) $\frac{I_C}{I_E}$
    (c) $\frac{I_C}{I_B}$ (d) $\frac{I_B}{I_E}$
28. The common emitter current gain $\beta$ is defined as
    (a) $\frac{I_E}{I_C}$ (b) $\frac{I_C}{I_E}$
    (c) $\frac{I_C}{I_B}$ (d) $\frac{I_E}{I_B}$
29. In a CE circuit, the BJT has
    (a) low current and voltage gain
    (b) high current and voltage gain
    (c) low current and high voltage gain
    (d) high current and low voltage gain

30. If the transistor operates in active region during common emitter configuration
   (a) $I_E = \beta I_B$ (b) $I_B = \beta I_C$
   (c) $I_B = \beta I_E$ (d) $I_C = \beta I_B$
31. The collector current in a common base circuit is
   (a) $I_C = I_E + \alpha I_{CO}$ (b) $I_C = \frac{\alpha}{1+\alpha} I_B + \frac{\mathbf{I}_{CO}}{1+\alpha}$
   (c) $I_C = \frac{\alpha}{1-\alpha} \mathrm{I}_B + \frac{\mathbf{I}_{CO}}{1-\alpha}$ (d) $I_C = \alpha I_E - I_{CO}$
32. With the positive probe on *n-p-n* base, an ohmmeter reading between the other transistor terminals should be
   (a) infinite (b) open
   (c) high resistance (d) low resistance
33. An *n-p-n* transistor with $\alpha = 0.99$ operates in common base configuration. If the emitter current is 50 mA and the reverse saturation current is 15$\mu$A, the collector current is
   (a) 0.485 mA (b) 15 $\mu$A
   (c) 49.515 mA (d) 49.5 mA
34. In a transistor amplifier circuit, $\alpha = 0.99$, $I_{CO} = 10\ \mu$A and the base current is 70 $\mu$A. The collector current is
   (a) 7.93 mA (b) 7 mA
   (c) 1 mA (d) none of these
35. The cut-off point on the dc load line is
   (a) $V_{CE} = \mathrm{V}_{CC}$, $I_C = 0$ (b) $V_{CE} = 0$, $I_C = 0$
   (c) $V_{BE} = V_{CC}$, $I_C = 0$ (d) $V_{BE} = 0$, $I_B = 0$
36. In basis compensation circuits, devices used are
   (a) thermistor (b) diode
   (c) diode and thermistor (d) none of these
37. The leakage current $I_{CBO}$ flows through
   (a) the emitter and base terminals
   (b) the collector and emitter terminals
   (c) the base and collector terminals
   (d) the emitter, base and collector terminals
38. The emitter of a transistor is heavily doped
   (a) to reduce injection efficiency
   (b) to increase injection efficiency
   (c) to drift change carriers in the base region
   (d) all of these

39. In a transistor, the reverse saturation current $I_{CO}$
    (a) becomes double for each 20°C rise in temperature
    (b) becomes double for each 50°C rise in temperature
    (c) becomes double for each 1°C rise in temperature
    (d) becomes double for each 10°C rise in temperature
40. Which configuration is called voltage buffer?
    (a) CE (b) CB
    (c) CC (d) None of these
41. In a transistor, the largest and smallest currents are
    (a) $I_E, I_B$ (b) $I_C, I_B$
    (c) $I_B, I_E$ (d) $I_B, I_C$
42. In a transistor operating in CE mode, the collector current changes from 4 $\mu$A to 4.5 μA when the collector emitter voltage changes from 15 mV to 20 mV. The output resistance is
    (a) 100 Ω (b) 10 Ω
    (c) 1 kΩ (d) 10 kΩ
43. A transistor shows amplifying property when operated in
    (a) reverse active region (b) forward active region
    (c) saturation region (d) cutoff region
44. The_________circuit of a BJT cannot be used for amplification.
    (a) CC (b) CB
    (c) CE (d) CE with base open
45. When the base width of a BJT becomes zero, the transistor is said to be in
    (a) cut-off (b) break down
    (c) saturation (d) punch through
46. The collector characteristics of a transistor in common emitter configuration can be used to find its
    (a) base current (b) voltage gain
    (c) output resistance (d) input resistance
47. The leakage current $I_{CEO}$ is equal to $I_{CBO}$ when
    (a) $\alpha = 0.1$ (b) $\alpha = 0.5$
    (c) $\alpha = 1$ (d) $\alpha = 0$
48. The region of a transistor with largest area is the
    (a) emitter (b) base
    (c) collector (d) collector-base junction
49. In a BJT, the CE mode has
    (a) high $r_i$ and low $r_o$ (b) low $r_i$ and low $r_o$
    (c) low $r_i$ and high $r_o$ (d) high $r_i$ and high $r_o$

50. The base current for an *n*-*p*-n transistor in CB configuration with emitter current of 8 mA and $\alpha = 0.975$ is
    (a) 0.2 mA (b) 7.8 mA
    (c) 10 mA (d) 8 mA
51. A transistor having a high input impedance and a low output impedance is operating in [WBUT 2012]
    (a) CB mode (b) CE mode
    (c) CC mode (d) inverted mode

## ANSWERS

| | | | | |
|---|---|---|---|---|
| 1. (b) | 2. (b) | 3. (b) | 4. (c) | 5. (c) |
| 6. (d) | 7. (a) | 8. (d) | 9. (a) | 10. (d) |
| 11. (a) | 12. (b) | 13. (a) | 14. (d) | 15. (b) |
| 16. (c) | 17. (a) | 18. (b) | 19. (c) | 20. (b) |
| 21. (a) | 22. (b) | 23. (a) | 24. (c) | 25. (b) |
| 26. (b) | 27. (d) | 28. (c) | 29. (b) | 30. (d) |
| 31. (c) | 32. (d) | 33. (c) | 34. (a) | 35. (a) |
| 36. (c) | 37. (c) | 38. (b) | 39. (d) | 40. (c) |
| 41. (a) | 42. (d) | 43. (b) | 44. (a) | 45. (d) |
| 46. (c) | 47. (d) | 48. (c) | 49. (c) | 50. (a) |
| 51. (c) | | | | |

## UNIVERSITY QUESTIONS WITH ANSWERS

1. A diode is used with a capacitor across ac supply. Why is the voltage across the diode clamped to a fixed polarity? [WBUT 2006]
   *Answer:* The charging time of a capacitor is small. Capacitor is charged instantaneously to the voltage of the circuit.
2. A full-wave bridge rectifier is fed from a 15 V rms ac source and is connected across a 100 ohm load. Calculate (i) PIV (ii) rms current drawn from the supply, and (iii) average dc current across the load.
   *Answer:* [WBUT 2006, 2013]
   Given: $V_{\text{rms}} = V = 15$ V, and $R_L = 100$ ohms
   Then maximum input voltage is

$$V_m = \sqrt{2}\,V = \sqrt{2} \times 15 \text{ V} = 21.21 \text{ V}$$

The $\quad \text{PIV} = V_m = \sqrt{2} \times 15 \text{ V} = 21.21 \text{ V}$

The peak value of current is

$$I_m = \frac{V_m}{R_L} = \frac{21.21}{100} = 0.2121 \text{ A}$$

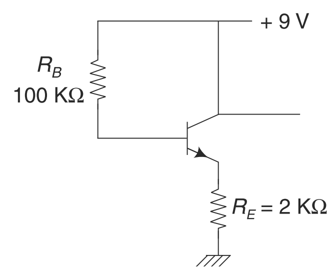
The rms value of current is

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} = \frac{0.2121}{\sqrt{2}} = 0.15 \text{ A}$$

The dc load current is

$$I_{\text{dc}} = \frac{2I_m}{\pi} = \frac{2 \times 0.2121}{\pi} = 0.135 \text{ A}$$

3. Find the values of (i) $I_B$, (ii) $I_E$, (iii) $V_{CE}$, (iv) $V_E$ and $V_B$ for the following circuit. Assume $\beta = 49$ and $V_{BE} = 0.7$ V. [WBUT 2011]




*Answer:* The circuit can be redrawn as

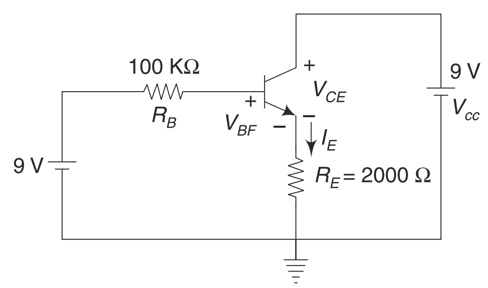



**Fig. 1**

$$9 = 100{,}000\, I_B + I_E \times 2000 + V_{BE}$$

or, $$9 = 100{,}000\, \frac{I_c}{\beta} + \frac{I_c}{\alpha} \times 2000 + 0.7$$

$$\beta = 49$$

∴ $$\alpha = \frac{\beta}{\beta + 1} = \frac{49}{50} = 0.98$$

∴ $$I_C = 2.0335 \times 10^{-3} \text{ A}$$

∴ $$I_B = \frac{I_c}{\beta} = 4.15 \times 10^{-5} \text{ A}$$

and $$I_E = \frac{I_c}{\alpha} = 2.075 \times 10^{-3} \text{ A}$$

$V_E = I_E R_E = 4.15$ V

$V_B = I_B R_B = 4.15$ V

$V_{CE} = -R_E I_F + V_{CC} = 4.85$ V

4. (a) What is thermal runaway? Can we interchange the emitter and collector of a transistor? In what region of the characteristic curve of a transistor? In what region of the characteristic curve does a transistor operate when it is used as a switch [WBUT 2011, 2013, 2014]

*Answer:* Refer Article 3.14.

A transistor consists of three differently doped semiconductor regions: emitter, base and collector region. The amount of doping in an emitter is much higher than that of the collector. Moreover, the collector-base junction has much larger area than the emitter-base junction. The collector surrounds the emitter region making the value of $\alpha$ close to unity and a large value of $\beta$. As the emitter is heavily doped and the collector is lightly doped, a large reverse bias voltage is required to break down the collector base junction. Hence in normal operation, the collector-base junction is reverse biased. If the collector and emitter junction is interchanged, the transistor operation is shifted from forward active mode to reverse mode due to decrease in the value of $\alpha$ and $\beta$.

A transistor operates in the cut-off region when it is used as a switch.

(b) What do you mean by load line for a transistor circuit?

*Answer:* Refer Article 3.10.

5. (a) Discuss the static characteristics of a transistor in CB configuration. 5 [WBUT 2012]

*Answer:* Refer Article 3.6.2.

6. (a) The reverse saturation current of an *NPN* transistor operating in CB configuration is 10 $\mu$A. For an emitter current of 2.4 mA, the collector current is 2.26 mA. Calculate the current gain and base current. 5 [WBUT 2012]

*Answer:* $I_E = 2.4$ mA

$I_C = 2.26$ mA

$I_{CBO} = 0.01$ mA

$I_C = \alpha I_E + I_{CBO}$

∴ Current gain $\alpha = \dfrac{I_C - I_{CBO}}{I_E} = 0.9375$

$I_E = I_C + I_B$

∴ $I_B = 0.14$ mA

7. The three terminals of a p-n-p transistor are
   (a) Emitter Base Drain (b) Anode Cathode Neutral
   (c) Source Base Drain (d) Emitter Base Collector
   [WBUT 2013]

   *Answer:* (d)

8. (a) Derive relation between α and β of a transistor
   (b) Find α and $I_C$ of a transistor with β = 49 and $I_E$ = 12 mA [WBUT 2013]

   *Answer:* (a) Refer Article 3.6.3.

$$\text{(b) } \alpha = \frac{\beta}{1+\beta} = \frac{49}{1+49} = 0.98$$

$$I_C = \alpha I_E = 0.98 \times 12 \text{ mA} = 11.76 \text{ mA}$$

9. (a) Compare the characteristics of transistors in CE, CC and CB mode
   (b) Draw and explain the input and output characteristics of CE connection of transistors [WBUT 2013]

   *Answer:* Refer Article 3.9 and 3.6.4

10. Write short note on Dc load line and $Q$ point [WBUT 2013, 2014]

    *Answer:* Refer Article 3.10 and 3.11

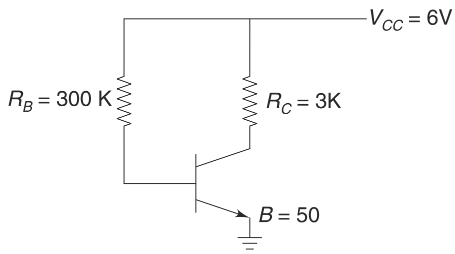
11. If α = 0.98 then β is [WBUT 2014]
    (a) 0.49 (b) 49
    (c) 50 (d) 0.5

    *Answer:* (b)

12. Draw the circuit diagram and output characteristics of a common emitter transistor showing different regions [WBUT 2014]

    *Answer:* Refer article 3.6.3 and 3.6.4

13. Calculate $V_{CE}$ and $I_C$ in the circuit below Assume $V_{BE}$ = 0.7 V.




[WBUT 2014]

*Answer:* Apply KVL in the base emitter circuit

$$V_{CC} = I_B R_B + V_{BE}$$

$$6 = I_B \times 300 \times 10^3 + 0.7$$

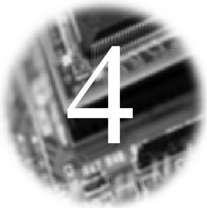
$$\therefore \quad I_B = \frac{6 - 0.7}{300 \times 10^3}\ \text{A} = 0.0177\ \text{mA}$$

$$I_C = \beta I_B = 50 \times 0.0177\ \text{mA} = 0.885\ \text{mA}$$

$$V_{CE} = V_{CC} - I_C R_C$$

$$= 6 - 0.885 \times 10^{-3} \times 3 \times 10^3$$

$$= 3.345\ \text{V}$$



# 4 JUNCTION TRANSISTORS: MODELLING, AMPLIFICATION AND BIASING

## 4.1 HYBRID PARAMETER EQUIVALENT CIRCUIT

Figure 4.1 shows a four-terminal (or two-port) network. Out of the four quantities $v_1$, $i_1$, $v_2$ and $i_2$ we can take any two as independent variables and express the remaining two in terms of these independent variables. If $i_1$ and $v_2$ are taken as independent variables we get the hybrid parameter equations, i.e.,

$$v_1 = h_{11}i_1 + h_{12}v_2 \tag{4.1}$$

$$i_2 = h_{21}i_1 + h_{22}v_2 \tag{4.2}$$

The quantities $h_{11}$, $h_{12}$, $h_{21}$ and $h_{22}$ are known as hybrid (i.e. $h$) parameters and defined as under.

$h_{11} = \left.\dfrac{v_1}{i_1}\right|_{v_2=0}$ = input impedance with output short circuited, and expressed in ohm, (4.3)

$h_{12} = \left.\dfrac{v_1}{v_2}\right|_{i_1=0}$ = reverse open circuit voltage transfer ratio; it has no unit, (4.4)

$h_{21} = \left.\dfrac{i_2}{i_1}\right|_{v_2=0}$ = forward short circuit current transfer ratio or short circuit current gain; it has also no unit, (4.5)

$h_{12} = \left.\dfrac{i_2}{v_2}\right|_{i_1=0}$ = output admittance with input open circuited; it is expressed in siemens (4.6)

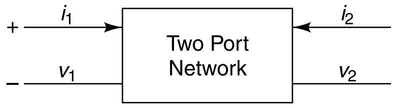



**Fig. 4.1** ***Two-port network***

If the two-port network does not contain any reactive element (i.e., $L$ and or $C$), $h_{11}$, $h_{12}$, $h_{21}$ and $h_{22}$ will be real numbers. However, if the network contains reactive elements and the excitation is sinusoidal, the voltages and currents would be represented by phasors $V_1$, $I_1$, $V_2$, $I_2$ and the $h$ parameters would be functions of frequency. Figure 4.2 shows the equivalent circuit of a transistor amplifier based on $h$ parameters.

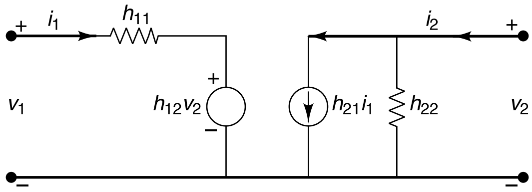



**Fig. 4.2** ***h parameter equivalent circuit of transistor amplifier***

## 4.2 TRANSISTOR *h* PARAMETER EQUIVALENT CIRCUIT FOR DIFFERENT CONFIGURATIONS

When the variations about the quiescent operating point are small, the transistor parameters can be assumed to be constant and the transistor can be represented by an equivalent circuit. In a vacuum tube, the input current (i.e., grid current) is negligible and therefore the equivalent circuit for a tube amplifier circuit is simple. However, in a transistor, current exists at the input terminals. Therefore an equivalent circuit for a transistor amplifier is more complex than that for a tube amplifier. The $h$ parameters, for a transistor, are the easiest to measure and the manufacturers quote the $h$ parameters. Therefore a transistor is usually represented by $h$ parameter equivalent circuit. The following convenient alternative subscript notation has been suggested by IEEE:

11 = input ($i$), 22 = output ($o$)
21 = forward transfer ($f$), 12 = reverse transfer ($r$).

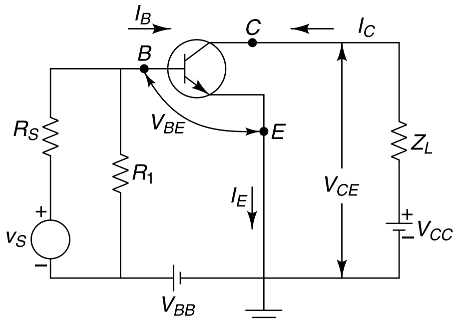
When the above subscripts are used for transistors, another subscript ($b$, $e$ or $c$) is added to denote the configuration, e.g.,

$h_{ie} = h_{11e}$ = input impedance in common emitter configuration.
$h_{fb} = h_{21b}$ = short circuit forward current gain in common base configuration.

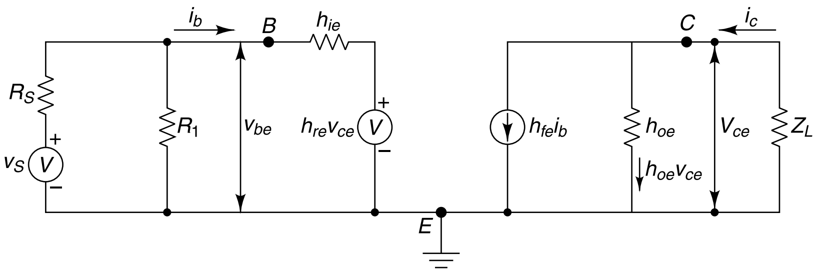
Figure 4.3(a) shows the common emitter amplifier circuit. The voltage $V_{BE}$ is a function $f_1$ of $i_B$ and $v_{CE}$ and the current $i_c$ is another function of $f_2$ of $i_B$ and $v_{CE}$, i.e.

$$v_{BE} = f_1(i_B, v_{CE}) \tag{4.7}$$

$$i_C = f_2(i_B, v_{CE}) \tag{4.8}$$




***(a) [B or b stands for base; C or c stands for collector and E or e stands for emitter of the junction transistor]***




***(b)***

**Fig. 4.3** ***CE amplifier (a) circuit (b) equivalent circuit***

Equation (4.7 and 4.8) can be expanded by Taylor's series around the quiescent values $I_B$ and $V_{CE}$. Neglecting the higher order terms we can write:

$$\Delta v_{BE} = \left.\frac{\delta f_1}{\delta i_B}\right|_{V_{CE}} \Delta i_B + \left.\frac{\delta f_1}{\delta i_{CE}}\right|_{I_B} \Delta v_{CE} \tag{4.9}$$

$$\Delta i_C = \left.\frac{\delta f_2}{\delta i_B}\right|_{V_{CE}} \Delta i_B + \left.\frac{\delta f_2}{\delta v_{CE}}\right|_{I_B} \Delta v_{CE} \tag{4.10}$$

While taking the partial derivatives, $V_{CE}$ or $I_B$ are kept constant as indicated in the above equations.

The quantities $\Delta v_{EB}$, $\Delta v_{CE}$, $\Delta i_B$ and $\Delta i_C$ are incremental voltages and currents. Then we can write equations (4.9 and 4.10) as:

$$v_{be} = h_{ie}i_b + h_{re}v_{ce} \tag{4.11}$$

$$i_c = h_{fe}i_b + h_{oe}v_{ce} \tag{4.12}$$

where $h_{ie} = \left.\dfrac{\delta f_1}{\delta i_B}\right|_{V_{CE}}$ ; $h_{re} = \left.\dfrac{\delta f_1}{\delta V_{CE}}\right|_{I_B}$

$$h_{fe} = \frac{\delta f_2}{\delta i_B}\Big|_{V_{CE}} \qquad h_{oe} = \frac{\delta f_2}{\delta V_{CE}}\Big|_{I_B}$$

Equations (4.11 and 4.12) are $h$ parameter equations for a two port network. Therefore the $CE$ circuit of Fig. 4.3(a) can be replaced by the equivalent circuit of Fig. 4.3(b).

Figure 4.4(a) shows the common base ($CB$) amplifier circuit. The terminal equations for this configuration are

$$v_{eb} = h_{ib}i_e + h_{rb}v_{cb} \tag{4.13}$$

$$i_c = h_{fb}i_e + h_{ob}v_{cb} \tag{4.14}$$

Figure 4.4(b) shows the $h$-parameter equivalent circuit for common base configuration.

Figure 4.5(a) shows the common collector ($CC$) amplifier circuit.

The terminal equations for this $CC$ amplifier circuit are

$$v_{bc} = h_{ic}i_b + h_{rc}v_{ec} \tag{4.15}$$

$$i_e = h_{fc}i_b + h_{oc}v_{ec} \tag{4.16}$$

It is sometimes more convenient to write $h$ parameter equation in generalized notation $i_1$, $v_1$, etc. The correlation of generalized notation to the circuit configuration has been shown in Table 4.1. The equivalent circuits are valid for both $p$-$n$-$p$ and $n$-$p$-$n$ transistors.

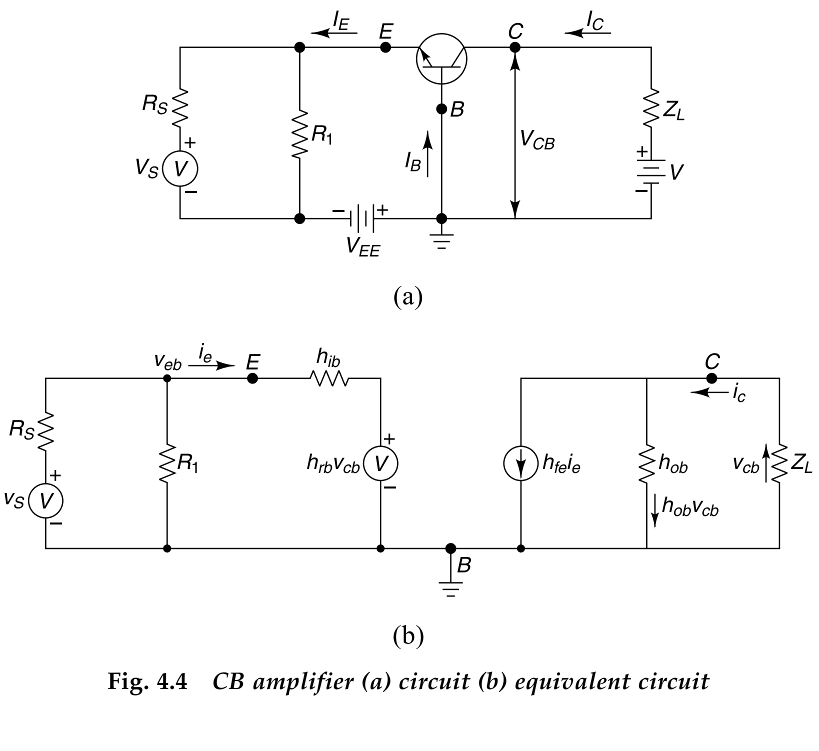
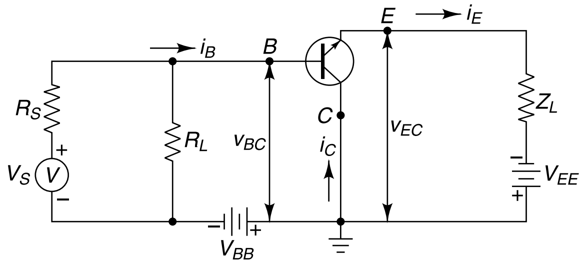
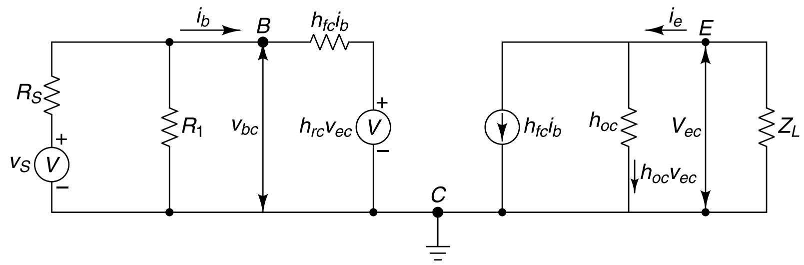



**Fig. 4.4** ***CB amplifier (a) circuit (b) equivalent circuit***




(a)




(b)

**Fig 4.5** ***CC amplifier (a) circuit (b) equivalent circuit***

**Table 4.1** *Correlation of generalized two-port notation to transistor amplifier configurations*

| *Generalized notation* | *Units* | *CE* | *CB* | *CC* |
|---|---|---|---|---|
| $i_1$ | A | $i_b$ | $i_e$ | $i_b$ |
| $v_1$ | V | $v_{be}$ | $v_{eb}$ | $v_{bc}$ |
| $i_2$ | A | $i_c$ | $i_c$ | $i_e$ |
| $v_2$ | V | $v_{ce}$ | $v_{cb}$ | $v_{ec}$ |
| $h_{11}$ | ohms | $h_{ie}$ | $h_{ib}$ | $h_{ic}$ |
| $h_{12}$ | — | $h_{re}$ | $h_{rb}$ | $h_{rc}$ |
| $h_{21}$ | — | $h_{fe}$ | $h_{fb}$ | $h_{fc}$ |
| $h_{22}$ | siemens | $h_{oe}$ | $h_{ob}$ | $h_{oc}$ |

The transistor data supplied by manufacturers gives the $h$ parameters for only one configuration. Sometimes the parameters specified are $h_{fe}$, $h_{ib}$, $h_{ob}$ and $h_{rb}$. These can be easily be converted to $h$ parameters for the desired configuration from the definitions of these parameters given in Eqs (4.7 to 4.10). Table 4.2 gives these conversion formulae.

From the definitions of $h$ parameters we note that

$$h_{fb} = \alpha = \frac{i_c}{i_e} \quad \text{and} \quad h_{fe} = \beta$$

**Table 4.2** *Conversion formulae for h parameters*

| | CE | CC | CB | CC |
|---|---|---|---|---|
| $h_{11}$ | $h_{ie} \approx \dfrac{h_{ib}}{1+h_{fb}}$ | $h_{ic} \approx \dfrac{h_{ib}}{1+h_{fb}}$ | $h_{ib} \approx \dfrac{h_{ie}}{1+h_{fe}}$ | $h_{ic}$ |
| $h_{12}$ | $h_{re} \approx \dfrac{\Delta h_b - h_{rb}}{1+h_{fb}}$ | $h_{rc} \approx 1$ | $h_{rb} \approx \dfrac{\Delta h_e - h_{re}}{1+h_{fe}}$ | $h_{rc}$ |
| $h_{21}$ | $h_{fe} \approx \dfrac{-h_{fb}}{1+h_{fb}}$ | $h_{fc} \approx \dfrac{-1}{1+h_{fb}}$ | $h_{fb} \approx \dfrac{-h_{fe}}{1+h_{fe}}$ | $h_{fc}$ |
| $h_{22}$ | $h_{oe} \approx \dfrac{-h_{ob}}{1+h_{fb}}$ | $h_{oc} \approx \dfrac{-h_{ob}}{1+h_{fb}}$ | $h_{ob} \approx \dfrac{-h_{oe}}{1+h_{fe}}$ | $h_{oc}$ |

where $\Delta h_b = h_{ib}h_{ob} - h_{fb}h_{rb}$

$\Delta h_e = h_{ie}h_{oe} - h_{fe}h_{re}$

In the *CE* configuration the parameter $h_{12}$ (i.e., $h_{re}$) is usually very small. Hence the voltage source $h_{re}\, v_{ce}$ in Fig. 4.3(b) can be considered as a short circuit.

## 4.3 TRANSISTOR LINEAR CIRCUIT ANALYSIS

A transistor amplifier can be replaced by its $h$ parameter equivalent circuit for small values of signals. The equivalent circuit is used to relate only the ac components of the voltage and current which are caused by the signal. The direct voltage and current levels produced by the power supply $V_{CE}$ need not be included in the equivalent circuit because the total response is the sum of dc and ac components. Therefore the equivalent circuit includes only those elements which affect the values of ac components. The steps in formulating the equivalent circuit are:

1. Replace the transistor by its $h$ parameter equivalent circuit depending on the configuration.
2. Connect the elements, which are external to the transistor and which affect the ac values, to the appropriate terminals as in the circuit diagram.

We can write generalized equations for the five figures of merit, viz. current gain $A_i$, and voltage gain $A_v$, power gain $A_p$, input impedance $Z_i$ and output impedance $Z_o$. The $h$ parameter equations are:

$$v_1 = h_{11}i_1 + h_{12}v_2 \tag{4.17}$$

$$i_2 = h_{21}i_1 + h_{22}v_2 \tag{4.18}$$

We can write Eq. (4.18) as:

$$0 = h_{21}i_1 + \left(h_{22} + \frac{1}{Z_L}\right)v_2 \tag{4.19}$$

We can solve Eqs (4.17 and 4.19) to find $i_1$ and $v_2$, the results are

$$i_1 = \frac{[h_{22} + (1/Z_L)]\,v_1}{h_{11}h_{22} - h_{12}h_{21} + (h_{11}/Z_L)} \tag{4.20}$$

$$v_2 = \frac{-h_{21}v_1}{h_{11}h_{22} - h_{12}h_{21} + (h_{11}/Z_L)} \tag{4.21}$$

***(a) Current Gain*** Since $v_2 = -Z_L i_2$ we can write Eq. (4.19) as:

$$0 = h_{21}i_1 - h_{22}Z_L i_2 - i_2$$

Current Gain $\quad (A_i) = \dfrac{-i_2}{i_1} = \dfrac{-h_{21}}{1 + h_{22}Z_L}$

***(b) Voltage Gain*** From Eq. (4.21):

Voltage gain $\quad (A_v) = \dfrac{v_2}{v_1} = \dfrac{-h_{21}Z_L}{h_{11} + Z_L(h_{11}h_{22} - h_{12}h_{21})} \quad (4.22)$

***(c) Power Gain*** The power gain ($A_p$) is the ratio of output power to input power

$$\therefore \quad A_p = \frac{-v_2 i_2}{v_1 i_1} = A_v A_i \tag{4.23}$$

***(d) Input Impedance*** The input impedance is defined as the impedance seen at the input terminals looking towards the load with load impedance $Z_L$ connected across output terminals. Therefore,

$$Z_i = \frac{v_1}{i_1} = \frac{Z_L(h_{11}h_{22} - h_{12}h_{21}) + h_{11}}{h_{22}Z_L + 1} \tag{4.24}$$

It is seen from Eq. (4.24) that input impedance depends on the load impedance. This is because in a transistor amplifier circuit the input is not isolated from the load circuit.

***(e) Output Impedance*** The output impedance ($Z_o$) is defined as the impedance at the output terminals looking backwards towards the source with the source resistance connected but the source voltage short circuited, i.e.,

$$Z_o = \left.\frac{v_2}{i_2}\right|_{v_s = 0} \tag{4.25}$$

From equations (4.17) and (4.18), we can write

$$0 = (R_s' + h_{11})i_1 + h_{12}v_2 \quad (4.26)$$

$$i_2 = h_{21}i_1 + h_{22}v_2 \quad (4.27)$$

where $R_s' = R_s R_1/(R_s + R_1)$ [See Fig. 4.3, 4.4 and 4.5]

The solution of Eqs (4.26 and 4.27) gives

$$Z_o = \frac{v_2}{i_2} = \frac{R_s' + h_{11}}{R_s'h_{22} + h_{11}h_{22} - h_{12}h_{21}} \quad (4.28)$$

The characteristics of the parameters of *CE*, *CB* and *CC* configurations are as under.

| *Characteristic* | *CE* | *CB* | *CE* |
|---|---|---|---|
| Current Gain ($A_i$) | High | Low | Highest of all |
| Voltage Gain ($A_v$) | High | High | Low |
| Input Resistance ($R_i$) | Medium | Low | High |
| Output Resistance ($R_o$) | Medium | High | Low |

In an actual amplifier circuit some external impedance in addition to those shown in Figs 4.3 to 4.5 may also be present. These must also be taken into account in calculating the figures of merit for the circuit.

**4.1** In the common base amplifier circuit of Fig. 4.7 $R_S$ = 1200 ohms $R_1$ = 400 ohms $Z_L = R_L = 50 \times 10^3$ ohms. The parameters of the transistor, as referred to common base are, $h_i$ = 20 ohms $h_f$ = –0.92, $h_r = 4 \times 10^{-4}$ and $h_o = 0.5 \times 10^{-6}$ siemens. Find the current gain, voltage gain, power gain, input resistance and output resistance for the common base circuit.

***Solution***

$h_{11} = h_{ib}$ = 20 ohms

$h_{12} = h_{rb} = 4 \times 10^4$

$h_{21} = h_{fb} = -0.92$

$h_{22} = h_{ob} = 0.5 \times 10^{-6}$ siemens

$h_{11}h_{22} - h_{12}h_{21} = 20 \times 0.5 \times 10^{-6} - 4 \times 10^{-4}\,(-0.92) = 3.78 \times 10^{-4}$

Current gain $A_i = \dfrac{-h_{21}}{1 + h_{22}R_L} = \dfrac{0.92}{1 + 0.5 \times 10^{-6} \times 50 \times 10^3} = 0.89$

Voltage gain

$$A_v = \frac{-h_{21}R_L}{R_L(3.78 \times 10^{-4}) + h_{11}} = \frac{-(-0.92)(50 \times 10^3)}{(50 \times 10^3 \times 3.78 \times 10^{-4}) + 20} = 1182$$

Power gain $A_p = A_vA_i$ = (1182) (0.89) = 1051.9

$$Z_i = R_i = \frac{R_L(3.78 \times 10^4) + h_{11}}{h_{22}R_L + 1} = \frac{50 \times 10^3 \times 3.78 \times 10^{-4} + 20}{0.5 \times 10^{-6} \times 50 \times 10^3 + 1} = 37.95 \text{ ohms}$$

Because of the presence of resistance $R_1$, the input resistance seen by the source is

$$R_i' = \frac{37.95 \times 400}{37.95 + 400} = 34.66 \text{ ohms}$$

$$Z_o = R_o = \text{output resistance} = \frac{R_s' + h_{11}}{R_s' h_{22} + 3.78 \times 10^{-4}}$$

$$R_s' = \frac{R_s R_1}{R_s + R_1} = \frac{1200 \times 400}{1200 + 400} = 300 \text{ ohm.}$$

**4.2** The common emitter amplifier circuit uses the transistor having $h$ parameters of Example 4.1. Find: (a) $h$ parameters for $CE$ circuit; (b) current gain, voltage gain, power gain, input resistance and output resistance. The values of $R_s$, $R_1$ and $R_L$ are the same as in Example 4.1.

***Solution***

(a) Using Table 4.2, the $h$ parameters for $CE$ circuit are

$$h_{11} = h_{ie} = \frac{h_{ib}}{1 + h_{fb}} = \frac{20}{1 + (-0.92)} = 250 \text{ ohms}$$

$$h_{12} = h_{re} = \frac{h_{ib} h_{ob} - h_{fb} h_{rb} - h_{rb}}{1 + h_{fb}}$$

$$= \frac{20 \times 0.5 \times 10^{-6} + 0.92 \times 4 \times 10^{-4} - 4 \times 10^{-4}}{1 + (-0.92)} = 5.25 \times 10^{-4}$$

$$h_{21} = h_{fe} = \frac{-h_{fb}}{1 + h_{fb}} = \frac{0.92}{1 - 0.92} = 11.5$$

$$h_{22} = h_{oe} = \frac{h_{ob}}{1 + h_{fb}} = \frac{0.5 \times 10^{-6}}{1 + (-0.92)} = 6.25 \times 10^{-6} \text{ siemens}$$

(b) $$A_i = \frac{-h_{21}}{1 + h_{22} R_L} = \frac{-11.5}{1 + 6.25 \times 10^{-6} \times 50 \times 10^3} = -8.761$$

$$h_{11} h_{22} - h_{12} h_{21} = 250 \times 6.25 \times 10^{-6} - 5.25 \times 10^{-4} \times 11.5 = -44.755 \times 10^{-4}$$

$$A_v = \frac{-h_{21} R_L}{R_L(-44.755 \times 10^{-4}) + h_{11}} = \frac{-11.5 \times 50 \times 10^3}{50 \times 10^3 \times (-44.755 \times 10^{-4}) + 250}$$

$$= -21.86 \times 10^3$$

The minus sign indicates that output voltage is 180º out of phase with respect to input voltage.

$A_p = A_i A_v = (-8.761)(-21.86 \times 10^3) = 191510$

$$Z_i = R_i = \frac{R_L(-44.755 \times 10^{-4}) + h_{11}}{h_{22} R_L + 1} = \frac{50 \times 10^3 \times (-44.755 \times 10^{-4}) + 250}{6.25 \times 10^{-6} \times 50 \times 10^3 + 1}$$

$= 19.977$ ohms

Because of the presence of $R_1$ the input resistance as seen by the sources is:

$$R_i' = \frac{19.97 \times 400}{19.97 + 400} = 18.838 \text{ ohms}$$

$$Z_o = R_o = \frac{R_s' + h_{11}}{R_s' h_{22} + (-44.755 \times 10^{-4})} = \frac{300 + 250}{300 \times 6.25 \times 10^{-6} + (-44.755 \times 10^{-4})}$$

$$= -21.15 \text{ ohm}$$

**4.3** The common collector configuration of Fig. 4.5 uses the transistor of Example 4.2. (a) Find $h$ parameters based on common collector circuit. (b) If $R_s = 60 \times 10^3$ ohm, $R_1 = 30 \times 10^3$ ohm and $R_L = 1000$ ohm, find current, voltage and power gains and input and output impedance.

***Solution***

(a) Using the formulae in Table 4.2:

$h_{11} - h_{ic} = h_{ie} = 1080$ ohms
$h_{12} - h_{rc} = 1 - h_{re} = 1 - 24 \times 10^{-4} \approx 1$
$h_{21} = h_{fc} = -1 - h_{fe} = -1 - 49 = -50$
$h_{22} = h_{oc} = h_{oe} = 25 \times 10^{-6}$ siemens

(b) $$A_i = \frac{-h_{21}}{1 + h_{22}R_L} = \frac{50}{1 + 25 \times 10^{-6} \times 1000} = 48.78$$

$$h_{11}h_{22} - h_{12}h_{21} = 1080 \times 25 \times 10^{-6} - (1)(-50) = 50.027$$

$$A_v = \frac{-h_{21}R_L}{R_L(50.027) + h_{11}} = \frac{50 \times 1000}{(1000)(50.027) + 1080} = 0.978$$

$$A_p = A_i A_v = 48.78 \times 0.978 = 47.7$$

$$Z_i = R_i = \frac{R_L(50.027) + h_{11}}{h_{22}R_L + 1} = \frac{1000 \times 50.027 + 1080}{25 \times 10^{-6} \times 1000 + 1} = 49860 \text{ ohm}$$

Because of the presence of $R_1$, the input resistance seen by the source is

$$R_i' = \frac{49860 \times 30 \times 10^3}{49860 + 30 \times 10^3} = 18730.2 \text{ ohm}$$

$$R_s' = \frac{R_s R_1}{R_s + R_1} = \frac{60 \times 10^3 \times 30 \times 10^3}{60 \times 10^3 + 30 \times 10^3} = 20 \times 10^3 \text{ ohms}$$

$$Z_o = R_o = \frac{R_s' + h_{11}}{R_s' h_{22} + 50.027} = \frac{20 \times 10^3 + 1080}{20 \times 10^3 \times 25 \times 10^{-6} + 50.027}$$

$$= 417.2 \text{ ohms}$$

**4.4** Derive expressions for the amplifier's characteristics like current gain, voltage gain, input impedance and output impedance for a *CE* amplifier configuration fed from a source having an internal resistance if ($R_s$) and feeding a load ($R_L$) in terms of $h$-parameters. From the results obtained, briefly describe the effects of source resistance ($R_s$) and the load resistance ($R_L$) on the amplifier's characteristics.

***Solution***

The hybrid equivalent circuit for a *CE*-amplifier alongwith the source and the load is shown in Fig. 4.6. The equivalent circuit of Fig. 4.6 is similar to that shown in Fig. 4.3 and only $i_b$ and $i_c$ are replaced by $I_1$ and $I_2$ as well as $v_{be}$ and $v_{ce}$ by $V_1$ and $V_2$.

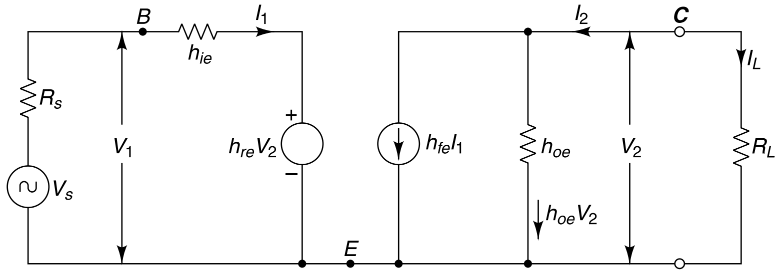



**Fig. 4.6**

*Current Gain*

$$\text{Current gain} = \frac{I_L}{I_1} = -\frac{I_2}{I_1}$$

From the equivalent circuit of Fig. 10.20

$$I_2 = h_{fe} I_1 + h_{oe} V_2$$

Now $V_2 = I_L R_L = -I_2 R_L$

Therefore, $I_2 = I_{fe} I_1 + h_{oe}(-I_2 R_L) = h_{fe} I_1 - h_{oe} I_2 R_L$

or $I_2(1 + h_{oc} R_L) = h_{fe} I_1$

or $I_2/I_1 = h_{fe}/(1 + h_{oe} R_L)$

The current gain ($A_i$) is therefore given by

$$A_i = \frac{-I_2}{I_1} = -\frac{h_{fe}}{(1 + h_{oe} R_L)}.$$

*Input Impedance (or Input Resistance)*

Input resistance, $R_i = \dfrac{V_1}{I_1}$

From the equivalent circuit of Fig. 4.6

$$V_1 = h_{ie} I_1 + h_{rc} V_2$$

or $$\frac{V_1}{I_1} = h_{ie} + h_{re}\left[\frac{V_2}{I_1}\right]$$

Now $V_2 = -I_2 R_L$

Therefore $$\frac{V_2}{I_1} = -\frac{I_2}{I_1} R_L = A_i R_L$$

Substituting the value of ($V_2/V_1$), we get the value of input resistance as

$$R_i = \frac{V_1}{I_1} = (h_{ie} + h_{re} A_i R_L)$$

*As* $h_{re} \cong 0, \quad R_i \cong h_{ie}$

*Voltage Gain*

Voltage gain, $A_v = \dfrac{V_2}{V_1}$

From the equivalent circuit of Fig. 4.6

$$V_2 = -I_2 R_L = -\left[\frac{I_2}{I_1}\right] R_L I_1 = A_i R_L I_1$$

$$\frac{V_2}{V_1} = \frac{A_i R_L I_1}{V_1} = \frac{A_i R_L}{(V_1 / I_1)} = \frac{A_i R_L}{R_i}$$

or Voltage gain $A_v = \dfrac{V_2}{V_1} = \dfrac{A_i R_L}{R_i} = -\dfrac{h_{fe}}{(1 + h_{oe} R_L)} \cdot \dfrac{R_L}{h_{ie}}$

*Output Impedance (or Output Resistance)*

Output resistance, $R_o = \dfrac{V_2}{I_1}$

Writing mesh equations on input and output sides in the equivalent circuit of Fig. 4.6, we get

$$V_s = (R_s + h_{ic})\, I_1 + h_{re}\, V_2 \qquad (1)$$

$$I_2 = h_{fe}\, I_1 + h_{oc}\, V_2 \qquad (2)$$

From Eq. (1) $I_1 = \dfrac{V_s - h_{re} V_2}{(R_s + h_{ie})}$

Therefore, from Eq. (2),

$$I_2 = \frac{V_s - h_{fe}}{R_s + h_{ie}} - \frac{h_{fe} h_{re} V_2}{R_s + h_{ie}} + h_{oe} V_2$$

or

$$I_2 = V_2\left[h_{oe} - \frac{h_{fe} h_{re}}{R_s + h_{ie}}\right] + \frac{V_s h_{fe}}{R_s + h_{ie}}$$

Now

$$V_s = \frac{V_1 (h_{ie} + R_s)}{h_{ie}}$$

This gives

$$I_2 = V_2\left[h_{oe} - \frac{h_{fe} h_{re}}{R_s + h_{ie}}\right] + \frac{V_1 (h_{ie} + R_s) h_{fe}}{h_{ie}(R_s + h_{ie})}$$

$$= V_2\left[h_{oe} - \frac{h_{fe} h_{re}}{R_s + h_{ie}}\right] + \frac{V_1 h_{fe}}{h_{ie}}$$

or $I_2/V_2 = G_o\ (= 1/R_o) = \left(h_{oe} - \dfrac{h_{fe} h_{re}}{R_s + h_{ic}}\right) + \dfrac{V_1 h_{fe}}{V_2 h_{ie}}$

$$\cong \left( h_{oe} = \frac{h_{fe} h_{re}}{R_s + h_{ie}} \right) \text{as} \frac{V_1 h_{fe}}{V_2\ h_{ie}} \cong 0 \text{ [where } R_o \text{ is the output resistance} = (1/G_o)]$$

It is clear from the above expressions that the current gain ($A_i$), voltage gain ($A_v$), input resistance ($R_i$) are dependent upon the load resistance ($R_L$). Also, the output resistance ($R_o$) is a function of source resistance ($R_s$).

## 4.4 TRANSISTOR BIASING

The basic function of a transistor is amplification (i.e., amplitude of a weak signal being amplified). Usually, a weak signal is given to the base circuit of a transistor and an amplified output is obtained at the collector. Shape of the signal must remain the same during amplification and this amplification of the signal without the change on its shape is known as *faithful amplification*.

In order to achieve faithful amplification, the emitter-base junction is kept in forward-biased while the collector-base junction is kept in the reverse-biased condition during all parts of the signal. This is known as *transistor biasing*.

In fact, to obtain faithful amplification, the *quiescent (Q)* point is set usually in the middle of the dc load line. To achieve this *transistor biasing* is needed.

## 4.5 CONCEPT OF FAITHFUL AMPLIFICATION

We have just indicated that the process by which the strength of a weak signal is amplified without changing its general shape is known as *faithful amplification.* The emitter junction remains in forward-biased and the collector junction in the reverse-based condition during the process of faithful amplification. This can be ensured by satisfying the following basic conditions:

(i) Minimum zero signal collector current
(ii) Minimum base-emitter voltage
(iii) Minimum collector-emitter voltage

The first two conditions [(i) and (ii)] ensure that the emitter-base junction is forward-biased during all parts of the signal, whereas, the third condition ensures that the collector-base junction is reversed-biased at all the times.

***(i) Minimum Zero Signal Collector Current*** When no signal is applied at the input, the current flowing through the collector is known as zero signal collector current ($I_C$). For faithfull amplification, the value of zero signal collector current must be more than or equal to the peak value of collector current due to signal alone, i.e.

Zero signal collector current ≥ peak value of collector current due to signal alone $I_C \geq i_c$ (peak)

***(ii) Minimum Base-Emitter Voltage*** In order to achieve faithful amplification, the potential barrier at the base-emitter junction must be wiped off. The value of potential barrier for Si transistor is 0.7 V and for Ge transistor it is 0.3 V. Therefore, the base-emitter voltage $V_{BE}$ should not fall below 0.7 V for Si and 0.3 V for Ge transistor. In fact, the base current $I_B$ is very small until the input voltage overcomes the potential barrier at the base-emitter junction. Once the potential barrier is overcome, the base current and hence the collector current increases quickly (Fig. 4.7).

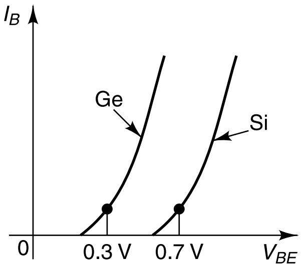



**Fig. 4.7** $I_B$ ***vs*** $V_{BE}$ ***Characteristics***

However, if at any part of the signal, the base-emitter voltage falls below the above-said values, then that part of the signal will not be amplified to the required extent and there will be distortion in the amplified signal.

***(ii) Minimum Collector-Emitter Voltage*** In order to achieve faithful amplification, the collector-emitter voltage $V_{CE}$ should not fall below 1.0 V for Si transistor and 0.5 V for Ge transistor, called *knee voltage*. When the value of $V_{CE}$ falls below the above-said values, the collector junction will not be properly reversed biased. Therefore, the collector will not be able to attract the charge carriers emitted by the emitter and hence a greater portion of them goes to the base. This decreases the collector current while the base current increases. Hence, the value of $\beta$-falls.

Hence, if the value of $V_{CE}$ falls below $V_{knee}$ during any part of the signal, that part will be less amplified due to reduced $\beta$. Thus, a distorted output signal is obtained.

## 4.6 DEFINITION OF TRANSISTOR BIASING

The process by which proper flow of zero signal collector current is obtained with the maintenance of proper collector-emitter voltage during the passage of signal is known as *transistor biasing*. The circuitry which provides the necessary conditions of transistor biasing is known as the biasing circuit.

Transistor biasing is done to keep the emitter junction forward-biased and the collector junction properly reverse biased during the application of signal. The biasing can be achieved either by using bias batteries $V_{BB}$ and $V_{CC}$. The latter method is frequently employed since it is more efficient. Transistor parameters (such as $\beta$ and $V_{BE}$) vary between wide limits even among different units of the same type. This inherent variation in the transistor parameters of the same type is because it may not be possible to control the base width. A minute variation in the base width changes its parameters.

The inherent variation of the transistor parameters may change the operating point, resulting in unfaithful amplification (i.e. distorted output signal). Therefore, while designing the biasing circuit, this property of transistors is taken into consideration. The biasing circuit is designed which can work with all the transistors of one type whatever may be the value of $\beta$ or $V_{BE}$.

## 4.7 STABILIZATION

The quiescent point is represented by zero signal collecetor current $I_C$ and $V_{CE}$. If either of the two changes, the operating point is shifted.

A transistor amplifier is said to be unstabilised when its operating point is shifted. Generally, it occurs because of the change in collector current. The collector current in a transistor changes rapidly when

(i) The temperature changes (it affects $I_C$).
(ii) The transistor is replaced by another one of the same type. This is due to the inherent variation of the transistor parameters (i.e. $\beta$).

***Effect of Temperature on $I_C$*** The collector current given by the relation;

$$I_C = \beta I_B + I_{CEO} \text{ where } I_{CEO} = (\beta + 1)\, I_{CBO}$$

$$\therefore \quad I_C = \beta I_B + (\beta + 1)\, I_{CBO} \qquad (4.29)$$

The collector leakage current $I_{CBO}$ is greatly influenced by the temperature. Under working conditions, the flow of collector current produces heat within the transistor. This raises the temperature of the transistor and increases $I_{CBO}$. This in turn increase the collector current $I_C$ as $I_C = \beta I_B + (\beta + 1)\, I_{CBO}$. The rise in collector current further increases the temperature resulting in further increase in $I_{CBO}$. Such a cumulative effects leads to thermal runway.

The self destruction of an unstabilized transistor due to rise in temperature is called thermal runway.

***Effect of Inherent Variations of Transistor Parameters*** When a transistor is replaced by another of the same type, that may have different parameters. Since collector current $I_C = \beta I_B + (\beta + 1)\, I_{CBO}$, the value of $I_C$ will be different if the new transistor parameters shift the operating point when it is not properly stabilized.

In order to avoid thermal runway (*i.e.* self-destruction of transistor because of rise in temperature) and to nullify the effect of inherent variations of transistor parameters, it is very essential to stabilize the operating point. In practice, it is done by selecting a suitable biasing circuit which decreases $I_B$ automatically with the rise in temperature (or rise in $\beta$ while replacing the transistor). Then decrease in $\beta I_B$ will compensate for the increase in $(\beta + 1)\, I_{CBO}$, keeping $I_C$ nearly constant.

Thus the process of making operating point independent of temperature changes or inherent variations in transistor parameters is called *stabilization.*

## 4.8 REQUIREMENTS OF A TRANSISTOR BIASING CIRCUIT

The biasing circuit should meet the following requirement:

(i) It should ensure proper zero signal collector current and $V_{CE}$.
(ii) $V_{CE}$ should not fall the required limit (i.e. 1.0 V for Si and 0.5 V for Ge transistors).
(iii) It should ensure the stabilization of collector current against temperature variations.
(iv) The operating point should be independent of transistor parameters.

## 4.9 STABILITY FACTOR

The stability of the operating point means that the value of $I_C$ remains constant even though there is variation in $I_{CBO}$ (or $I_{CO}$ ) because of rise in temperature. The extent to which a biasing circuit achieves this goal is measured by the *stability factor*. It may be defined as under:

The rate of change of collector current $I_C$ with respect to the rate of change of collector leakage current $I_{CBO}$ (or $I_{CO}$ ) at constant $I_B$ and $\beta$ is called *stability factor.*

i.e. Stability factor, $S = \dfrac{dI_C}{dI_{CO}}$ at constant $I_B$ and $\beta$

The stability factor shows the change in collector current $I_C$ because of the change in collector leakage current $I_{CO}$. The lower the value of stability factor, greater is the thermal stability of the transistor. The ideal value of the stability factor is 1 but it is never achieved in practice. However, the values of $S$ below 25 results in satisfactory performance.

***General Expression of Stability Factor for CE Configuration*** The collector current is expressed as

$$I_C = \beta I_B + (\beta + 1)\, I_{CBO}$$

Differentiating the above expression with respect to $I_C$, we get

$$1 = \beta \frac{d}{dI_C} I_B + (\beta + 1) \frac{d}{dI_C} I_{CO} = \beta \frac{dI_B}{dI_C} + \frac{(\beta + 1)}{dI_C / dI_{CO}} = \beta \frac{dI_B}{dI_C} + \frac{(\beta + 1)}{S}$$

or $$\frac{(\beta + 1)}{S} = 1 - \beta \frac{dI_B}{dI_C}$$

$$\therefore \quad S = \frac{(\beta + 1)}{1 - \beta \dfrac{dI_B}{dI_C}} \tag{4.30}$$

## 4.10 TRANSISTOR BIASING METHODS FOR CE CONFIGURATION

It has been shown in the various transistor amplifier circuits that the biasing was done by applying two batteries $V_{BB}$ in the input and $V_{CC}$ in the output circuit. However, in the interest of simplicity and economy, this biasing is never applied in the actual circuits. In the actual practice, a single source of supply ($V_{CC}$) is employed with the necessary biasing circuitry. The basic principle of the biasing circuitry is to obtain the required value of $I_B$ (and hence $I_C$) from $V_{CC}$ in the zero signal conditions. At the same time the value of collector load resistor $R_C$ is selected such that $V_{CE}$ should never fall below 1 V for silicon and 0.5 V for germanium transistors.

The following are the most commonly used biasing arrangements employing single supply source:

(i) Base resistor biasing

(ii) Feedback resistor biasing
(iii) Emitter resistor biasing
(iv) Voltage divider biasing

## 4.11 BASE RESISTOR BIASING

The base resistor biasing for an *npn* transistor is shown in Fig. 4.8. Here, a high resistance $R_B$ (of several hundred kΩ) is connected between the positive end of the supply and the base. The required zero signal base current $I_B$ is provided by $V_{CC}$. The supply also keeps the base positive with respect to emitter and hence makes the base-emitter junction forward biased.

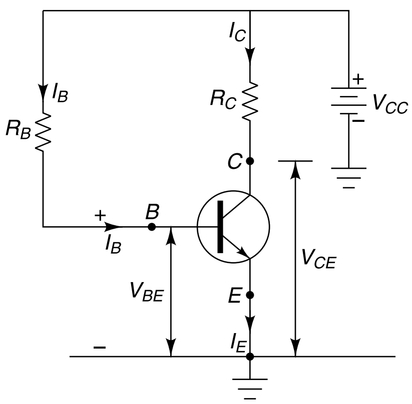



**Fig. 4.8** ***Base resistor biasing for npn transistor***

***Circuit Analysis*** While analyzing the circuit, generally we make the equations to determine the value of $R_B$ for the required zero signal collector current. Let $I_C$ be the required zero signal collector current. Then

$$I_B = \frac{I_C}{\beta} \tag{4.31}$$

Considering the input circuit shown in Fig. 4.8 and applying Kirchhoff's voltage law (KVL) to the base-emitter loop, we get,

$$V_{CC} = I_B R_B + V_{BE}$$

or $$I_B R_B = V_{CC} - V_{BE}$$

or $$R_B = \frac{V_{CC}\, V_{BE}}{I_B} \tag{4.32}$$

The value of $R_B$ can be determined from the expression (4.32) as the value of $V_{CC}$ and $I_B$ are known and $V_{BE}$ can be seen from the transistor manual.

Since the value of $V_{BE}$ is quite small as compared to $V_{CC}$, it is generally neglected to simplify the expression. Then

$$R_B \cong \frac{V_{CC}}{I_B}$$

It may be noted that $V_{CC}$ is a fixed known value and $I_B$ is fixed by the operating conditions, therefore, this biasing methods is sometimes called *fixed biasing methods.*

As the operating point is represented by $I_C$; $V_{CE}$, therefore, for the selected value of $I_C$, we determine the value of $V_{CE}$. For this consider only the output circuit shown in Fig. 4.8.

Applying Kirchhoff's voltage law (KVL) to the collector-emitter loop, we get,

$$V_{CC} = I_C R_C + V_{CE}$$

or

$$V_{CE} = V_{CC} - I_C R_C \quad (4.33)$$

***Stability Factor*** It is given by the expression

$$S = \frac{(\beta + 1)}{1 - \beta \dfrac{dI_B}{dI_C}}$$

In this biasing arrangement, $I_B$ is independent of $I_C$. Therefore $dI_B/dI_C = 0$. Substituting this value in the above expression, we get,

$$S = \beta + 1$$

Thus, the value of stability factor in fixed biasing method is very high which shows its poor thermal stability.

***Advantages***

(i) In this biasing methods calculations are simple and easy to obtain the required condition.
(ii) Simple in construction as only one resistor $R_B$ is required to set the conditions.
(iii) There is no loading of the source as no resistor is employed across base-emitter junction.

***Disadvantages***

(i) It provides poor thermal stability (high stability factor) and there are chances of thermal runway.
(ii) This method provides poor stability against inherent variations of transistor parameters. For instant, if $\beta$ increases due to transistor replacement, then $I_C$ also increases by the same factor as $I_C = \beta I_B$ where $I_B$ is constant.

In fact, in this biasing arrangement no means are provided to check a self increase in collector current due to temperature rise and individual variations. Due to these disadvantage, this method is rarely employed in practical circuits.

## 4.12 FEEDBACK RESISTOR BIASING

The feedback resistor biasing is also known as collector to base resistor biasing. This biasing arrangement for an *npn* transistor is shown in Fig. 4.9. Here, a high resistor $R_B$ is connected between collector and base. The required zero signal base current is determined by collector-base voltage $V_{CB}$ and not by $V_{CC}$. Hence, the base-emitter junction is forward biased and base current $I_B$ flows through $R_B$.This in turn causes the zero signal collector current $I_C$ to flow in the circuit.

It may be noted that the collector (output) is connected to the base (input) through $R_B$. This means that feedback exist in the circuit. The base current depends upon the collector voltage. This circuit is also called a voltage feedback biasing circuit.

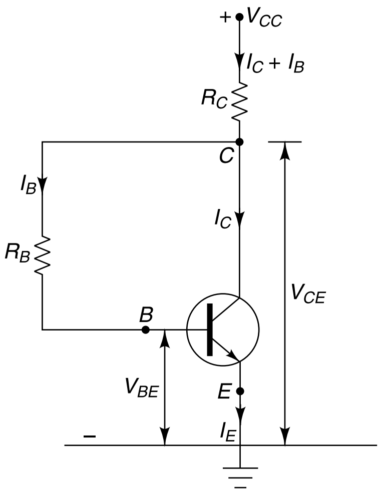



**Fig. 4.9** ***Feedback resistor biasing***

***Circuit Analysis*** While analyzing the circuit, generally we make the equations to determine the value of $R_B$ for the required zero signal collector current $I_C$.

Referring Fig. 4.9, the loop equation for the input circuit is

$$V_{CC} = R_C(I_C + I_B) + I_B R_B + V_{BE} \quad (4.34)$$

or $$V_{CC} = R_C I_C + I_B (R_C + R_B) + V_{BE}$$

or $$I_B(R_C + R_B) = V_{CC} - I_C R_C - V_{BE},$$

$$R_B = \frac{(V_{CC} - I_C R_C) - V_{BE}}{I_B} - R_C \quad (4.35)$$

Writing loop equation for the output circuit, we get

$$V_{CC} = R_C (I_C + I_B) + V_{CE} \quad (4.36)$$

$$V_{CE} \cong V_{CC} - R_C I_C \text{ (neglecting } I_B \text{ since } I_B \nwarrow I_C) \quad (4.36a)$$

Substituting this value in Eq. (4.35), we get,

$$R_B = \frac{(V_{CE} - V_{BE})}{I_B} R_C \cong \frac{V_{CE} - V_{BE}}{I_B} \quad (4.37)$$

$$R_B = \frac{V_{CB}}{I_B} \text{ where } I_B = \frac{I_C}{\beta}$$

For determining the operating point, consider Eq. (4.34) and put $I_C = \beta I_B$. We get

$$V_{CC} = I_B(R_C + R_B) + \beta I_B R_C + V_{BE}$$

or $$V_{CC} = I_B[R_B + (\beta_B + 1)R_C] + V_{BE}$$

or $$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)\, R_C} \cong \frac{V_{CC}}{R_B + \beta R_C} \quad (4.38)$$

For determining $V_{CE}$, consider Eq. (4.36);

Here, $V_{CC} = R_C (I_C + I_B) + V_{CE}$

or $V_{CE} = V_{CC} - R_C (I_C + I_B) \cong V_{CC} - I_C B_C$ (4.39)

***Stability Factor*** The stability factor in this arrangement is less than $(\beta + 1)$, i.e.

$$S < (\beta + 1)$$

### *Advantages*

(i) It is simple biasing arrangement as only one resistor $R_B$ is required.

(ii) This biasing circuit provides some stabilization of the operating point as explained below:

Suppose temperature increases, this causes increase in leakage current and at the same time there is increase in $\beta$ due to inherent variation in parameters. This increases the collector current since $I_C = \beta I_B + (\beta + 1) I_{CBO}$. As the collector current increases, the voltage $V_{CE}$ decreases as $V_{CE} = V_{CC} - R_C I_C$. The decrease in $V_{CE}$ causes decrease in bias current since $I_B = \frac{V_{CE} - V_{BE}}{R_B}$. This decrease in bias current in turn decrease the collector current to bring it to original value. Hence, there is a tendency of this circuit to stabilize the operating point.

### *Disadvantages*

(i) The stability factor of this biasing method is quite high. Hence, the operating point does change, although to lower extent, due to temperature rise or inherent variation in parameters.

(ii) The resistor $R_B$ has not only dc feedback for stabilization of operating point, but it also causes ac feedback. This reduces the voltage gain of the amplifier which is not desirable. Because of this drawback, this circuit is *seldom used in operational amplifiers*.

## 4.13 EMITTER RESISTOR BIASING

The emitter resistor biasing for *npn* transistor is shown in Fig. 4.10. It is just a modification to the base resistor biasing circuit. In this biasing an additional resistor $R_E$ is connected in the emitter. Hence this circuit contains three resistors $R_B$, $R_E$ and $R_C$.

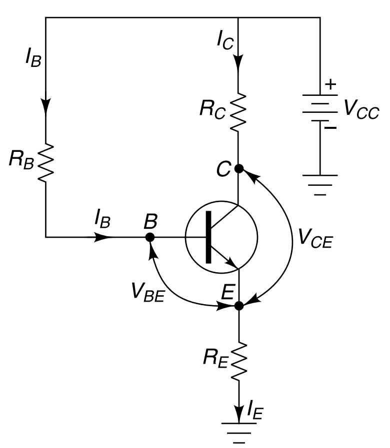



**Fig. 4.10** ***Emitter resistor biasing***

***Circuit Analysis*** The circuit may be analysed for determining the operating point *Q*. To do this equation can be framed for the input and output circuits.

Writing the loop equation for the input circuit, we get,

$$V_{CC} = I_B R_B + I_E R_E + V_{BE}$$

or $\quad V_{CC} = I_B R_B + (\beta + 1)\, I_B R_E + V_{BE}$ $\quad$ [since $I_E = (\beta + 1)\, I_B$]

or $\quad I_B = \dfrac{V_{CC} - V_{BE}}{R_B + (b+1)\, R_E}$

Neglecting $V_{BE}$ (since it is very small)

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (b+1)\, R_E} \tag{4.40}$$

$$\text{Collector current, } I_C = \beta I_B = \frac{\beta V_{CC}}{R_B + \beta R_E} = \frac{V_{CC}}{R_E + R_B/\beta} \tag{4.41}$$

Writing loop equation for the output circuit, we get,

$V_{CC} = R_C I_C + I_E R_E + V_{CE}$

or $\quad V_{CE} = V_{CC} - (R_C + R_E)\, I_C \quad (\therefore\ I_C \cong I_E)$ $\quad$ (4.42)

Thus the operating point can be obtained by usimg Eq. (4.41) and (4.42). Similarly, for the given operating point, the value of $R_B$ and $R_E$ can be determined by using same equations provided the other data *viz* $V_{CC}$, $R_C$ and $\beta$ are known.

***Stabilisation*** Let us see the effect of increase in temperature on the operating point $Q$. For this rewrite the loop equation for input circuit

$$V_{CC} = I_B R_B + I_E R_E + V_{BE}$$

or
$$I_B = \frac{V_{CC} - I_E R_E - V_{BE}}{R_B}$$

or
$$I_B \cong \frac{V_{CC} - I_E R_E}{R_B} \text{ (since } V_{BE} \text{ is very small)} \tag{4.43}$$

As the temperature increases, the leakage current increases. This increase is in the collector current $I_C$, as well as emitter current $I_E$. As a result voltage drop $I_E R_E$ increases which reduces the numerator of equation (4.43) and hence the current $I_B$ reduces. This results in reduction of collector current. Hence, the stability of the operating point is improved to large extent by inserting a resistor in the emitter circuit.

The stabilization is improved by increasing $R_E$ in the emitter circuit. This resistor is present in the output side as well as the input side. Therefore, a feedback occurs through this resistor. The feedback voltage is proportional to the emitter current $I_E$. Hence, the circuit is also called *current feedback biasing* circuit.

*Stability Factor* The stability factor of this type of biasing circuit is quite less than $(\beta + 1)$.

***Advantages***

(i) Although three resistors $R_B$, $R_E$ and $R_C$ are employed in the circuit, still the circuit is quite simple to explain.

(ii) It provides far better stabilization than feedback resistor biasing.

***Disadvantages***

(i) The stability factor of this biasing is to some extent high. Hence the operating point shifts, although to lower extent, due to temperature rise or inherent variations in parameters.

(ii) The dc feedback helps in the stabilization of operating point but at same time ac feedback reduces the voltage gain of the amplifier. This is an undesirable feature. However, this drawback can be remedied by putting a capacitor $C_E$ across the resistor $R_E$. The capacitor $C_E$ offers very low impedance to the ac current (signal) and does not allow it to pass through $R_E$ and hence ac feedback is required. Thus the process of amplification remains unaffected.

This biasing circuit is also not used practically because of the following reasons;

Since $I_C = \dfrac{V_{CC}}{R_E + {}^{R_B}\!/_{\beta}}$, for stability, the denominator should be independent of $\beta$. It is only possible if $R_E \gg R_B/\beta$. This condition can only be obtained either using $R_E$ of very large value or by using $R_B$ of very small value. Now, a large value of $R_E$ will cause a large voltage drop across it and to obtain a required $Q$ point we have to apply a voltage source $V_{CC}$ of high value. On the other hand if $R_B$ is of very low value we have to use a separate voltage source of low value for base circuit. Both these alternative are quite impractical. Hence, this biasing circuit is not used practically.

## 4.14 VOLTAGE-DIVIDER BIASING

The voltage divider biasing for an *npn* transistor is shown in Fig. 4.11. In this biasing circuit two resistors $R_1$ and $R_2$ are connected across the supply voltage $V_{CC}$ and provide the necessary biasing. A resistor connected in the emitter circuit $R_E$ provides stability. Since the resistors $R_1$ and $R_2$ form the voltage divider hence the name voltage divider biasing. The voltage drop across $R_2$ forward biases the base-emitter junction and causes the base current. Hence, collector current flows which set up the zero signal conditions. This is the most widely used method of providing biasing and stabilization to transistor since the operating point, in this case, can be made almost independent of $\beta$.

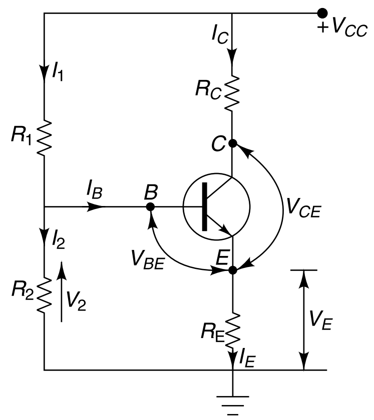



**Fig. 4.11** ***Voltage divider biasing [$I_1 \simeq I_2$ as $I_B$ is small]***

***Circuit Analysis*** The circuit can be analyzed either by using approximations or by accurate method (applying Thevenin theorem). Usually, approximate analysis is employed since it provides reasonable accuracy.

Consider the input section of the circuit. Let current $I_1$ flows through resistors $R_1$. As the base current $I_B$ is very small, therefore, it can be assumed that same current $I_1$ flows through the resistors $R_2$ (i.e., $I_1 \approx I_2 >> I_B$).

Applying Kirchhoff's voltage law to the input circuit, we get,

$$V_{CC} = I_1R_1 + I_1R_2$$

or $$I_1 = \frac{V_{CC}}{R_1 = R_2}$$

Voltage across resistor $R_2$,

$$V_2 = I_1R_2 = \left(\frac{V_{CC}}{R_1 + R_2}\right)R_2 \tag{4.44}$$

Writing loop equation in base emitter circuit, we get,

$$V_2 = V_{BE} + V_E$$

or $$V_2 = V_{BE} + I_ER_E \qquad \text{(Since } V_E = I_ER_E\text{)}$$

$$I_E = \frac{V_2 - V_{BE}}{R_E}$$

Since $I_C \cong I_E$ [$I_B$ being very small]

$$\therefore \quad I_C = \frac{V_2 - V_{BE}}{R_E}$$

In all practical circuits $V_2 >> V_{BE}$, therefore $V_{BE}$ in comparison to $V_2$ is neglected

Thus $$I_C = \frac{V_2}{R_E} \tag{4.45}$$

Applying Kirchhoff's voltage law to the output (collector) side, we get,

$$V_{CC} = I_CR_C + I_ER_E + V_{CE}$$

or $$V_{CC} = I_CR_C + I_CR_E + V_{CE} \qquad (I_C \cong I_E)$$

or $$V_{CE} = V_{CC} - I_C(R_C + R_E) \tag{4.46}$$

***Stabilization*** In the above analysis, we have seen that $\beta$ does not appear in any of the expressions. If reveals that the operating point is independent of this parameters ($\beta$). It means that if we change the transistor with same type having different value of $\beta$, the operating point will not be affected at all. Hence, a good stabilization is achieved.

On the other hand, if the collector current increases due to rise in temperature this will cause greater voltage drop across the emitter resistance ($I_CR_E$). As the voltage drop across $R_2$ (i.e. $V_2$) is independent of $I_C$, therefore $V_{BE}$ decreases (since $V_{BE} = V_2 - I_CR_E$). This in turn causes to decrease of $I_B$. The reduction in $I_B$ restores the original value of collector current $I_C$.

***Stability Factor*** It can be shown mathematically that the stability factor of this biasing circuit is 1, which is an ideal one. However, in actual practice, its stability factor is around 10.

***Advantages*** The voltage divider biasing circuit provides good stabilization; therefore, this biasing circuit is invariably employed for transistor biasing.

**4.5** A base resistor biasing circuit is shown in Fig. 4.12. Determine (i) The collector current $I_C$ and collector-emitter voltage $V_{CE}$. Neglect small $V_{BE}$ and assume $\beta = 50$. (ii) If $R_B$ in this circuit is changed to 200 kΩ and transistor is considered to be silicon *npn*.

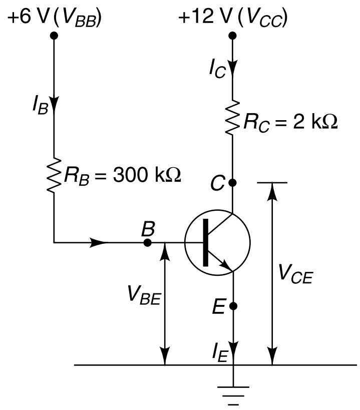



**Fig. 4.12**

***Solution***

(i) Applying Kirchhoff's voltage law to base-emitter circuit, we get,
$V_{BB} = I_B R_B + V_{BE}$

$$I_B = \frac{V_{BB}}{R_B} = \frac{6}{300} = 20\,\mu\text{A}$$

Collector current, $I_C = \beta I_B = 50 \times 20 = 1000\ \mu\text{A} = 1.0$ mA. (Ans.)
Applying Kirchhoff's voltage law to collector-emitter circuit, we get,

$$V_{CC} = I_C R_C = V_{CE}$$

or $$V_{CE} = V_{CC} - I_C R_C$$

$$= 12 - 1.0 \text{ mA} \times 2 \text{ k}\Omega = 12 - 2.0 = 10.0 \text{ V (Ans.)}$$

(ii) When $R_B = 200$ kΩ and $V_{BE} = 0.7$ V (for silicon transistor)

Base current, $$I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{6 - 0.7}{200} = 26.5\,\mu\text{A}$$

Collector current, $I_C = \beta I_B = 50 \times 26.5 = 1325\ \mu\text{A} = 1.325$ mA (Ans.)
Collector-emitter voltage, $V_{CE} = V_{CC} - I_C R_C = 12 - 1.32 \text{ mA} \times 2 \text{ k}\Omega$
$= 9.36$ V (Ans.)

**4.6** Calculate the collector current and the collector to emitter voltage for the circuit shown in Fig. 4.13. Assume $\beta = 50$.

***Solution***

The base current,

$$I_B = \frac{V_{CC} - V_{BE}}{R_B}$$

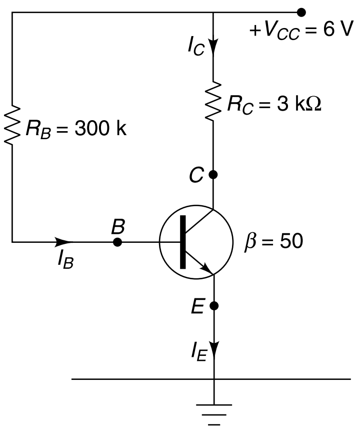



**Fig. 4.13**

$$I_B = \frac{V_{CC}}{R_B} = \frac{6}{300\,\text{k}\Omega} = 20\,\mu\text{A} \quad [\text{Neglecting } V_{BE}]$$

Collector current, $I_C = \beta I_B = 50 \times 20 = 1000\ \mu A = 1.0$ mA. (Ans.)
This value of the collector current must be less than the collector saturation current.

[Check: Collector saturation current $I_{C(\text{sat})} = \frac{V_{CC}}{R_C} = \frac{6\text{V}}{3\,\text{k}\Omega} = 2\,\text{mA}$. $I_{C(\text{sat})}$ is more than the actual collector current calculated above, thus the transistor is not in saturation]

Collector-emitter voltage,

$$V_{CE} = V_{CC} - I_C R_C = 6 - 1.0\text{ mA} \times 3\text{ k}\Omega = 6 - 3 = 3.0\text{ V (Ans.)}$$

**4.7** In a fixed biasing circuit, a supply of –6 V and a load resistance of 1 kΩ is used. Determine the value of base resistor $R_B$ so that a germanium transistor with $\beta = 50$ and $I_{CBO} = 2\ \mu A$ draws a collector current 2 mA. What will be the error if we neglect the leakage current $I_{CBO}$ and base emitter voltage $V_{BE}$? Also determine the value of $I_C$ if the transistor parameters are changed to $\beta = 60$ and $I_{CBO} = 8\ \mu A$ due to rise in temperature. Comment on the result.

***Solution***

The required circuit is shown in Fig. 4.14.

We know, $I_C = \beta I_B + (\beta + 1)\, I_{CBO}$

Where $\beta = 50$; $I_C = 2$ mA; $I_{CBO} = 2\ \mu A$.

$\therefore\ 2000 = 50 \times I_B + (50 + 1) \times 2$ [$\because$ 2 mA = 2000 μA]

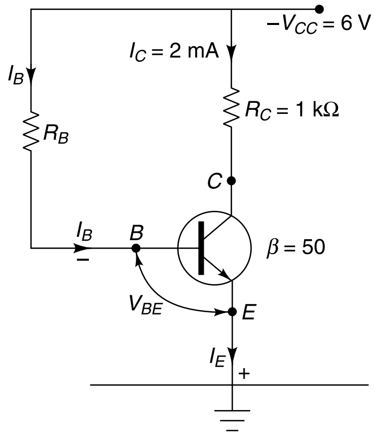



**Fig. 4.14**

or $$I_B = \frac{2000 - 102}{50} = 37.96\,\mu A$$

Now, base current, $I_B = \dfrac{V_{CC} - V_{BE}}{R_B}$

or $$R_B = \frac{V_{CC} - V_{BE}}{I_B}$$

$$= \frac{6 - 0.3}{37.96} = 150.15\,k\Omega \text{ (Ans.)}$$

If $I_{CBO}$ is neglected

$$I_C = \beta I_B$$

$$I_B = \frac{I_C}{\beta} = \frac{2\,mA}{50} = 40\ \mu A.$$

Also neglecting $V_{BE}$,

$$R_B = \frac{V_{CC}}{I_B} = \frac{6}{40\,\mu A} = 150\,k\Omega.$$

$$\%\ \text{error} = \frac{150 - 150.15}{150.15} \times 100 = -0.09\% \text{ (Ans.)}$$

*Comments:* This error is too small and should not be bothered at all. Moreover the resistors available in the market ordinarily have tolerance ± 10%. Hence, it is not very incorrect to neglect $V_{BE}$ and $I_{CBO}$ while doing calculations.

When $\beta$ changes to 60 and $I_{CBO}$ to 8 μA due to rise in temperature;

$$I_C = \beta I_B + (\beta + 1)\ I_{CBO} = 60 \times 37.96\ \mu A + (60 + 1) \times 8\ \mu A$$
$$= 2097.6 + 488 = 2585.68\ \mu A = 2.585\ mA \text{ (Ans.)}$$

*Comments:* It may be noted that the collector current has increased by more than 25% due to change in parameters by the rise in temperature. Hence, it is concluded that the operating point is shifted due the rise in temperature in the biasing circuit.

**4.8** A silicon transistor biased by feedback resistor method is shown in Fig. 4.15. Determine the operating point.

***Solution***

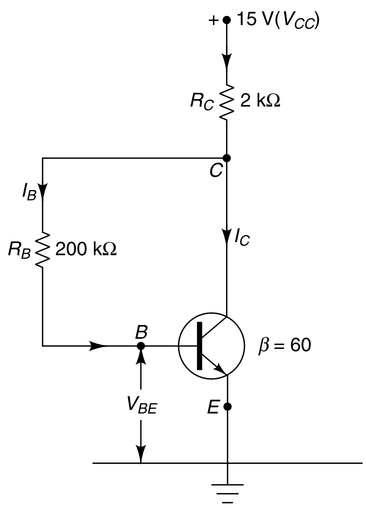



**Fig. 4.15**

Base current,

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta R_C} = \frac{15 - 0.7}{200\ \text{k}\Omega + 60 \times 2\ \text{k}\Omega} = \frac{14.3\ \text{V}}{320\ \text{k}\Omega} = 44.68\ \mu\text{A}$$

Collector current, $I_C = \beta I_B = 60 \times 44.68\ \mu\text{A} = 2680\ \mu\text{A} = 2.68$ mA. (Ans.)

Collector-emitter voltage,

$V_{CE} = V_{CC} - I_C R_C = 15\ \text{V} - 2.68\ \text{mA} \times 2\ \text{k}\Omega = 15 - 5.36 = 9.64$ V (Ans.)

Hence the operating point is 9.64 V, 2.68 mA (Ans.).

**4.9** A transistor is biased with feedback resistor $R_B = 150$ kΩ. If the collector load resistance is of 1.5 kΩ and the supply voltage is of 10 V, determine the upper and lower limit of collector current when $\beta$ varies as $50 < \beta < 200$. Comment upon the result.

***Solution***

A simplified circuit is shown in Fig. 4.16.

When $\beta = 50$, base current,

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta R_C} \approx \frac{V_{CC}}{R_B + \beta R_C} = \frac{10\ \text{V}}{150\ \text{k}\Omega + 50 \times 1.5\ \text{k}\Omega} = 44.44\ \mu\text{A}$$

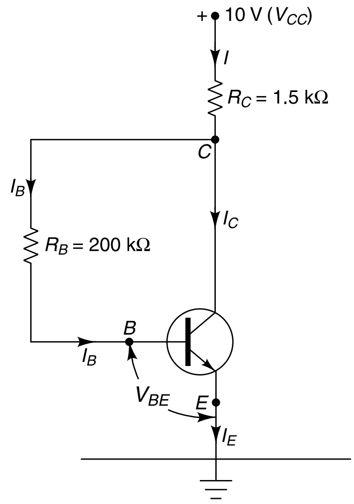



**Fig. 4.16**

Collector current, $I_C = \beta I_B = 50 \times 44.444\ \mu A = 2.22$ mA. (Ans.)

When $\beta = 200$, base current, $I_B = \dfrac{10\ V}{150\ k\Omega + 200 \times 1.5\ k\Omega} = 22.222\ \mu A$

Collector current, $I_C = \beta I_B = 200 \times 22.222\ \mu A = 4.444$ mA. (Ans.)

*Comments:* It may be noted that when $\beta$ increases to *four times*, the collector current increases to *two times*. This shows that this biasing circuit does not provide perfect stabilisation. However, if there would have been bias resistor circuit then collector current would have been increased to four times. Hence, this method of biasing is better than the base bias resistor method.

**4.10** Calculate the value of $R_B$ in the biasing circuit shown in Fig. 4.17, so that the operating point is fixed at $I_C = 8$ mA and $V_{CE} = 3$ V.

***Solution***

Base current,

$$I_B = \frac{I_C}{\beta} = \frac{8\ mA}{100} = 80\ \mu A$$

Now, $$V_{CC} = I_B R_B + V_{BE} + (\beta + 1) I_B R_E$$

or $$R_B = \frac{V_{CC} - (\beta + 1) I_B R_E}{I_B} \quad \text{(neglecting } V_{BE}\text{)}$$

$$= \frac{10 - (100 + 1) \times 80 \times 10^{-6} \times 600}{80 \times 10^{-6}}$$

$$= \frac{10 - 4.84}{80 \times 10^{-6}} = 64.5\ k\Omega \text{ (Ans.)}$$

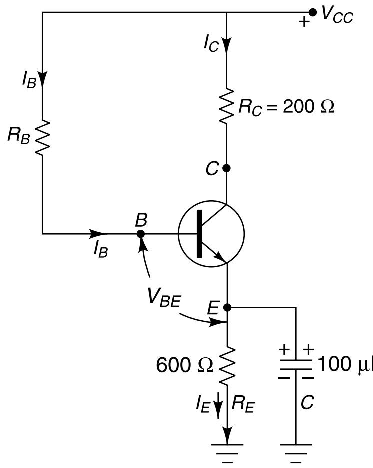



**Fig. 4.17** *[$I_E = I_B + I_C = I_B\ (\beta + 1)$]*

**4.11** A voltage divider biasing circuit is shown in Fig. 4.18. Determine emitter current, collector-emitter voltage and collector voltage.

***Solution***

Base voltage i.e., voltage across $R_2$,

$$V_2 = \frac{V_{CC}}{R_1 + R_2} \times R_2 = \frac{18}{7+2} \times 2 = 4\ \text{V}$$

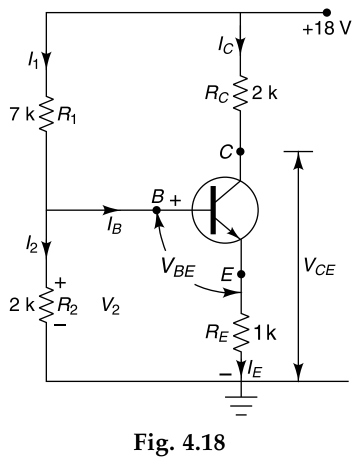



**Fig. 4.18**

Loop equation for the base-emitter circuit,

$$V_2 = V_{BE} + I_E R_E$$

or
$$I_E = \frac{V_2}{R_E} = \frac{4\ \text{V}}{1\ \text{k}\Omega} = 4\ \text{mA}\ \ \text{(Ans.) (neglecting } V_{BE}\text{).}$$

$$I_C = I_E = 4\ \text{mA.}$$

Collector voltage, $V_C = V_{CC} - I_C R_C = 18 - 4\text{ mA} \times 2\text{ k}\Omega = 10\text{ V}$ (Ans.)

Collector-emitter voltage, $V_{CE} = V_{CC} - I_E R_E = 10 - 4\text{ mA} \times 1\text{ k}\Omega = 6\text{ V}$ (Ans.)

**4.12** A potential divider biasing circuit of an *npn* germanium transistor is shown in Fig. 4.19. To fix the operating point at $I_C = 2.5$ mA, $V_{CE} = 6$ V, find the values of $R_1$, $R_2$ and $R_E$.

Assuming the other circuit component and transistor parameters to be $V_{CC} = 12$ V, $R_C = 3$ kΩ, $\beta = 100$ and $I_1 = 10\ I_B$, $V_{BE} = 0.3$ V.

***Solution***

Here, $V_{CC} = 12$ V, $R_C = 3$ kΩ, $\beta = 100$, $I_C = 2.5$ mA, $V_{CE} = 3$ V and $I_I = 10\ I_B$

$V_{BE} = 0.3$ V (for Ge transistor).

Base current,

$$I_B = \frac{I_C}{\beta} = \frac{2.5\text{ mA}}{100} = 0.025\text{ mA}$$

Since $I_B$ is very small as compared to $I_1$, for reasonable accuracy it may be assumed that same current $I_1$ flows through $R_1$ and $R_2$

$$I_1 = 10\ I_B = 10 \times 0.025 = 0.25\text{ mA}.$$

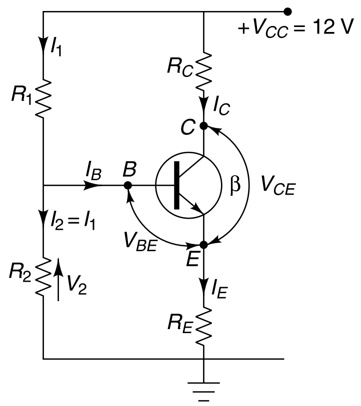



**Fig. 4.19**

Now, $$I_1 = \frac{V_{CC}}{R_1 + R_2}$$

$$\therefore \quad R_1 + R_2 = \frac{V_{CC}}{I_1} = \frac{12}{0.5\text{ mA}} = 24\text{ k}\Omega.$$

Writing loop equation for the collector-emitter (output) circuit, we get,

$$V_{CC} = I_C R_C + V_{CE} + I_E R_E$$

$$V_{CC} = I_C R_C + V_{CE} + I_C R_E$$

$(\because I_C = I_E)$

or $$12 = 2.5\text{ mA} \times 3\text{ k}\Omega + 6 + 2.5\text{ mA} \times R_E$$

$$\therefore \quad R_E = \frac{12 - 7.5 - 6}{2.5\text{ mA}} = \frac{-1.5\text{ V}}{2.5\text{ mA}} = 600\,\Omega \text{ (Ans.)}$$

Voltage across $R_2$, $V_2 = V_{BE} + I_C R_E = 0.3 + 2.5 \text{ mA} \times 600\ \Omega = 0.3 + 1.5 = 1.8$ V.

Resistance, $R_2 = \dfrac{V_2}{I_1} = \dfrac{1.8}{0.5\,\text{mA}} = 3.6\,\text{k}\Omega$ (Ans.)

Resistance, $R_1 = 24 - 3.6 = 20.4\ \text{k}\Omega$ (Ans.)

**4.13** If $V_{CC} = 12$ V, find $V_{CE}$ and $I_E$ in Fig. 4.20 and comment on your findings.

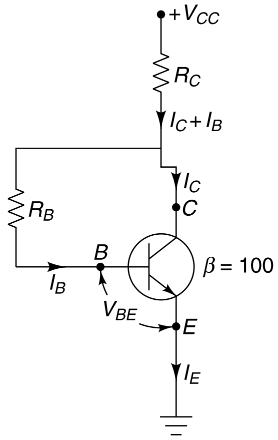



**Fig. 4.20**

***Solution***

In collector loop,

$$\begin{aligned} V_{CC} &= (I_C + I_B)\, R_C + I_B R_B + V_{BE} \\ &= I_C R_C + I_B\,(R_C + R_B) + V_{BE} \\ &= I_B\,[(\beta + 1)\, R_C + R_B] + V_{BE} \qquad [\because\ I_C = \beta I_B] \end{aligned}$$

$$\therefore \qquad I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta+1)R_C} \cong \frac{V_{CC}}{R_B + BR_C}\ [\therefore V_{BE} \text{ is smell}]$$

Given: $V_{CC} = 12$ V; $R_B = 500$ kΩ; $\beta = 100$; $R_C = 0.5$ kΩ

$$\therefore \qquad I_B = \frac{12}{(500 + 100 \times 0.5)} \times 10^{-3} = 0.0218 \times 10^{-3} = 21.8\ \mu\text{A}$$

$$I_C = \beta I_B = 100 \times 21.8\ \mu\text{A} = 2.18 \text{ mA}$$

$$\begin{aligned} I_E &= I_C + I_B \simeq I_C\ \ [I_B \text{ being small, it may be neglected}] \\ &= 2.18 \text{ mA} \end{aligned}$$

$$\begin{aligned} \therefore \qquad V_C &= V_{CE} = V_{CC} - I_C R_C = 12 - 2.18 \times 10^{-3} \times 0.5\ \text{k}\Omega \\ &= 10.91 \text{ V} \end{aligned}$$

It may be noted that $V_{CE}$ being 10.91 V, it is very close to the value of $V_{CC}$. Hence the operating point is very near to the cut off region.

**4.14** A transistor is biased with feedback resistor $R_B = 100$ kΩ. If the collector load resistance is 1 kΩ and the supply voltage is 10 V, find the upper and lower limit of collector current when $b$ is in the range of $50 \le \beta \le 150$.

***Solution***

Refer Fig. 4.21.

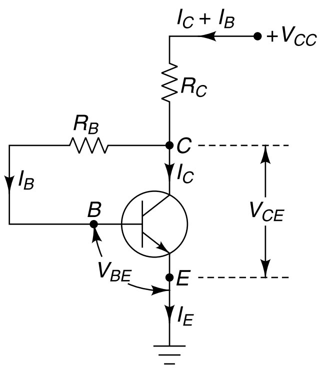



**Fig. 4.21**

Let us assume first $\beta = 50$

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta R_C} \simeq \frac{V_{CC}}{R_B + \beta R_C}$$

$$= \frac{10}{100 \times 10^3 + 50 \times 10^3} = 66.66\ \mu\text{A}.$$

$$I_C = \beta I_B = 50 \times 66.66 \times 10^{-6} = 3.33\ \text{mA}$$

When $\beta = 150$,

$$I_B = \frac{10}{100 \times 10^3 + 150 \times 10^3} = 40\ \mu\text{A}.$$

$$\therefore \quad I_C = \beta I_B = 150 \times 40\ \mu\text{A} = 6\ \text{mA}.$$

If may be noted with $\beta$ increasing to 3 times, the collector current increases to 1.8 times. The biasing circuit will not provide perfect stabilization. For better stabilization, we need to increase the value of $I_C$ further. This is possible by using base bias resistor.

**4.15** A transistor operating as *CE* amplifier couples a source having internal resistance of 3 kΩ to a load of 30 kΩ. Determine the input and output resistances if $h_{re} = 3 \times 10^{-4}$, $h_{fe} = 200$, $h_{ie} = 1.5$ kΩ and $h_{oe} = 30 \times 10^{-6}$.

***Solution***

$$\text{Current gain } (A_I) = \frac{-h_{fe}}{1 + h_{oe}R_L} = \frac{-200}{1 + 30 \times 10^{-6} \times 3 \times 10^3}$$

$$= \frac{-200}{1 + 90 \times 10^{-3}} = \frac{-200}{1.09} = -183.486$$

Input resistance

$(R_i) = h_{ie} + A_I\, h_{re}\, R_L = 1.5 \times 10^3 - 183.486 \times 3 \times 10^{-4} \times 3 \times 10^3$

$= 1.33 \times 10^3 = 1330\ \Omega$

Output resistance

$$R_o = \frac{R_s' + h_{ie}}{R_s' + h_{oe} + h_{ie}h_{oe} - h_{fe}h_{re}} \quad \text{where } R_s' \text{ is the source resistance}$$

$$= \frac{R_s R_1}{R_s + R_1}$$

$$\therefore \quad R_o = \frac{3 \times 10^3 + 30 \times 10^3}{3 \times 10^3 \times 30 \times 10^{-6} + 1.5 \times 10^3 \times 30 \times 10^{-6} - 200 \times 3 \times 10^{-4}}$$

$$= \frac{33 \times 10^3}{(90 + 45 - 60) \times 10^{-3}} = 0.44 \times 10^6\,\Omega = 440k\Omega$$

**4.16** The $h$-parameter values for the transistor used in the *CB* circuit are as follows: $h_{ie} = 4k$, $h_{re} = 7 \times 10^{-4}$, $h_{fe} = 100$, $h_{oe} = 50\ \mu$ mhos (Fig. 4.22).

Determine the following.

(a) The current gain taking source and load resistances into account.
(b) The input resistance.
(c) The voltage gain taking source and load resistances into account.
(d) The output resistance.
(e) The power gain.

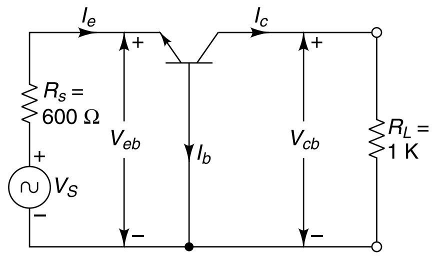



**Fig. 4.22**

***Solution***

The $h$-parameter values given in the problem are for *CB* configuration. Hence we can write

$$h_{ib} = \frac{h_{ie}}{1 + h_{fe}} = \frac{4000}{1 + 100} \cong 40 \text{ ohms}$$

$$h_{rb} = \frac{h_{ie}h_{oe}}{1 + h_{fe}} - h_{re} = \frac{4000 \times 50 \times 10^{-6}}{1 + 100} - 7 \times 10^{-4}$$

$$= \frac{0.2}{101} - 0.0007 = 0.0013$$

$$h_{fb} = \frac{-h_{fe}}{1 + h_{fe}} = \frac{-100}{101} \cong -1$$

$$h_{ob} = \frac{h_{oe}}{1 + h_{fe}} = \frac{50 \times 10^{-6}}{101} \cong 0.5\ \mu \text{ mho.}$$

*(a) Current Gain*

The current gain ($A_i$) without taking source resistance into account is given by

$$A_i = \frac{-h_{fb}}{1 + h_{ob} R_L}$$

$$= \frac{-(-1)}{1 + 0.5 \times 10^{-6} \times 1000}$$

$$= \frac{1}{1.0005} = 1$$

The current gain ($A_{is}$) taking source resistance into account is then given by Fig. 4.23(a)

$$A_{is} = \frac{A_i R_s}{R_s + R_i}$$

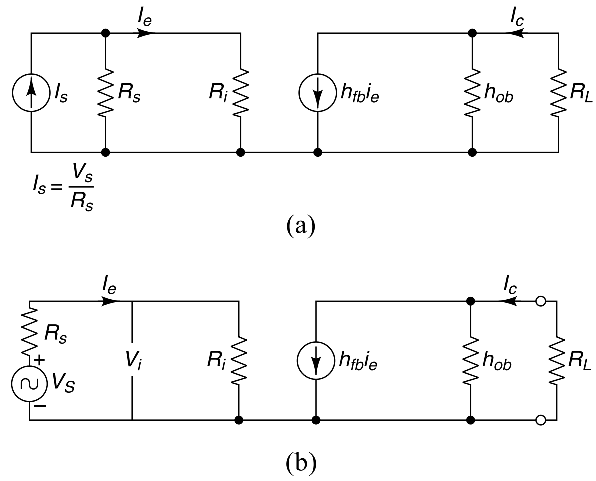



**Fig. 4.23**

*(b) Input Resistance*

The input resistance ($R_i$) is given by

$$R_i = h_{ib} + h_{rb} A_i R_L$$
$$= 40 + 0.0013 \times 1 \times 1000$$
$$= 41.3 \text{ ohms.}$$

The current gain ($A_{is}$) is therefore given by

$$A_{is} = \frac{1 \times 600}{600 + 41.3} = \frac{600}{641.3} \cong 0.93$$

*(c) Voltage Gain*

The voltage gain ($A_v$) without taking source resistance into account is given by [Fig. 4.23(b)]

$$A_v = \frac{A_i R_L}{R_i} = \frac{1 \times 1000}{641.3} \equiv 1.5$$

Voltage gain ($A_{vs}$) taking source resistance ($R_s$) into account is given by

$$A_{vs} = \frac{A_v R_i}{R_i + R_s} = \frac{1.5 \times 41.3}{600 + 41.3} \equiv \frac{1.5 \times 41.3}{641.3} \cong 0.09$$

(d) *Output Resistance*

The output conductance ($G_O$), where $G_O = 1/R_O$, is given by

$$G_O = h_{ob} - \frac{h_{fb} h_{rb}}{R_s + R_i} = 0.5 \times 10^{-6} - \frac{(-1)(0.0013)}{641.3}$$

$$= 0.5 \times 10^{-6} + \frac{0.0013}{641.3} = 2.5 \times 10^{-6} \text{mhos}$$

The output resistance ($R_O$) is therefore given by

$$R_o = \frac{1}{2.5 \times 10^{-6}} = 400 \text{ K.}$$

(e) *Power Gain*

The power gain is given by

$$A_p = A_i \times A_v$$
$$= 1 \times 1.5 = 1.5$$

If we take the source resistance into consideration the power gain will reduce.

**4.17** Determine the values of $\beta_9$ $V_{CC}$ and $R_B$ for the circuit shown in Fig. 4.24.

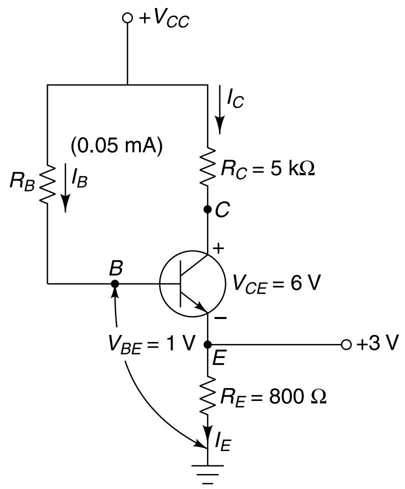



**Fig. 4.24**

***Solution***

The emitter current $I_E$ which passes through $R_E$ is obtained as

$$I_E = \frac{3}{800} = 3.75 \text{ mA}$$

The base current flowing through $R_B$ is

$$I_B = 0.05 \text{ mA (given)}$$

∴ The collector current $I_C$ is

$$I_C = I_E - I_B = 3.75 - 0.05 = 3.7 \text{ mA}$$

$$\beta = \frac{I_C}{I_B} = \frac{3.7}{0.05} = 74$$

Now, $$V_{CC} = I_C R_C + V_{CE} + 3 = 3.7 \times 5 + 6 + 3 = 27.5 \text{ V}$$

The voltage across $R_B$ is $V_{CC} - (V_{BE} + 3) = 27.5 - (1 + 3) = 23.5$ V

∴ $$R_B = \frac{23.5}{0.05 \times 10^3} = 470 \text{ k}\Omega.$$

**4.18** Calculate the values of the collector, base and emitter currents.

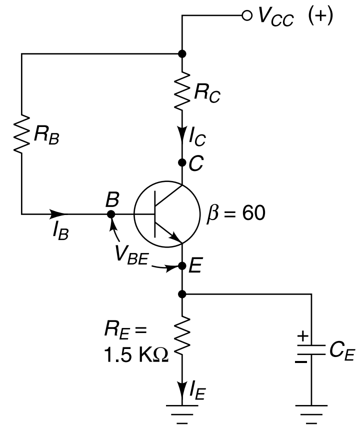



**Fig. 4.25**

$$[V_{CC} = 12 \text{ V}; R_C = 2.0 \text{ k}\Omega$$
$$R_B = 1.5 \text{ m}\Omega, \beta = 60,$$
$$R_E = 1.5 \text{ k}\Omega]$$

***Solution***

$$V_{CC} = I_B R_B + V_{BE} + I_E R_E$$

where $$I_E = I_C + I_B = \beta I_B + I_B = (\beta + 1)\, I_B$$

∴ $$V_{CC} = I_B R_B + V_{BE} + (\beta + 1)\, I_B R_E$$

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1) R_E}$$

Neglecting $V_{BE}$,

$$I_B = \frac{V_{CC}}{R_B + (\beta + 1)R_E}$$

$$= \frac{12}{1.5 \times 10^6 + (60 + 1)\,1.5 \times 10^3}$$

$$= 7.54\ \mu\text{A}$$

$$I_C = \beta I_B = 60 \times 7.54 \times 10^{-6} = 452.4\ \mu\text{A}$$

$$I_E = I_C + I_B = 452.4 + 7.54 = 459.94\ \mu\text{A}$$

**4.19** In the circuit shown in Fig. 4.26, $\alpha = 0.97$ and $V_{BE} = 1$ V. Neglecting $I_{CO}$, determine the resistance for an emitter current $I_E = 4$ mA

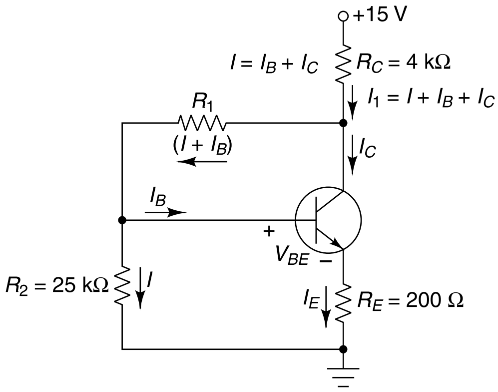



**Fig. 4.26**

***Solution***

$$\alpha = 0.97$$

$$\therefore \quad \beta = \frac{\alpha}{1-\alpha} = \frac{0.97}{1-0.97} = 32.33$$

Now $\quad I_C = \beta I_B = 32.33\ I_B$

and $\quad I_E = I_C + I_B = 32.33\ I_B + I_B = 33.33\ I_B$

or $\quad 4 \times 10^{-3} = 33.33\ I_B$

or $\quad I_B = 0.12$ mA

$\therefore \quad I_C = 32.33 \times 0.12 = 3.88$ mA

Applying Kirchhoff's voltage law in the base emitter circuit.

$V_{BE} + I_E \times 200 = 25 \times 10^3\ I$ ($I_E$ is the emitter current)

or $$I = \frac{1 + 4 \times 10^{-3} \times 200}{25 \times 10^3} = \frac{0.8}{25}\ \text{mA} = 0.032\ \text{mA}$$

Now applying Kirchhoff's voltage law in the loop containing $R_1$, $R_2$, $R_C$ and 15 V source

$$(I + I_B + I_C)\,R_C + (I + I_B)\,R_1 + IR_2 = 15$$

or $(0.032 + 0.12 + 3.88) \times 4 + (0.032 + 0.12)\, R_1 + 0.032 \times 25 = 15$

or $R_1 = \dfrac{16.128 + 0.8}{0.152} = 111.36\,\Omega$

**4.20** What will happen if the biasing resistor $R_2$ is open circuited or short circuited in a potential divider biasing circuit?

***Solution***

If $R_2$ is opened (Fig. 4.27), the emitter base junction will be heavily forward biased through $R_1$. This will raise the collector current $I_C$ to a high value, the transistor will go on saturation. $V_{CE}$ will be almost zero. If $R_2$ is shorted, the base will be effectively grounded. The emitter base junction will be no more be forward biased and the transistor will be cut off. The output will then be nil.

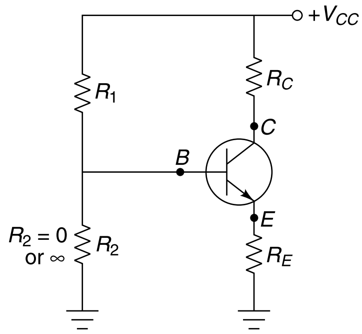



**Fig. 4.27**

**4.21** What will happen if the biasing resistor $R_1$ is open circuited or short circuited in a potential divider biasing circuit?

***Solution***

With reference to the figure shown in the previous example, if $R_1$ is open circuited, emitter base junction will be no more forward biased and the transistor will be cutoff producing no output.

On the other hand if $R_1$ is shorted, emitter base junction will be highly forward biased as the voltage $V_{CC}$ will appear at the base. This will lead to high value of $I_C$ and the transistor will be saturated $V_{CE}$ will be almost zero.

**4.22** Refer the circuit shown in Fig. 4.28. Given $\beta = 98$ and $V_{BE} = 0.6$. Determine the quiescent values of $I_B$, $I_C$, $I_E$ and $V_{CE}$. If $\beta$ increases by 30%. What is the change in $I_C$?

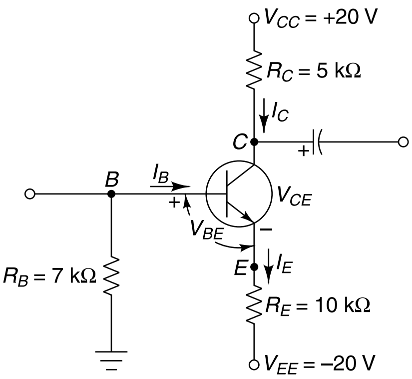



**Fig. 4.28**

***Solution***

Applying Kirchhoff's voltage law in the base emitter loop

$$I_B R_B = V_{BE} + I_E R_E + V_{EE} = 0$$

or $$7 \times 10^3\, I_B + 0.6 + I_E \times 10 \times 10^3 = 20 \qquad \text{(i)}$$

Again $$I_E = (\beta + 1)\, I_B = (98 + 1)\, I_B = 99\, I_B \qquad \text{(ii)}$$

Solving (i) and (ii)

$$I_B = 0.0194 \text{ mA and } I_E = 1.926 \text{ mA},$$

$$I_C = \beta I_B = 98 \times 0.0194 = 1.9 \text{ mA}$$

Applying Kirchhoff's voltage law in the output circuit

$$I_C R_C + V_{CE} + I_E R_E = V_{CE} - V_{EE}$$

$$1.9 \times 10^{-3} \times 5 \times 10^3 + V_{CE} + 1.926 \times 10^{-3} \times 10 \times 10^3 = 20 + 20$$

or $$V_{CE} = 11.24 \text{ V}$$

The new value of $\beta$ is, as per the given problem

$$\beta' = 98 \times 1.3 = 127.4$$

$\therefore$ $$I_E = (127.4 + 1)\, I_B = 128.4\, I_B \qquad \text{(iii)}$$

and $$7 \times 10^3\, I_B + 0.6 + 10 \times 10^3\, I_E = 20 \qquad \text{(iv)}$$

$\therefore$ $$I_B = 0.015 \text{ mA and } I_E = 1.926 \text{ mA}$$

$\therefore$ $$I_C = \beta' I_B = 127.4 \times 0.015 = 1.911 \text{ mA}$$

$\therefore$ % increase in $I_C$ is $\dfrac{1.911 - 1.9}{1.9} \times 100\% = 0.57\%$

The circuit provides a good stabilization against changes in $\beta$ as with 30% increase in $\beta$, $I_C$ changes by a mere amount of 0.57%.

**4.23** Determine the stability factor for potential divider bias circuit.

***Solution***

In Fig. 4.29(a),

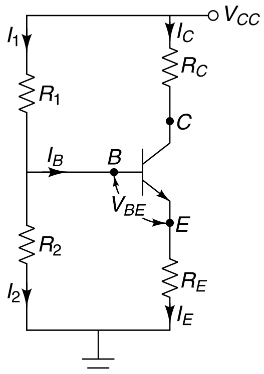



**Fig. 4.29 (a)**

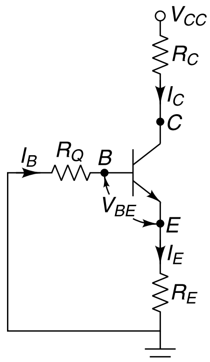



**Fig. 4.29 (b)**

$$V_{CC} = I_B R_Q + V_{BE} + (I_B + I_C)\, R_E$$

when $R_Q$ is parallel combination $R_1$ and $R_2$ [see Fig. 4.29(b)]

Differentiating this expression with respect to $I_C$ and assuming $V_{BE}$ as constant, we have

$$0 = \frac{dI_B}{dI_C} \cdot R_Q + R_E \left(1 + \frac{dI_B}{dI_C}\right)$$

and
$$\frac{dI_B}{dI_C} = -\frac{R_E}{R_E + R_Q}$$

Substituting the value of $\frac{dI_B}{dI_C}$ in the general expression for $S$, the stability factor given by

$$S = \frac{\beta + 1}{1 - \beta \frac{dI_B}{dI_C}}, \text{ we get}$$

$$S = \frac{\beta + 1}{1 - \beta \frac{R_E}{R_E + R_Q}}.$$

This is the expression for stability factor in potential divider circuit.

**4.24** A feedback biasing circuit is shown in Fig. 4.30(a). Given $R_B$ = 100 kΩ; $R_C$ = 1 kΩ; $V_{CC}$ = 10 V; $\beta$ = 50. Obtain the co-ordinates of the operating point. If the same transistor is replaced by another transistor having $\beta$ = 150, what are the coordinates of the new operating point?

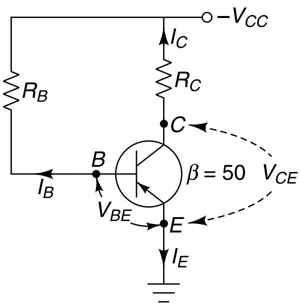



**Fig. 4.30(a)**

***Solution***

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} \simeq \frac{V_{CC}}{R_B} \text{ (neglecting } V_{BE})$$

$$= \frac{10 \text{ V}}{100 \text{ k}\Omega} = 100 \,\mu\text{A}$$

$I_C = \beta I_B = 50 \times 100 = 5000 \,\mu\text{A} = 5$ mA.

Collector saturation current,

$$I_{C(\text{sat})} = \frac{V_{CC}}{R_C} = \frac{10}{1 \times 10^3} = 10 \text{ mA}$$

The transistor with $\beta$ = 50 will not go for saturation as $I_C < I_{C(\text{sat})}$. Next we find $V_{CE}$. Here, $V_{CE} = V_{CC} - I_C R_C = 12 - 5 \times 10^{-3} \times 1 \times 10^{-3} = 7$ V.

The co-ordinates of the operating point are them (7 V, 5 mA). The load line (dc) is shown in Fig. 4.30(b).

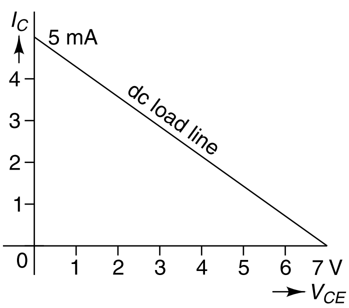



**Fig. 4.30(b)**

If this transistor is replaced by another one with $\beta = 150$,

$$I_C = \beta I_B = 150 \times 100\ \mu\text{A}$$
$$= 15000\ \mu\text{A} = 15\ \text{mA}$$

Here $I_C > I_{C(\text{sat})}$ and hence this goes to saturation

$$\therefore \quad V_{CE} = 0;\ I_C = I_{C(\text{sat})} = 10\ \text{mA}$$

**4.25** Figure 4.31 represents a potential divider biasing circuit.

Here, $R_1 = 30$ kΩ and $R_2 = 15$ kΩ. $V_{CC} = 10$ V, $R_C = 1$ kΩ and $R_E = 2$ kΩ. Determine $I_C$ for $V_{BE} = 0.6$ V and 0.3 V respectively.

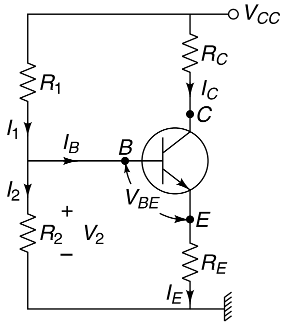



**Fig. 4.31**

***Solution***

Let the voltage across $R_2$ be $V_2$.

Here,
$$V_2 = \frac{R_2}{R_1 + R_2} . V_{CC} = \frac{15}{30 + 15} \times 10$$
$$= 3.33\ \text{V}$$
$$I_E = \frac{V_2 - V_{BE}}{R_E} = \frac{3.33 - 0.6}{2 \times 10^3} = 1.365\ \text{mA}$$

when $V_{BE} = 0.3$ V, we can write

$$I_C = \frac{V_2 - V_{BE}}{R_E} = \frac{3.33 - 0.3}{2 \times 10^3} = 1.515\ \text{mA}.$$

**4.26** A *CE* amplifier is biased by a potential divider circuit. Here, $R_1 = 60$ kΩ; $R_2 = 10$ kΩ, $\beta = 60$.

Find the value of the stability factor when $R_E = 1$ kΩ and 3 kΩ.

***Solution***

Since $R_1$ and $R_2$ in the potential divider bias circuit are in parallel with the supply $V_{CC}$, hence the total resistance due to both,

$$R_Q \equiv \frac{R_1 R_2}{R_1 + R_2} = \frac{60 \times 10^3 \times 10 \times 10^3}{(60 + 10)10^3} \simeq 8571\,\Omega.$$

Using the expression of the stability factor for the potential divider circuit, we have

$$S = \frac{\beta + 1}{1 + \beta \dfrac{R_E}{R_E + R_Q}}$$ [See Ex. 4.23]

with $R_E = 1\ \text{k}\Omega$, we get

$$S = \frac{60 + 1}{1 + 60 \dfrac{1000}{1000 + 8571}} = 8.39$$

with $R_E = 3000\ \Omega$,

$$S = \frac{60 + 1}{1 + 60 \times \dfrac{3000}{3000 + 8571}} = 3.684.$$

Hence we see with increase of $R_E$, the stability factor decreases.

**4.27** Find the maximum current for the silicon transistor shown in Fig. 4.32 that can be allowed to flow through it during the application of signal for faithful amplification. Also find the value of minimum zero signal collector current and the peak value of base current when $\beta = 75$

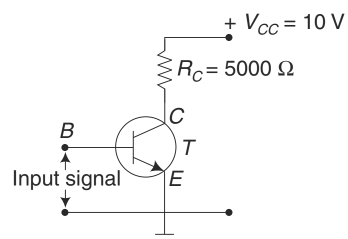



**Fig. 4.32**

***Solution***

$$\boldsymbol{V}_{CC} = 10\ \text{V}$$
$$R_C = 5000\ \Omega$$

For faithful amplification $V_{CE} = 1$ V for silicon transistor.

Hence the maximum voltage that can be allowed across $R_C$ is (10 – 1) is 9 V i.e.,

The maximum collector current that can be allowed is $\left(\dfrac{9}{5000}\right)$ i.e., 1.8 mA

Hence maximum current of 1.8 mA should be allowed.

When the signal is applied the collector current can be at the most allowed to fall to zero during the negative peak of the signal as shown in Fig. 4.33.

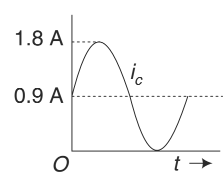



**Fig. 4.33**

Hence zero signal collector current is $\left(\dfrac{1.8}{2}\right) = 0.9$ mA

Peak value of signal collector current is bieng 0.9 mA, peak value of signal base current in

$$i_\beta = \frac{i_c}{\beta} = \frac{0.9}{50} = 18\ \mu\text{A}$$

**4.28** Determine the collector current $I_C$ and collector emitter voltage $V_{CE}$ for the base resistor biasing circuit shown in Fig. 4.34 is Assunce $\beta = 50$ and neglect $V_{BE}$

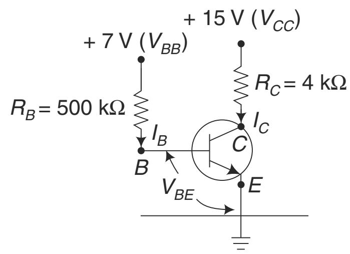



**Fig. 4.34**

***Solution***

Applying KVL to the base emitter circuit

$$V_{BB} = I_B R_B + V_{BE}$$

or, $$V_{BB} = I_B R_B \qquad (\because V_{BE} = 0)$$

$$I_B = \frac{V_{BB}}{R_B} = \frac{7}{500 \times 10^3} = 14 \ \mu\text{A}$$

The collector current $$I_C = \beta I_B = 50 \times 14 \ \mu\text{A}$$

$$= 700 \ \mu\text{A}$$

$$= 0.7 \text{ mA}$$

Now applying KVL to the collector emitter circuit we have

$$V_{CC} = I_C R_C + V_{CE}$$

or $$V_{CE} = V_{CC} - I_C R_C$$

i.e., $$V_{CE} = 15 - 0.7 \times 10^{-3} \times 4 \times 10^3$$

$$= 15 - 2.8 = 12.2 \text{ V}$$

**4.29** Determine the collector current and the collector to emitter voltage for the circuit shown in the Fig. 4.35 with $\beta = 60$.

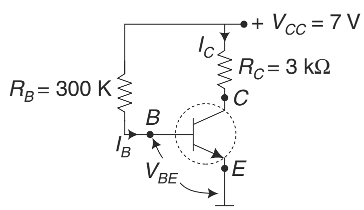



**Fig. 4.35**

***Solution***

The base current $$I_B = \frac{V_{CC} - V_{BE}}{R_B}$$

Neglecting $V_{BE}$ we have

$$I_B = \frac{7}{300 \times 10^3} = 23.33\ \mu A$$

Collector current ($I_C$) is obtained as

$$I_C = \beta I_B = 60 \times 23.33\ \mu A$$
$$= 1.4\ mA$$

The collector saturation current is given as

$$I_{C(sat)} = \frac{V_{CC}}{R_C} = \frac{7}{3 \times 10^3} = 3.5\ mA.$$

As the collector current is much less than the collector saturation current the transistor is not in saturation.

Hence collector to emitter voltage is given as

$$V_{CE} = V_{CC} - I_C R_C$$
$$= 7 - 1.4 \times 10^{-3} \times 3 \times 10^3$$
$$= 7 - 4.2 = 2.8\ V.$$

**4.30** For the fixed biasing transistor circuit shown in Fig. 4.36(a) draw the dc load line and locate the operating point. Also find out the stability factor for the circuit; $V_{BE}$ is given as 0.7 V and $\beta = 100$

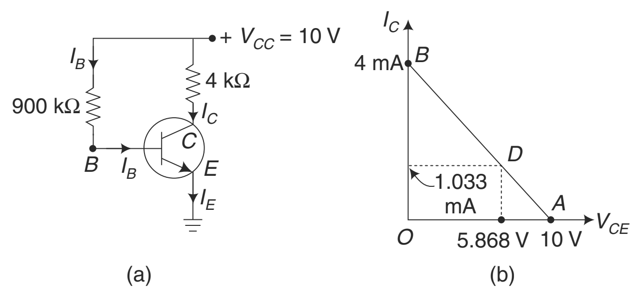



**Fig. 4.36**

***Solution***

The dc load line is shown in the Fig. 4.36(b) with $OA = V_{CE} = V_{CC} = 10$ V

and $$OB = I_C = \frac{V_{CC}}{R_C} = \frac{10}{4 \times 10^3} A = 2.5\ mA$$

*AB* is the dc load line

Base current, $$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{10 - 0.7}{900 \times 10^3} = 10.33\ \mu A$$

The collector current $I_C = \beta I_B = 100 \times 10.33 = 1.033$ mA

The collector emitter voltage

$$V_{CE} = V_{CC} - I_C R_C = 10 - 1.033 \times 10^{-3} \times 4 \times 10^3$$
$$= 5.868\ V$$

In the dc load line figure the operating point *P* is 5.868 V, 1.033 mA

Stability factor $S = \beta + 1 = 100 + 1 = 101$

**4.31** Determine the coordinates of the operating point in the fixed biasing circuit shown in Fig. 4.37.

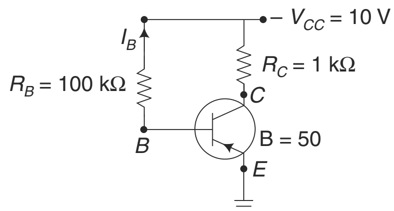



**Fig. 4.37**

***Solution***

Base current

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} \simeq \frac{V_{CC}}{R_B} = \frac{10}{100 \times 10^3} = 100\ \mu\text{A} \qquad \text{(Neglecting } V_{BE}\text{)}$$

The collector current

$$I_C = \beta I_B = 50 \times 100 = 5000\ \mu\text{A} = 5\text{ mA}$$

The collector saturation current

$$I_{C(sat)} = \frac{V_{CC}}{R_C} = \frac{10}{1000} = 10\text{ mA}$$

As the collector current is much less than the saturation current the transistor is not in saturation Hence collector to emitter voltage $V_{CE} = V_{CC} - I_C R_C$

$$= 10 - 5 \times 10^{-3} \times 1 \times 10^3$$

$$= 5\text{ V}$$

The co-ordinates of the operating point are (5 V, 5 mA).

**4.32** Find the value of the base resistor $R_B$ so that a germanium transistor with fixed biasing current (Fig. 4.38) draws a collector current of 3 mA. Given that the supply voltage is –5 V, load resistance 2 kΩ, $\beta$ = 30 and $I_{CBO}$ = 5 µA. Also find the error if leakage current $I_{CBO}$ and $V_{BE}$ are neglected. If $\beta$ = 50 and $I_{CBO}$ = 10 µA what will be the new value of $I_C$?

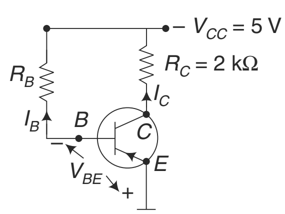



**Fig. 4.38**

***Solution***

We know that $\quad I_C = \beta I_B + (\beta + 1)\, I_{CBO}$

or, $\quad 3 \times 10^{-3} = 30 \times I_B + (50 + 1) \times 5 \times 10^{-6}$

$$\therefore \quad I_B = \frac{3 \times 10^{-3} - 0.255 \times 10^{-3}}{30} = 0.0915 \text{ μA}$$

$$= 91.5 \text{ μA}$$

Again $$I_B = \frac{V_{CC} - V_{BE}}{R_B}$$

or, $$R_B = \frac{V_{CC} - V_{BE}}{I_B} = \frac{5 - 0.3}{91.5 \times 10^{-6}} = 51.36 \text{ kΩ}$$

Now Neglecting $I_{CBO}$

$$I_C = \beta I_B$$

or, $$I_B = \frac{I_C}{\beta} = \frac{3 \times 10^{-3}}{30} = 100 \text{ μA}$$

Neglecting $V_{BE}$

$$R_B = \frac{V_{CC}}{I_B} = \frac{5}{100 \times 10^{-6}} = 50 \text{ kΩ}$$

$$= \frac{51.36 - 50}{51.36} \text{ error} = 26.48\%$$

with $\beta = 50$ and $I_{CBO} = 10$ μA,

New value of $I_C$ is obtained as follows

$$I_C = \beta I_B + (\beta + 1) I_{CBO}$$

$$= 50 \times 91.5 \times 10^{-6} + (50 + 1) \times 10 \times 10^{-6}$$

$$= 5085 \text{ μA} = 5.085 \text{ mA}$$

**4.33** Find the emitter current and the collector voltage in the circuit shown in Fig. 4.39

***Solution***

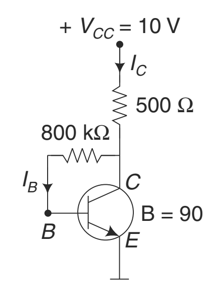



**Fig. 4.39**

Applying KVL we get

$$V_{CC} = (I_C + I_B) R_C + I_B R_B + V_{BE}$$

Neglecting $V_{BE}$ we have

$$V_{CC} = (I_C + I_B) R_C + I_B R_B$$

or, $$V_{CC} = \beta I_B R_C + I_B R_C + I_B R_B$$

or, $$10 = (\beta + 1) I_B R_C + I_B R_B$$

or, $$10 = (90 + 1) I_B \times 500 + I_B \times 800 \times 10^3$$

or, $$I_B = \frac{10}{500 \times 91 + 800\ 10^3} = 11.827 \text{ μA}$$

Collector currents, $I_C = \beta I_B = 90 \times 11.827 = 1.064$ mA

Emitter current, $I_E = I_C + I_B = (1.064 + 0.01183)$ mA

$= 1.076$ mA

Hence collector voltage

$$V_C = V_{CE} = V_{CC} - I_C R_C$$

$$= 10 - 1.064 \times 10^{-3} \times 500$$

$$= 9.468 \text{ V}$$

**4.34** Determine the operating point for the circuit shown in the figure below (Fig. 4.40)

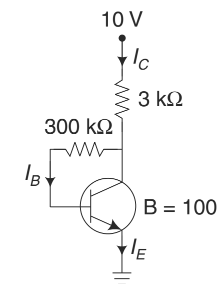



**Fig. 4.40**

***Solution***

Base current $$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta R_C}$$

Neglecting $V_{BE}$

$$I_B = \frac{10}{300 \times 10^3 + 100 \times 2000} = 20\ \mu\text{A}$$

The collector current, $I_C = \beta I_B = 100 \times 20 \times 10^{-6}$

$= 2$ mA

The collector emitter voltage is given as

$$V_{CE} = V_{CC} - I_C R_C = 10 - 2 \times 10^{-3} \times 3 \times 10^3$$

$= 10 - 6 = 4$ V

The operating point is (4 V, 2 mA).

**4.35** A transistor is biased with feedback resistor $R_B = 100$ kΩ If the supply voltage is 15 V and the collector load resistance is 2 kΩ (Fig. 4.41) find the upper and lower limit of collector when $\beta$ varies from 60 to 150

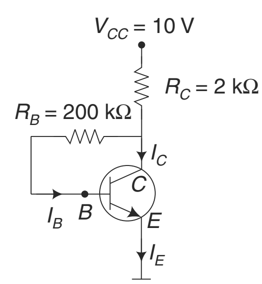



**Fig. 4.41**

***Solution***

The simplified circuit is redrawn in Fig. 4.41.

When $\beta = 60$

Base current $$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta R_C}$$

$[\because V_{BE} + I_B R_B + (\beta I_b) R_C = V_{CC}]$

Neglecting $V_{BE}$, $$I_B = \frac{10}{200 \times 10^3 + 60 \times 2 \times 10^3} = 31.25\ \mu\text{A}$$

$$I_C = \beta I_B = 60 \times 31.25 = 1.875\ \text{mA}$$

When $\beta = 150$

$$I_B = \frac{10}{200 \times 10^3 + 150 \times 2 \times 10^3} = 20\ \mu\text{A}$$

$\therefore$ $$I_C = \beta I_B = 150 \times 20 \times 10^{-6} = 3\ \text{mA}$$

Hence the upper and low limit of collector current are 3 mA and 1.875 mA respectively.

**4.36** A transistor is biased with a feedback resistor (Fig. 4.42) given that $I_C = 2$ mA, $V_{CE} = 15$ V, $V_{CC} = 20$ V, $V_{BE} = 0.7$ and $\beta = 90$, find the value of $R_C$ and $R_B$.

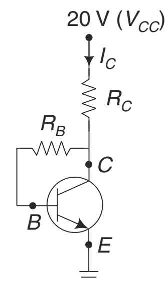



**Fig. 4.42**

***Solution***

$$V_{CC} = V_{CE} + I_C R_C$$

Hence, $$R_C = \frac{V_{CC} - V_{CE}}{I_C}$$

i.e., $$R_C = \frac{20 - 15}{2 \times 10^{-3}} = \frac{5}{2}\ \text{k}\Omega = 2.5\ \text{k}\Omega$$

Again $$I_C = \beta I_B$$

or $$I_B = \frac{2 \times 10^{-3}}{90} = 22.22\ \mu\text{A}$$

$\therefore$ $$R_B = \frac{V_{CE} - V_{BE}}{I_B} = \frac{15 - 0.7}{22.22 \times 10^{-6}} = 643.56\ \text{k}\Omega$$

## EXERCISES

1. What do you understand by transistor biasing? Why is it needed?
2. What do you mean by "faithful application"? What are the basic requirements to achieve faithful application of a transistor?
3. Explain what do you mean by stabilization, stability factor in transistor characteristics.
4. Explain the statement "a transistor is equivalent to a two point network".
5. Sketch a hybrid model for a transistor in $C_E$ configuration.
6. Develop the *h*-parameter equivalent circuit for $C_B$ and $C_C$ mode of operation of a transistor amplifier.
7. Find expressions for input impedance, current gain and voltage gain from a hybrid model.
8. Explain a fixed bias circuit. State the advantages and disadvantages.
9. Explain a feedback resistor bias circuit. Why this circuit is not commonly used.
10. Explain a voltage divider biasing circuit. What is the expression for the stability factor for such a circuit?
11. A transistor in $C_E$ configuration has base resistor biasing circuit. If $V_{CC} = 15$ V, $V_{BE} = 0.5$ V, $V_{CE} = 10$ V, $b = 100$ and collector current ($I_C$) is 2 mA, find the collector resistance $R_C$. What is the value of $R_B$?

   [Hint: $V_{CC} = V_{CE} + I_C R_C$; $R_C = \dfrac{V_{CC} - V_{CE}}{I_C} = 2.5\ \text{k}\Omega$

   $V_{CC} = V_{BE} + I_B R_B \qquad \therefore R_B = \dfrac{V_{CC} - V_{BE}}{I_B} = \dfrac{\beta (V_{CC} - V_{BE})}{I_C}$

   $= 725\ \text{k}\Omega$] [Ans. $R_C = 2.5\ \text{k}\Omega$; $R_B = 725\ \text{k}\Omega$]

12. A transistor is operated at zero signal collector current of 2 mA. If $\beta = 100$, what will be the value of $R_B$ if $V_{CC} = 10$ V? If the original transistor is replaced by a new one with $\beta = 60$, what will be the new value of zero signal collector current? What is the stability factor? Assume $V_{BE} = 0.3$ V

[Ans. $R_B = 485$ kΩ; $I_C$ (zero signal) = 1.2 mA, $S = 61$]

[Hint: $I_B$ (zero signal) $= \dfrac{I_C}{\beta} = 0.02$ mA

$V_{CC} = I_B R_B + V_{BE}$ $\quad\therefore I_B = 0.02$ mA (for the new transistor)

$I_C$ (zero signal) $= \beta I_B = 60 \times 0.02 = 1.2$ mA

$S = \beta + 1 = 61$].

## MULTIPLE CHOICE QUESTIONS

1. The expression of stability factor with respect to variation of $I_{CO}$ is

   (a) $S = \dfrac{1+\beta}{1+\beta\dfrac{\delta I_B}{\delta I_C}}$ (b) $S = \dfrac{1-\beta}{1-\beta\dfrac{\delta I_B}{\delta I_C}}$

   (c) $S = \dfrac{1-\beta}{1-\beta\dfrac{\delta I_B}{\delta I_C}}$ (d) $S = \dfrac{1-\beta}{1+\beta\dfrac{\delta I_B}{\beta I_C}}$

2. Bias stability means stability

   (a) cut-off point (b) voltage gain

   (c) saturation point (d) $Q$-point

3. The stability factor of an ideal biasing circuit is

   (a) 1000 (b) 100

   (c) 1 (d) 0.5

4. Stability factor is the rate of variation of collector current with respect to

   (a) collector leakage current (b) emitter current

   (c) base current (d) temperature

5. The thermal runaway problem in a transistor can be best avoided in

   (a) collector feedback bias circuit

   (b) fixed bias circuit

   (c) emitter feedback bias circuit

   (d) potential divider bias circuit

6. The disadvantage of a fixed bias circuit is that it

   (a) has no resistance across the emitter base junction

   (b) is a complicated circuit

   (c) has very high stability factor

   (d) provides negative feedback through base resistor

7. With the increase of base current, $Q$-point shifts
   (a) down the load line
   (b) up the load line
   (c) towards lower value of collector current
   (d) towards the saturation region
8. Transistor biasing is needed to fix up the value of
   (a) alternating current (b) direct current
   (c) current gain (d) alternating voltage
9. The stability factor of collector feedback bias circuit is
   (a) $> (\beta + 1)$ (b) $< (\beta + 1)$
   (c) $< (\beta - 1)$ (d) $(\beta + 1)$
10. The voltage divider biasing circuit is widely used in amplifiers because it
    (a) reduces the cost of the circuit
    (b) reduces gain of the amplifier
    (c) makes $Q$ point independent of $\beta$
    (d) involves less number of resistors
11. The stability factor of a common base transistor circuit is
    (a) $S = \beta$ (b) $S = 1 - \beta$
    (c) $S = 1 + \beta$ (d) $S = 1$
12. The stability factor of a common emitter transistor circuit is
    (a) $S = \beta$ (b) $S = 1 - \beta$
    (c) $S = 1 + \beta$ (d) $S = 1$
13. The self-bias arrangement provides a better $Q$-point stability if
    (a) $R_E$ is large (b) $R_E$ is small
    (c) $R_E$ is large and $\beta$ is small (d) both $R_E$ and $\beta$ is large
14. The voltage gain of a single-stage CE amplifier is
    (a) $A_v = -\frac{r_e}{e_L}$ (b) $A_v = -r_L$
    (c) $A_v = -r_e r_L$ (d) $A_v = -\frac{r_L}{r_e}$
15. The amplifier circuit with highest voltage gain is
    (a) common base (b) common collector
    (c) common emitter (d) none of these
16. Cascade amplifiers are used for
    (a) video amplifiers (b) power amplifiers
    (c) tuned amplifier (d) voltage amplification
17. In a common emitter amplifier circuit with emitter feedback, the input impedance is equal to
    (a) $R_E$ (b) $\frac{h_{fe}}{R_E}$
    (c) $h_{fe} R_E$ (d) $h_{fe}$

18. A transformer-coupled amplifier provides
    (a) impedance matching (b) maximum current gain
    (c) large bandwidth (d) maximum voltage gain
19. The $h_{11}$ parameter of a transistor with output shorted is given by the ratio
    (a) $\frac{i_2}{v_2}$ (b) $\frac{v_1}{i_1}$
    (c) $\frac{i_1}{v_1}$ (d) $\frac{v_1}{v_2}$
20. The dimensionless *h*-parameters are
    (a) $h_{11}$ and $h_{22}$ (b) $h_{21}$ and $h_{22}$
    (c) $h_{11}$ and $h_{12}$ (d) $h_{12}$ and $h_{21}$
21. The unit of $h_{OE}$ parameter is
    (a) ampere (b) volt
    (c) ohm (d) mho
22. Which of the following *h*-parameters has largest value?
    (a) $h_{ie}$ (b) $h_{fe}$
    (c) $h_{oe}$ (d) $h_{re}$
23. The *h*-parameter with smallest value is
    (a) $h_{ie}$ (b) $h_{fe}$
    (c) $h_{oe}$ (d) $h_{re}$
24. The input impedances with output shorted in CB and CE configurations of a transistor are
    (a) $h_{ib} = \frac{h_{ie}}{1 + h_{fe}}$ (b) $h_{ib} = \frac{1 + h_{fe}}{h_{ie}}$
    (c) $h_{ib} = \frac{h_{ie}}{1 + h_{fe}}$ (d) $h_{ib} = \frac{h_{ie}}{h_{fe}}$
25. The typical value of $h_{ib}$ for a transistor is
    (a) 1 Ω (b) 20 Ω
    (c) 100 kΩ (d) 1 kΩ
26. Which of the following is incorrect?
    (a) $h_{ic} = h_{ie}$ (b) $h_{rc} = h_{rb}$
    (c) $h_{fc} = h_{fe}$ (d) $h_{ic} = h_{ib}$
27. The *h*-parameter values do not depend on
    (a) transistor configuration (b) operating point
    (c) transistor type (d) transistor characteristics
28. The *h*-parameters are called hybrid parameters because these parameters are
    (a) independent of operating point
    (b) unique parameters
    (c) mixed with *y* and *z* parameters
    (d) used both in open and short-circuit terminations

## ANSWERS

| | | | | |
|---|---|---|---|---|
| 1. (c) | 2. (d) | 3. (c) | 4. (a) | 5. (d) |
| 6. (c) | 7. (b) | 8. (b) | 9. (b) | 10. (c) |
| 11. (d) | 12. (c) | 13. (b) | 14. (d) | 15. (c) |
| 16. (b) | 17. (c) | 18. (a) | 19. (b) | 20. (d) |
| 21. (d) | 22. (a) | 23. (c) | 24. (a) | 25. (b) |
| 26. (a) | 27. (b) | 28. (d) | | |

## UNIVERSITY QUESTIONS WITH ANSWERS

1. (a) What is biasing?

   (b) Draw the circuit diagram of self biased n-p-n transistor and explain how lias stability is achieved in this case

   (c) Derive the stability factors of self bias [WBUT 2013]

   *Answer:* (a) Refer Article 4.4 (b) & (c) Refer article 4.14

2. Write short note on Stability factors [WBUT 2014]

   *Answer:* Refer Article 4.9


# FIELD EFFECT TRANSISTOR

## 5.1 INTRODUCTION

A transistor is a linear semiconductor device that controls current with the application of a low power signal. Transistors may be roughly grouped into two major divisions: bipolar and field-effect. We have already studied bipolar transistors, which utilize a small current to control a large current. In this chapter, we introduce the general concept of the field effect transistor—a device utilizing a small voltage to control and then focus on one particular type: the junction field-effect transistor. We will explain the types of field-effect transistor, the insulated gate variety.

All field-effect transistors are *unipolar* rather than *bipolar* devices. That is, the main current through them comprised either of electrons through an *N*-type semiconductor or holes through a *P*-type semiconductor.

## 5.2 THE FIELD EFFECT TRANSISTOR

In the Bipolar Junction Transistor (BJT) the output collector current is determined by the amount of current flowing into the base terminal of the device and thereby making the Bipolar Transistor a current operated device. The *Field Effect Transistor*, or simply *FET* however, uses the voltage that is applied to their input terminal to control the output current, since their operation is governed by the electric field (hence the name field effect) generated by the input voltage. This makes the Filed Effect Transistor a voltage operated device.

The *Field Effect Transistor* is a *unipolar* device that has very similar properties to those of the *Bipolar transistor*, i.e., high efficiency, instant operation, robust and cheap, and they can be used in most circuit applications that uses the equivalent BJT transistor and along with their low power consumption and dissipation make them ideal for use in integrated circuits such as the CMOS range of chips.

We know that there are two basic types of Bipolar transistor construction, *NPN* and *PNP*, which basically describes the physical arrangement of the *P*-type and *N*-type semiconductor materials from which they are made. There are also two basic types of Field Effect Transistor, *N*-channel and *P*-channel. As their name implies, Bipolar Transistors are "Bipolar" devices because they operate with both types of charge carriers, Holes and Electrons. The Field Effect Transistor on the other hand is a "*Unipolar*" device that depends only on the conduction of Electrons (*N*-channel) or Holes (*P*-channel).

The Field Effect Transistor has one major advantage over its standard bipolar transistor cousins, in that their input impedance is very high (thousands of ohms) making them very sensitive to input signals. But this high sensitivity also means that they can be easily damaged by static electricity. There are two main types of field effect transistor, the *Junction Field Effect Transistor* or *JFET* and the *Insulated-gate Field Effect Transistor* or *IGFET*, which is more commonly known as the standard *Metal Oxide Semiconductor Field Effect Transistor* or MOSFET.

## 5.3 THE JUNCTION FIELD EFFECT TRANSISTOR

We saw previously that a bipolar junction transistor is conducted using two *PN* junctions in the main current path between the emitter and the collector terminals. The Field Effect Transistor has no junctions but instead has a narrow "Channel" of *N*-type or *P*-type silicon with electrical connections at either end commonly called the drain and the source respectively. Both *P*-channel and *N*-channel FET's are available. Within this channel there is a third connection which is called the GATE and this can be a *P* or *N*-type material forming a *PN* junction and these connections are compared below.

| *Bipolar Transistor* | *Field Effect Transistor* |
|---|---|
| Emitter-(E) | Source-(S) |
| Base-(B) | Gate-(G) |
| Collector-(C) | Drain-(D) |

The semiconductor "channel" of the Junction Field Effect Transistor is a resistive path through which a voltage $V_{ds}$ causes a current $I_d$ to flow. A voltage gradient is thus formed down the length of the channel with this voltage becoming less positive as we go from the drain terminal to the source terminal. The *PN* junction therefore has a high reverse bias at the drain terminal and a lower reverse bias at the source terminal. This bias causes a "depletion layer" to be formed within the channel and whose width increases with the bias. FETs control the current flow through them between the drain and source terminals by controlling the voltage applied to the gate terminal. In an *N*-channel JFET this gate voltage is negative while for a *P*-channel JFET the gate voltage is positive.

## 5.4 BIAS ARRANGEMENT FOR AN *N*-CHANNEL JFET AND CORRESPONDING CIRCUIT SYMBOLS

The cross-sectional diagram (Fig. 5.1(a)) shows an *N*-type semiconductor channel with a *P*-type region called the gate diffused into the *N*-type channel forming a reverse biased *PN* junction and it is this junction that forms the depletion layer around the gate area. This depletion layer restricts the current flow through the channel by reducing its effective width and thus increasing the overall resistance of the channel.

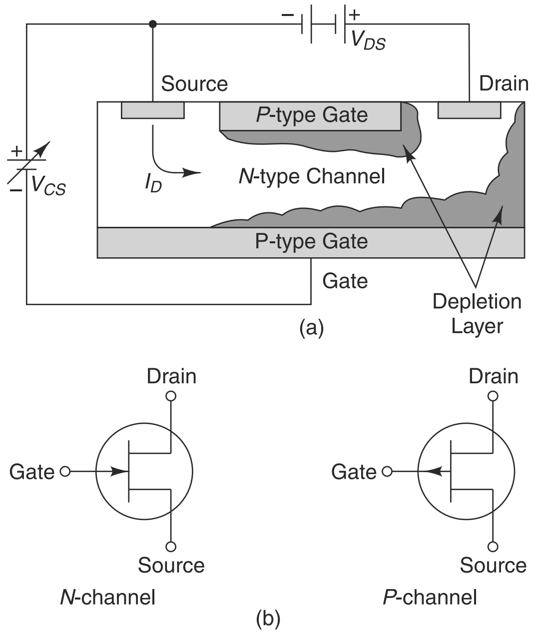



**Fig. 5.1** ***(a) Gross sectional diagram of n-type semiconductor channel (b) Digramatic representation of n-channel and p-channel JEFET***

When the gate voltage $V_g$ is equal to 0 V and a small external voltage ($V_{ds}$) is applied between the drain and the source maximum current ($I_d$) will flow through the channel slightly restricted by the small depletion layer. If a negative voltage ($V_{gs}$) is now applied to the gate the size of the depletion layer begins to increase reducing the overall effective areas of the channel and thus reducing the current flowing through it. As the gate voltage ($V_{gs}$) is made more negative, the width of the channel decreases until no more current flows between the drain and the source and the FET is said to be "pinched-off". In this pinch-off region the gate voltages, $V_{gs}$ controls the channel and $V_{ds}$ has little or no effect. The result is that the FET acts more like a voltage controlled resistor which has zero resistance when $V_{gs} = 0$ and maximum "ON" resistance ($R_{ds}$) when the gate voltage is very much negative.

## 5.5 OUTPUT CHARACTERISTIC (VOLTAGE-CURRENT CURVES) OF A TYPICAL JUNCTION FET

The voltage $V_{gs}$ applied to the gate controls the current flowing between the drain and the source terminals. $V_{gs}$ refers to the voltage applied between the gate and the source while $V_{ds}$ refers to the voltage applied between the drain and the source. That is why a Field Effect Transistor is a VOLTAGE controlled device. As no current flows into the gate the source current ($I_s$) flowing out of the device equals the drain current flowing into it and therefore ($I_d = I_s$).

The characteristics curves (Fig. 5.2) shows the four different regions of operation for a JFET and the biasing circuit diagram is shown in Fig. 5.2(a).

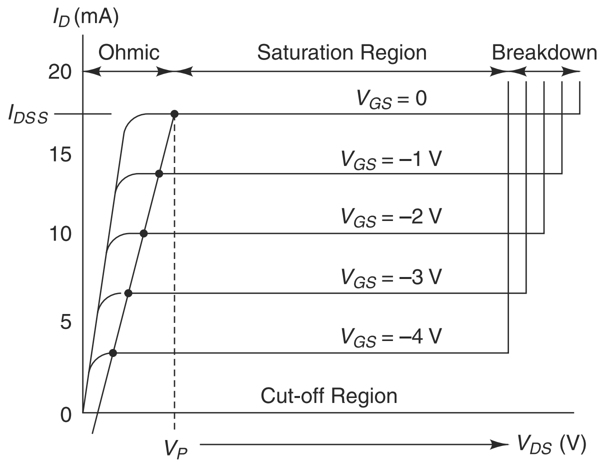



**Fig. 5.2** ***Characteristic curvs of JFET***

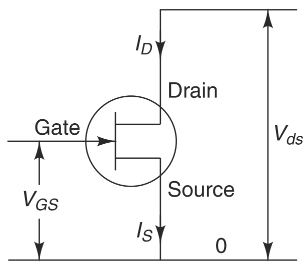



**Fig. 5.2(a)** ***Circuit diagram for JFET biasing***

***Ohmic Region*** The depletion layer of the channel is very small and the JEFET acts like a variable resistor.

***Cut-off Region*** The gate voltage is sufficient to cause the JFET to act as an open circuit as the channel resistance is at maximum.

***Saturation or Active Region*** The JFET becomes a good conductor and is controlled by the gate-source voltage, ($V_{gs}$) while the drain source voltage, ($V_{ds}$) has little or no effect.

***Breakdown Region*** The voltage between the drain and source, ($V_{ds}$) is high enough to cause the JFET's resistive channel to break down and pass current.

The control of the drain current by a negative gate potential makes the Junction Field Effect Transistor useful as a switch and it is essential that the gate voltage is never positive for an *n*-channel JEFET as the channel current will flow to the gate and not the drain resulting in damage to the JFET. The principles of operation for a *p*-channel JFET are the same as for the *n*-channel JFET, except that the polarity of the voltage needs to be reversed.

## 5.6 FET PARAMETERS

A FET has three important parameters (like vacuum tubes):

1. ac drain resistance
2. Transconductance
3. Amplification

***1. ac Drain Resistance ($r_d$)*** This is similar to ac plate resistance in vacuum tubes. The ratio of a small change in drain source voltage ($V_{DS}$) to the corresponding change in the drain current ($I_D$) at constant $V_{GS}$ is called ac resistance.

$$r_d = \frac{\Delta V_{DS}}{\Delta I_D} \text{ at constant } V_{GS}$$

Its value for FET ranges from 10 k to 1 MΩ.

***2. Transconductance ($g_m$ or $g_{fs}$)*** This is similar to the transconductance (or mutual conductance $g_m$) in vacuum tubes.

The ratio of small change in drain current ($\Delta I_D$) to the change in the gate source voltage ($\Delta V_{GS}$) at constant drain source voltage is called 'transconductance' of FET.

$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} \text{ at constant } V_{DS}.$$

Its value for a FET is about 3000–3500 $\mu$ mho.

In FET, the transconductance is sometimes represented by $g_{fs}$.

***3. Amplification Factor ($\mu$)*** A FET is a better amplifier than vacuum tube and also than solid state transistor.

The ratio of change in the drain source voltage ($\Delta V_{DS}$) to the corresponding change in the gate source voltage ($\Delta V_{GS}$) at constant drain current ($I_D$) is called 'amplification factor'.

$$\mu = \frac{\Delta V_{DS}}{\Delta V_{GS}} \text{ at constant } I_D.$$

The value of $\mu$ for FET is very high.

**Relation between $r_d$, $g_m$ and $\mu$**

We know that

$$\mu = \frac{\Delta V_{DS}}{\Delta V_{GS}}$$

$$= \frac{\Delta V_{DS}}{\Delta V_{GS}} \times \frac{\Delta I_D}{\Delta I_D}$$

$$= \frac{\Delta V_{DS}}{\Delta I_D} \times \frac{\Delta I_D}{\Delta V_{GS}}$$

$$\mu = r_d \times g_m$$

This is similar to $\mu = r_p \times g_m$ as in case of vacuum tubes.

## 5.7 FET AS AN AMPLIFIER

While going through the operation of a FET, we have seen that gate source voltage can control the gate current. A small change in the voltage at the gate ($V_{GS}$) can produce a great change in the drain current ($I_D$). This property makes FET suitable to be used as an amplifier.

The weak signal is applied between gate and source, and amplified output can be obtained between drain and source (Fig. 5.3). Usually, common source configuration is used as it gives good voltage gain and high input impedance. For biasing, a battery or 'biasing circuits' like 'potential divider method' can be used. FET amplifiers provide an excellent voltage gain with added features of high input impedance. They are also low power consumption devices with good frequency range and minimum size and weight.

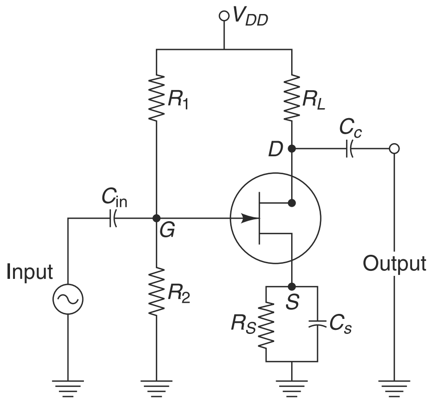



**Fig. 5.3** ***FET amplifier circuit***

An FET can be used as a linear as well as a digital amplifier in logic circuits. While the common source configuration is most popular providing an inverted amplified signal, we can also use common drain (source follower) circuits providing unity gain with no invertion (phase reversal). The common gate circuit also provides amplification without any inversion.

Figure 5.4 shows an equivalent circuit of a low frequency small signal FET amplifier (FET is also used for high frequency application).

The circuit has an output with a current source $g_m$. $V_{GS}$ which is proportional to the gate source voltage $V_{GS}$. The constant of proportionality is the transconductance ($g_m$). The output resistance is the drain resistance $r_d$.

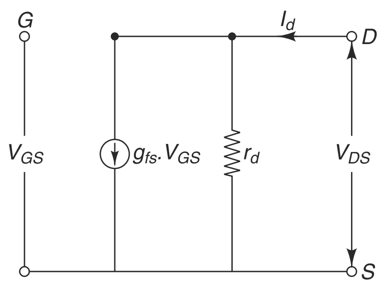



**Fig. 5.4** ***FET equivalent circuit***

***Voltage Gain of an FET Amplifier*** The voltage gain of a FET amplifier can be calculated as in case of vacuum tube.

Here, $$A_v = \frac{\mu R_L}{r_d + R_L}$$

With $\mu = r_d \cdot g_{fs}$,

From the above expression

$$A_v = \frac{r_d \cdot g_{fs} . R_L}{r_d + R_L}$$

As practically $r_d >> R_L$ (FET is a current source), we can neglect $R_{L.}$

$$\therefore \quad A_v = g_{fs} \times R_L \cong g_{fs} .$$

## 5.8 THE MOSFET

Similar to the Junction Field Effect Transistor, there is another type of Field Effect Transistor available whose gate input is electrically insulated from the main current carrying channel and is therefore called an *Insulated Gate Field Effect Transistor*. The most common type of insulated gate FET or IGFET as it is sometimes called is the *Metal Oxide Semiconductor Field Effect Transistor* or *MOSFET*.

The MOSFET type of field effect transistor has a "Metal Oxide" gate (usually, silicon dioxide commonly known as glass), which is electrically insulated from the main semiconductor *N*-channel or *P*-channel. This isolation of the controlling gate makes the input resistance of the MOSFET extremely high in the Mega-ohms region and almost infinite. As the gate terminal is isolated from the main current carrying channels no current flows into the gate and like the JFET, the MOSFET also acts like a voltage controlled resistor. Also like the JFET, this very high input resistance can easily accumulate large static charges resulting in the MOSFET becoming easily damaged unless carefully handled or protected.

## 5.9 BASIC MOSFET STRUCTURE AND SYMBOL (FIG. 5.5)

We have seen earlier that the gate of a JFET must be biased in such a way as to forward bias the *pn* junction but in a MOSFET device no such limitations apply. Hence it is possible to bias the gate in either polarity. This makes MOSFETs specially valuable as electronic switches or to make logic gates because with no bias they are normally non-conducting and the high gate resistance means that very little control current is needed. Both the *P*-channel and the *N*-channel MOSFET are available in two basic forms, the *enhancement type* and the *depletion type*.

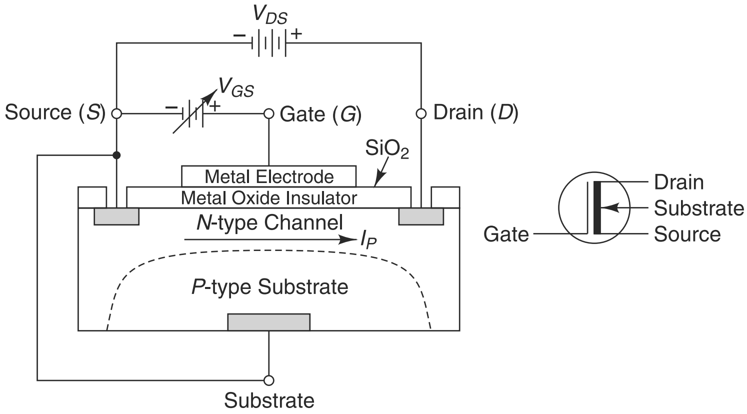



**Fig. 5.5** ***Basic structure of MOSFET and its symbol***

## 5.10 TYPES OF MOSFET

### 5.10.1 Depletion-Mode MOSFET

The depletion-mode MOSFET, which is less common than the enhancement types is normally switched 'ON' without a gate bias voltage but requires a gate to source voltage ($V_{gs}$) to switch the device 'OFF' similar to the JFET types. For *N*-channel MOSFET's a 'positive' gate voltage widens the channel, increasing the flow of the drain current and decreasing the drain current as the gate voltage goes more negative. The opposite is also true for the *P*-channel types. The depletion mode MOSFET is equivalent to a 'normally closed' switch.

Depletion-mode MOSFETs are constructed similar to their JFET transistor counterparts where the drain-source channel is inherently conductive with electrons and holes already present within the *N*-type or *P*-type channel. This doping of the channel produces a conducting path of low resistance between the drain and source with zero gate bias. The characteristics and symbols of depletion mode MOSFET are shown in Fig. 5.6.

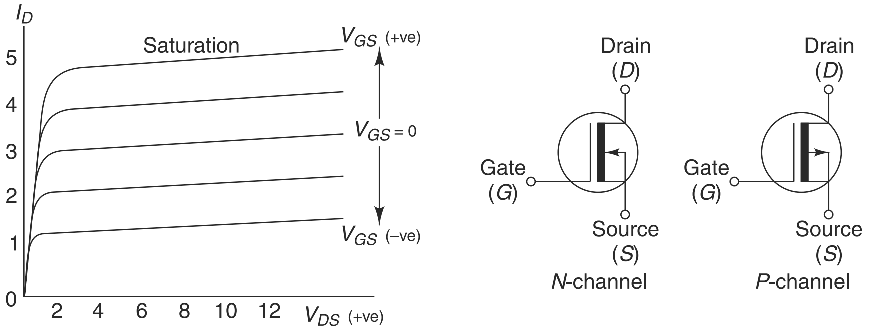



**Fig. 5.6** ***Characteristic of depletion mode MOSFET***

### 5.10.2 Enhancement-mode MOSFET

The more common enhancement mode MOSFET is the reverse of the depletion mode type. Here the conducting channel is lightly doped or even undoped making it non-conductive. This results in the device being normally 'OFF' when the gate bias voltage is equal to zero.

A drain current will only flow when a gate voltage ($V_{gs}$) is applied to the gate terminal. This positive voltage creates an electrical field within the channel attracting electrons towards the oxide layer and thereby reducing the overall resistance of the channel allowing current to flow. Increasing this positive gate voltage will cause an increase in the drain current, $I_d$ through the channel. Then the enhancement-mode device is equivalent to a 'normally open' switch.

Enhancement-mode MOSFETs make excellent electronics switches due to their low 'ON' resistance and extremely high 'OFF' resistance and extremely high gate resistance. Enhancement-mode MOSFETs are used in integrated circuits to produce CMOS type logic gates and power switching circuits as they can be driven by digital logic levels. Figures 5.7 and 5.8 are the characteristics and symbols of enhancement mode of MOSFET.

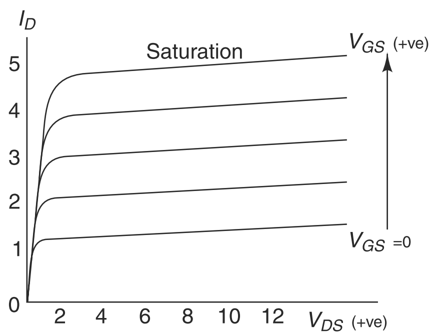



**Fig. 5.7** ***Characteristics of enhancement mode of MOSFET***

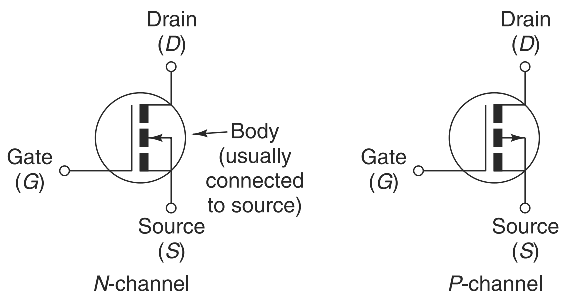



**Fig. 5.8** ***Symbols of enhancement mode of MOSFET***

# 5.11 WORKING OF AN *n*-MOS

## 5.11.1 As Depletion MOS

The circuit diagram (Fig. 5.9) shows the connections for the study of the working of an *n*-MOS. Unlike FET, a MOS has no gate diode but it forms a capacitor which has two metal electrodes (gate of aluminum and channel of *n*-type material) separated by a dielectric ($SiO_2$) layer.

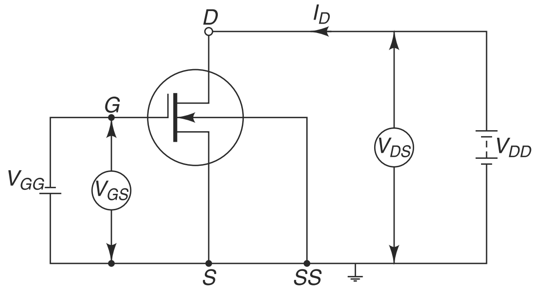



**Fig. 5.9** ***Circuit diagram with n-MOS***

When negative voltage is applied to the gate, it induces an equal and opposite positive charge on the opposite side (plate) of the capacitor, i.e., in the channel (Fig. 5.10). Therefore, the number of electrons available for current conduction through the channel will reduce. The gate current will reduce, as more and more negative potential is applied at the gate. Thus the MOS will work in depletion mode.

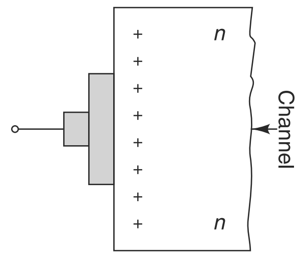



**Fig. 5.10** ***Capacitance formation at the n-MOS***

Figure 5.11 shows what actually happens inside the channel on applying negative bias at the gate. Figure 5.12 shows *V-I* characteristic of *n*-MOS working in depletion mode.

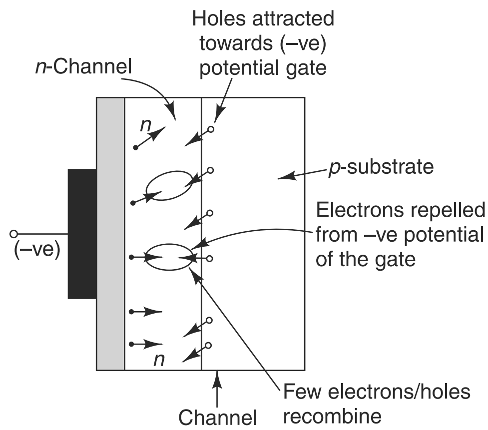



**Fig. 5.11** ***n-MOS channel with negative bias***

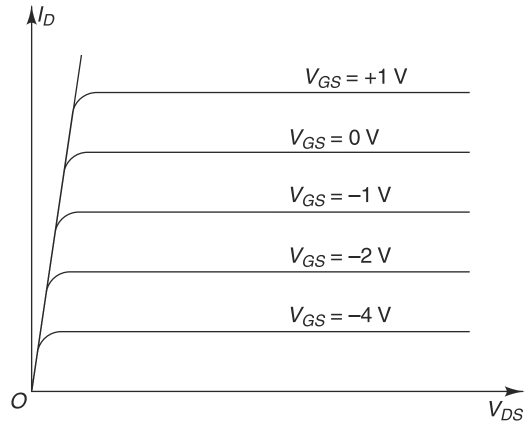



**Fig. 5.12** ***VI characteristic of n-MOS in depletion mode***

### 5.11.2 As Enhancement MOS

When positive voltage is applied at the gate, it induces an equal and opposite negative charge on the channel. Therefore, the number of electrons available for current conduction through the channel will increase. The gate current will increase as more and more positive potential is applied at the gate. In other words, the MOS will work in enhancement mode.

Similar explanations can be given for a *p*-MOS.

*Note*

(i) A MOS can be given any potential (positive or negative) at its gate and can be made to act in both the modes, i.e., depletion as well as enhancement.

(ii) The drain current in a MOSFET is controlled by the capacitor action of the gate unlike depletion layers in the FET.

(iii) The substrate (SS) terminal is usually grounded as shown (Fig. 5.9)

## 5.12 *n*-MOS VS *p*-MOS

1. The *p*-channel enhancement MOS is easier to fabricate than the *n*-channel MOS.
2. In MOS, the main contamination is due to positive ions trapped in $SiO_2$ layer between the gate and the substrate.

   In *n*-enhancement MOS the gate is usually kept positive with respect to the substrate. Hence, these positive ions make the device ON prematurely, whereas in *p*-enhancement MOS, these do not have any effect as the gate is usually kept negative with respect to the substrate.
3. The switching speed of *n*-MOS is higher than that of *p*-MOS. The switching speed is dependent on the RC constant of the internal capacitance (at the gate) of the device.
4. Though *n*-MOS is to some extent superior, its fabrication demands rigorous process control and is, therefore, costly. This is the reason that *p*-MOS is more popular.

## 5.13 MOSFET SUMMARY

The MOSFET has an extremely high input gate resistance and as such easily damaged by static electricity if not carefully protected. MOSFETs are ideal for use as electronic switches or common-source amplifiers as their power consumption is very small. Typical applications for MOSFETs are in microprocessors, memories, calculators and logic gates, etc. Also, notice that the broken lines within the symbol indicates a normally 'OFF' Enhancement types showing that no current can flow through the channel when zero gate voltage is applied and a continuous line within the symbol indicates a normally 'ON' Depletion type showing that current can flow through the channel with zero gate voltage. For *P*-Channel types the symbols are exactly the same for both types except that the arrow points outwards.

This can be summarized in the following switching table.

| *MOSFET type* | $V_{gs}$ = +ve | $V_{gs}$ = 0 | $V_{gs}$ = –ve |
|---|---|---|---|
| *N*-Channel Depletion | ON | ON | OFF |
| *N*-Channel Enhancement | ON | OFF | OFF |
| *P*-channel Depletion | OFF | ON | ON |
| *P*-Channel Enhancement | OFF | OFF | ON |

## 5.14 THE MOSFET AS A SWITCH

We have seen that the *N*-channel Enhancement-mode MOSFET operates using a positive input voltage and has an extremely high input resistance (almost infinite) making it possible to interface with nearly any logic gate or driver capable of producing a positive output. Also due to this very high input (Gate) resistance we can parallel together many different MOSFETs until we achieve the current handling limit required. Connecting together various MOSFETs may enable us to switch high current or high voltage loads, doing so becomes expensive and impractical in both components and circuit board space. To overcome this problem Power Field Effect transistors or Power FETs were developed.

We now know that there are two main differences between FETs, depletion-mode for JFETs and Enhancement-mode for MOSFETs. We will now look at using the Enhancement-mode MOSFET as a switch.

By applying a suitable drive voltage to the Gate of an FET the resistance of the drain-source channel can be varied from an 'OFF-resistance' of high values of KΩ (effectively an open circuit) to an 'ON-resistance' of less than 1 Ω (effectively a short circuit). We can also drive the MOSFET to turn 'ON' fast or slow, or to pass high currents or low currents. This ability to turn the power MOSFET 'ON' and 'OFF' allows the device to be used as a very efficient switch with switching speeds much faster than standard bipolar junction transistors.

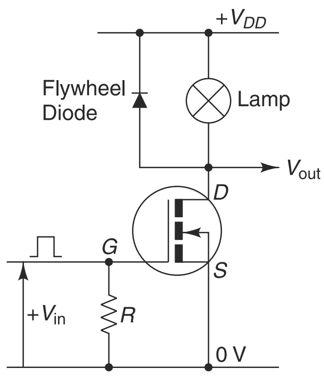
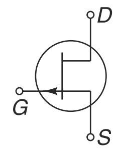
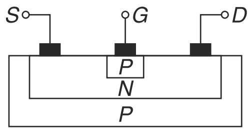
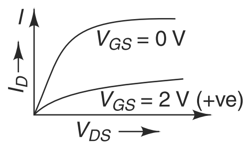
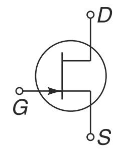
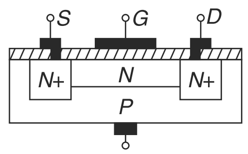
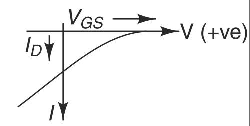
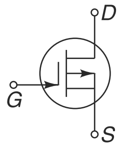
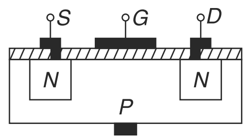
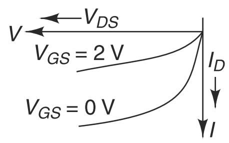
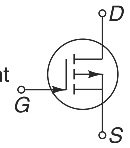
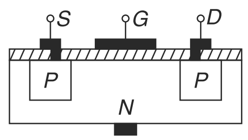
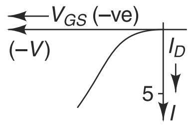
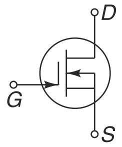
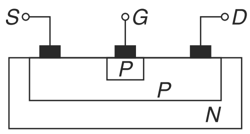
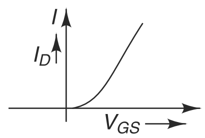
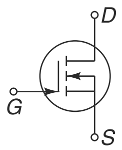
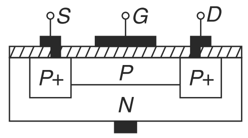
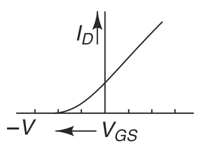



**Fig. 5.13** ***MOSFET as a switch***

In a circuit arrangement (Fig. 5.13) an enhancement-mode *N*-channel MOSFET is being used to switch a simple lamp 'ON' and 'OFF'. The gate input voltage $V_{GS}$ is taken to an appropriate positive voltage level to turn the device and the lamp either fully 'ON', ($V_{GS}$ = +ve) or a zero voltage level to turn the device fully 'OFF' ($V_{GS}$ = 0).

*Few particulars about JFET, and MOSFET*

| *SN.* | *Name* | *Symbol* | *Structure* | *VI characteristic* |
|---|---|---|---|---|
| 1. | *n*-channel JFET |  |  |  |
| 2. | *p*-channel JEFT |  |  |  |
| 3. | *n*-channel depletion MOSFET |  |  |  |
| 4. | *n*-channel Enhancement MOSFET |  |  |  |
| 5. | *p*-channel depletion MOSFET |  |  |  |
| 6. | *p*-channel ehancement MOSFET |  |  |  |

If the resistance load of the lamp was to be replaced by an inductive load such as a coil or solenoid, a 'Flywheel' diode would be required in parallel with the load to protect the MOSFET from any back-emf.

When using power MOSFET's to switch either inductive or capacitive loads some form of protection is required to prevent the MOSFET device from becoming damaged. Driving inductive loads has the opposite effect from driving a capacitive load. For example, a capacitor without an electrical charge is a short circuit, resulting a high 'inrush' of current and when we remove the voltage from an inductive load we have a large reverse voltage build-up as the magnetic field collapses, resulting in an induced back-emf in the winding of the inductor.

For the power MOSFET to operate as an analogue switching device, it needs to be switched between its 'Cut-off Region' where $V_{GS} = 0$ and its 'saturation region' where $V_{GS}$ (on) = +ve . The power dissipated in the MOSFET ($P_D$) depends upon the current flowing through the channel $I_D$ at saturation and also the 'ON-resistance' of the channel given as $R_{DS(\text{on})}$ .

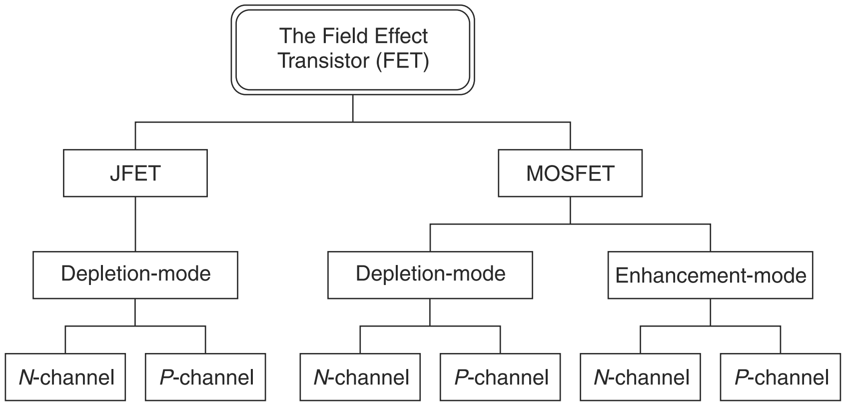
## 5.15 SALIENT POINTS OF BIPOLAR JUNCTION TRANSISTORS

- The Bipolar Junction Transistor (BJT) is a three layer device constructed from two semiconductor diode junctions joined together, one forward biased and one reverse biased.
- There are two main types of bipolar junction transistors, the NPN and the gate transistor.
- Transistors are *current operated devices* where a much smaller Base current causes a larger Emitter to Collector current, which themselves are nearly equal.
- The most common transistor connection is the Common-emitter configuration.
- Requires a Biasing voltage for AC amplifier operation.
- The Collector or output characteristics curve can be used to find either $I_b$, $I_C$ or $\beta$ to which a load line can be constructed to determine a suitable operating point, $Q$ with variations in base current determining the operating range.
- A transistor can also be used as an electronic switch to control devices such as lamps, motors and solenoids, etc.
- Inductive loads such as DC motors, relays and solenoids require a reverse biased "Flywheel" diode placed across the load. This helps to prevent any induced back emf's generated when the load is switched 'OFF' from damaging the transistor.
- The *NPN* transistor requires the Base to be more positive than the Emitter while the *PNP* type requires that the Emitter is more positive than the Base.

## 5.16 SALIENT POINTS OF FIELD EFFECT TRANSISTORS

- Field effect transistors, or FET's are *Voltage operated devices* and can be divided into two main types: junction-gate devices called JFETs and insulated gate devices called IGFET or more commonly known as MOSFET's.
- Insulated-gate devices can also be sub-divided into Enhancement types and Depletion types. All forms are available in both *N*-channel and *P*-channel versions.

- FETs have very high input resistance so very little or no current (MOSFET types) flows into the input terminal making them ideal for use as electronic switches.
- The input impedance of the MOSFET is even higher than that of the JFET due to the insulating oxide layer and therefore static electricity can easily damage MOSFET devices so care needs to be taken when handling them.
- FETs have very large current gain compared to junction transistors.
- They can be used as ideal switches due to their very high channel 'OFF' resistance, 'ON' resistance. The field effect transistor family tree.




Field effect transistors can be used to replace normal Bipolar Junction Transistors in electronic circuits and simple comparison between FETs and transistors stating both their advantages and their disadvantages is given below.

| | *Field Effect Transistor (FET)* | *Bipolar Junction Transistor (BJT)* |
|---|---|---|
| 1. | Low voltage gain | High voltage gain |
| 2. | High current gain | Low current gain |
| 3. | Very high input impedance | Low input impedance |
| 4. | High output impedance | Low output impedance |
| 5. | Low noise generation | Medium switching time |
| 6. | Fast switching time | Medium switching time |
| 7. | Easily damaged by static | Robust |
| 8. | Some require an input to turn it OFF | Requires zero input to turn it OFF |
| 9. | Voltage controlled device | Current controlled device |
| 10. | Exhibits the properties of a Resistor | |
| 11. | More expensive than bipolar | Cheap |
| 12. | Difficult to bias | Easy to bias |

## 5.17 SPECIAL MOSFETS

Two special MOS have been described below:

***1. C-MOS (Complementary MOS)*** We can have a C-MOS by using a complementary set of MOSFETs; the one is an *n*-channel MOS and the other is a *p*-channel MOS. This works as an 'Inverter'. When one device is ON, the other is OFF; thus it is analogous to a push-pull circuit.

When $V_{in}$ is low (Fig. 5.14) the lower MOS is OFF but the upper MOS is ON and the output voltage will be high. On the other hand when $V_{in}$ is high, the lower MOS is ON and upper MOS is OFF, the output will be low. Since the output voltage is always opposite in phase to the input voltage, the device works as an inverter.

The circuit has extremely low power consumption. As both devices are in series, the current is in nanoamperes and thus the total power consumption is also in nanowatts. Due to this reason the circuit is very popular in pocket calculators, digital wrist watches and satellites.

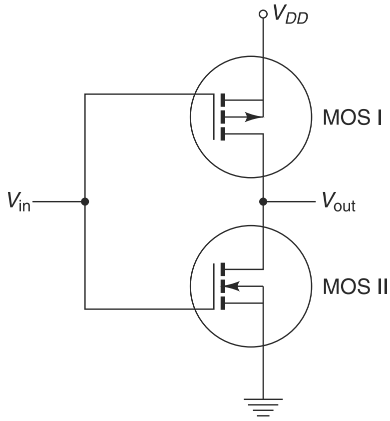



**Fig. 5.14** ***C-MOS***

***2. V-MOS (Vertical MOS)*** In an enhancement MOS the electrons flow from source to drain along a very narrow horizontal channel. The device has thus very small drain currents and can handle less than 1 W power.

In a V-MOS (Fig. 5.15) the channel is vertical and also wider. The device has two sources $S_1$ and $S_2$ at the top, which are interconnected. Moreover the substrate also acts like a drain (*D*). Now free electrons flow vertically downward from the two sources to the drain. As the channel now is much wider along both sides of the *V* groove, the current is also much larger and the device can handle larger powers than the conventional MOS. These are, therefore, used in R.F. and A.F. amplifiers at high loads. A V-MOS has a negative temperature coefficient. As the temperature increases, drain current decreases. This reduces power dissipation. A V-MOS, therefore is free from the risk of thermal runaway, which is a big problem in bipolar transistors.

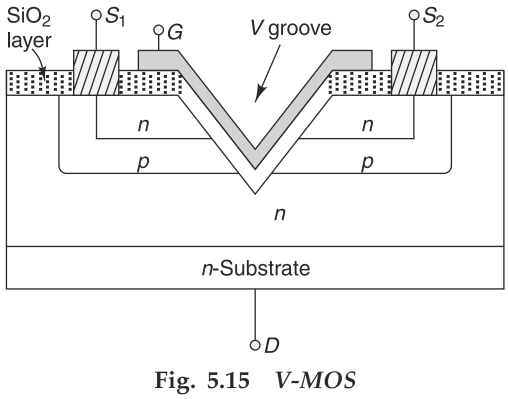



**Fig. 5.15** ***V-MOS***

## EXERCISES

1. What is a FET and what are the differences between a FET and BJT?
2. Describe the biasing arrangement of an N-channel JFET. Also draw the output characteristics of a JFET. Identify different regions of operation.
3. For a FET, define ac drain resistance, transconductance, amplification. Show that $\mu = r_d \times g_m$.
4. Briefly discuss the operation of a FET as an amplifier. What is its voltage gain?
5. What is a MOSFET? Show its basic structure and symbol.
6. What are different types of MOSFET? Discuss briefly.
7. Discuss the working of an *n*-MOS both as depletion MOS and enhencement MOS.
8. Compare *n*-MOS and *p*-MOS.
9. Show a circuit to illustrate the operation of *n*-MOS as a switch.
10. Write a short not on "Special Mosfets".
11. (a) Explain the principle of p-channel depletion MOSFET.
    (b) Write a short note on CMOS.

## MULTIPLE CHOICE QUESTIONS

1. A FET is a
   (a) current-controlled device
   (b) voltage-controlled device
   (c) resistance-controlled device
   (d) impedance-controlled device
2. Compared to BJT, a FET has a higher
   (a) voltage gain (b) current gain
   (c) input impedance (d) output impedance
3. The channel current in a MOSFET is controlled by the electric field of the
   (a) drain bias (b) gate bias
   (c) *p-n* junction diode (d) capacitor
4. The pinch-off voltage of an *n*-channel FET is
   (a) 4 V (b) – 4 V
   (c) 0.7 V (d) – 0.6 V
5. A depletion MOSFET differs from a FET in the sense that it has no
   (a) *p-n* junction (b) gate
   (c) channel (d) substrate
6. The gate-to-source diode of a JFET should be
   (a) forward biased
   (b) reverse biased
   (c) may be forward or reverse biased
   (d) none of these

7. CMOS stands for
   (a) common MOS (b) complementary MOS
   (c) *p*-channel MOS (d) *n*-channel MOS
8. The pinch-off voltage is equal to
   (a) gate voltage (b) drain-to-source voltage
   (c) gate-to-source voltage (d) gate-to-source cut-off voltage
9. The input impedance of a JFET is
   (a) about 500 Ω (b) about 10,000 Ω
   (c) about zero (d) about infinity
10. The value of transconductance of a FET is
    (a) 1000 to 25,000 $\mu$ mho (b) more than 1000 milli mho
    (c) 1 to 2 milli mho (d) 100 to 200 milli mho
11. Amplification factor in a FET is
    (a) $\frac{\Delta I_D}{\Delta V_{GS}}$ (b) $\frac{\Delta V_{DS}}{\Delta I_D}$
    (c) $\frac{\Delta V_{DS}}{\Delta V_{GS}}$ (d) $\frac{\Delta V_{GS}}{\Delta V_{DS}}$
12. In a FET, drain resistance varies from
    (a) 1 to 100 Ω (b) 1000 to 10,000 Ω
    (c) 10,000 to 1 MΩ (d) 0.5 to 1 Ω
13. A field effect transistor operates on
    (a) majority carriers only
    (b) minority carriers only
    (c) positive and negative ions
    (d) positive charged ions
14. Enhancement MOSFET biasing is done by
    (a) source bias (b) collector feedback bias
    (c) self-bias (d) voltage divider bias
15. The depletion region width in a biased FET is
    (a) narrow near the source and wide near the drain
    (b) wide near the source and narrow near the drain
    (c) narrow at both source and drain ends
    (d) narrow at both source and drain ends

## ANSWERS

| | | | | |
|---|---|---|---|---|
| 1. (b) | 2. (c) | 3. (d) | 4. (b) | 5. (a) |
| 6. (b) | 7. (b) | 8. (d) | 9. (d) | 10. (a) |
| 11. (c) | 12. (c) | 13. (a) | 14. (b) | 15. (b) |